56:271 Nonlinear Pgmg.
Midterm Exam 10/29/86

(1.) Consider the problem:

Minimize $f(x,y)$

where

$$f(x,y) = x^2 + 4y^2 - 18y - 2xy$$

(a.) Test whether $f(x,y)$ is convex, concave, both, or neither .

(b.) If we start with the initial "guess" $x^o=1$, $y^o=1$, and use the steepest descent algorithm, in what direction will we first search?

(c.) The optimal step size in the direction of part (b) is $t^o= 0.5$. What are the values of x and y at the beginning of the next iteration?

(d.) If we were to use the Fletcher-Reeves ("Conjugate Gradient") algorithm, what would be the first search direction? What would be the second search direction?

(e.) The Fletcher-Reeves method, applied to this problem, converges in only two iterations. Explain why this is not surprising.

(f.) If we were to use Newton's algorithm, what would be the initial search direction? What is the step size used in this method? How many iterations of Newton's method should be required for this problem?

(2.) Consider the optimization problem:

Minimize $x_1^2 + x_1x_2 + 2x_2^2 -12x_1 - 18x_2$

subject to

$$-3x_1 + 6x_2 \le 9$$
$$-6x_1 + x_2 \le 1$$
$$x_1 \ge 0, \quad x_2 \ge 0$$

Consult the APL output, which is attached, for the solution of this problem using the GRG algorithm. (The upper bounds specified for the variables were sufficiently large to be nonbinding.)

(a.) At iteration number 2, show how, once the gradient of f has been computed, the reduced gradient is computed.

(b.) Explain how, at iteration 2, the search direction is computed.

(c.) Write the Kuhn-Tucker conditions for this problem, using explicit Lagrange multipliers for the non-negativity constraints.

(d.) Are the Kuhn-Tucker conditions necessary, sufficient, both, or neither for optimality of this problem?

(3.) Consider the nonlinear programming problem

Minimize $-x$

subject to

$$x^2 + y^2 \leq 1$$

(no nonnegativity restrictions on x and y)

(a.) Sketch a <u>very rough</u> graph and indicate the optimum.

(b.) Write down the Kuhn-Tucker conditions for optimality.

(c.) Verify that the optimum indicated in (a) satisfies the Kuhn-Tucker conditions.

(d.) What is the objective function of the Lagrangian dual problem?

(e.) State the Lagrangian dual problem.

(f.) Without solving it, state the optimal objective function value and optimal dual variable for the dual problem. Is there a duality gap?