Name ____________________________

@@@@@@@@@@@@@@

56:271 Nonlinear Programming

Final Exam (Take-home) - Thursday, May 15, 1997

@@@@@@@@@@@@@@

Section A

Indicate whether each statement is *true* (+) or *false* (**o**). If false, briefly state why or give a counterexample.

_____ 1. A barrier function adds a penalty to the objective when the solution approaches a boundary of the feasible region.

_____ 2. The number of "dependent" variables of the GRG algorithm is equal to the number of equality constraints.

_____ 3. The cubic interpolation procedure for one-dimensional minimization requires the computation of second derivatives.

_____ 4. The function x^a is convex for x>0.

_____ 5. The feasible directions algorithm computes a search direction by solving an LP.

_____ 6. The Fibonacci method for one-dimensional minimization requires the computation of derivatives.

_____ 7. If all posynomials in a (posynomial) GP problem are condensed into single terms, then it is always possible to rewrite the resulting problem as an LP problem.

_____ 8. A linear function is neither convex nor concave.

_____ 9. The Gradient Projection method always follows the boundary of the feasible region, and never enters the interior.

_____ 10. The logarithm of the dual GP objective $v(\delta,\lambda)$ above is a concave function of δ and λ.

_____ 11. The logarithm of the dual GP objective $v(\delta,\lambda)$ above is not a concave function of both δ and λ, but becomes so if you eliminate λ_k using $\lambda_k = \sum_{i \in [k]} \delta_i$.

_____ 12. A locally optimal solution for a signomial geometric programming problem must be globally optimal.

_____ 13. The dual ("pseudo-dual") of a signomial GP problem may have multiple local maxima, but its global maximum will have the same objective value as the global minimum of the primal signomial GP.

_____ 14. Penalty functions may be used for both equality and inequality constraints, but barrier functions may be used only for inequality constraints.

Name ____________________________

_____ 16. If the Lagrangian function $L(x,\lambda)$ has a saddlepoint (x^o,λ^o), then there is zero duality gap between primal and dual optimal solutions.

_____ 17. Consider the problem: min $f(x)$ subject to $g_i(x)\leq 0$, $i=1,2\ldots m$, $x\geq 0$ (where g_i satisfy some constraint qualification) and its Lagrangian function

$$L(x,\lambda) = f(x) + \sum_{i=1}^{m} \lambda_i g_i(x)$$

Then if f and g_i are convex, the partial derivative of L with respect to each x_j must be zero at the optimum.

_____ 18. For the problem in the previous statement, $\lambda_i g_i(x)$ must be zero at the optimum.

_____ 19. For the problem in the previous statement, either x_j or $\frac{\partial L(x,\lambda)}{\partial x_j}$ (or possibly both) must be zero at the optimum.

_____20. The limit, as $w \rightarrow 0$, of the function $(c/w)^w$ is ∞.

_____21. $\lim_{x\rightarrow 0} \ln x = -\infty$, but $\lim_{x\rightarrow 0} x \ln x = 0$

_____22. If we define the function $\varphi(x) = x \ln x$, then $\lim_{x\rightarrow 0} \frac{d\varphi}{dx} = -\infty$

_____23. When the dual problem is solved, the primal variables of a posynomial GP problem are often computed by exponentiating the Lagrange multipliers of the orthogonality constraints.

_____24. The function $f(x,y) = xy$ is a concave function of x and y.

_____25. If an algorithm is applied to a minimization problem with optimal value zero, and the objective value is approximately halved at each iteration, then we would say that the algorithm has a linear rate of convergence.

_____26. The Hessian matrix of $(x^tQx + cx)$ is 2Q.

_____27. In the QC/LD problem, the objective function is assumed to be quadratic.

_____28. If $f_i(x)$ is linear for i=1,2,...n, then the function F(x) defined by $F(x)= \max\{f_i(x): i=1,2,...n\}$ is a convex function.

_____29. If a posynomial GP objective function continues to decrease as $x_j \rightarrow 0$ for some primal variable x_j, then the dual problem objective is unbounded.

_____30. If a posynomial GP objective function continues to decrease as $x_j \rightarrow +\infty$ for some primal variable x_j, then the dual problem is infeasible.

_____ 31. Every posynomial function is convex.

_____ 32. The dual of a constrained posynomial GP problem has only linear equality and nonnegativity constraints.

Name ____________________________

_____ 33. If a primal GP constraint is slack, then all the weights of the terms in that constraint must be zero.

_____ 34. In the QC/LD problem, the variables are restricted to be integer-valued.

_____ 35. The gradient of the function $f(x) = (x^tQx + cx)$ is $Qx + c$.

_____ 36. Solving the QC/LD problem requires a one-dimensional search at each iteration.

_____ 37. The posynomial constraint $g_i(x) \leq 1$ has a convex feasible region.

_____ 38. It is always possible (e.g., by a change of variable) to reformulate a posynomial GP problem so as to have a convex objective and convex feasible region.

_____ 39. The objective of the posynomial GP problem, i.e.,

$$v(\delta,\lambda) = \prod_i \left(\frac{c_i}{\delta_i}\right)^{\delta_i} \prod_k \lambda_k^{\lambda_k}$$

is a concave function of δ and λ.

_____ 40. In the "Golden Section Search" method, more than one-third of the interval of uncertainty is eliminated at each iteration (assuming that no function values are equal).

_____ 41. If all posynomials in a (posynomial) GP problem are condensed into single terms, then the resulting problem has a feasible region which includes the original feasible region.

_____ 42. The Hessian matrix of a quadratic function is constant.

_____ 43. In Wolfe's complementary pivoting algorithm for QP, if a single artificial variable is used, then the tableau does not require an objective row.

_____ 44. The gradient projection method computes Lagrange multipliers at each iteration, and stops when they have the appropriate sign.

_____ 45. "Quasi-Newton" search methods for unconstrained minimization require the computation of <u>second</u> partial derivatives.

_____ 46. Newton's method for unconstrained minimization requires the computation of <u>second</u> partial derivatives.

_____ 47. If you guess at the value of some primal GP variable and then fix it at this value, the dual GP problem becomes more difficult to solve.

_____ 48. In a "generalized linear programming" problem, the column of coefficients of the variables must be selected as well as the variables.

_____ 49. The function e^{ax} is convex *only* for $a \geq 0$.

_____ 50. The Davidon-Fletcher-Powell (DFP) algorithm requires the computation of partial derivatives.

_____ 51 . If a posynomial function is "condensed" into a monomial, the monomial function (if not equal) is an overestimate of the posynomial function.

Name ____________________________

_____ 52 . Wolfe's method for quadratic programming requires a one-dimensional search at every iteration.

_____ 53. If a constrained nonlinear programming problem satisfies a "constraint qualification", then a point which satisfies the Karush-Kuhn-Tucker conditions must be an optimal solution.

_____ 54. A barrier function allows a constraint to be violated, but adds a penalty if the constraint is violated.

_____ 55. The GRG algorithm requires the use of a one-dimensional search method.

_____ 56. The Feasible Directions algorithm requires the use of a one-dimensional search method.

_____ 57. If a constrained nonlinear programming problem satisfies a "constraint qualification", then the Karush-Kuhn-Tucker conditions must be satisfied by an optimal solution.

_____ 58. The function e^{ax} is convex for *all* values of a.

_____ 59. In Wolfe's complementary pivoting method for quadratic programming, the complementary slackness conditions are satisfied after each iteration.

_____ 60 . The tableau for Wolfe's method for quadratic programming includes columns for both primal and dual variables.

_____ 61. The function x^a is convex for $a \geq 0$.

_____ 62. If a nonlinear programming problem has only linear constraints, then any point which satisfies the Karush-Kuhn-Tucker conditions must be optimal.

_____ 63. The Fletcher-Reeves method is also known as a "Quasi-Newton" method.

_____ 64. The quadratic interpolation procedure for one-dimensional minimization requires the computation of derivatives.

_____ 65. If the GRG algorithm were applied to a LP problem, it would produce, at each iteration, the same solution as the simplex algorithm for LP.

_____ 66. A penalty function allows a constraint to be violated, but adds a penalty if the constraint is violated.

_____ 67. The Lagrangian dual of a convex quadratic programming problem is a quadratic programming problem with only nonnegativity constraints on the dual variables.

_____ 68. Powell's algorithm for unconstrained minimization requires the computation of partial derivatives.

_____ 69. The Davidon-Fletcher-Powell method is also known as the "Conjugate Gradient" method.

_____ 70. In the QC/LD problem, the objective function is assumed to be convex.

Name ___________________________

Section B

Below is a list of references concerning Geometric Programming and applications. Part I lists papers which explicity use GP, while Part II lists papers whose applications appear to be good candidates for GP.

Part I: Selected References to Geometric Programming Applications

56:271 Spring '97
D. Bricker

~~~~~~

Paul, H. (1982). “An Application of Geometric Programming to Heat Exchanger Design.” Computers and Industrial Engineering **6**(2): 103-114.

Phillips, D. T. and C. S. Beightler (1970). “Optimization in Tool Engineering Using Geometric Programming.” AIIE Transactions **2**(4): 355-360.

Corstjens, M. and P. Doyle (1981). “A Model for Optimizing Retail Space Allocations.” Management Science **27**(7): 822-833.

Balachandran, V. and D. Gensch (1973). Solving the "Marketing Mix" Problem Using Geometric Programming. Northwestern University.

Dajani, J. S., Y. Hasit, et al. (1977). “Geometric Programming in Sewer Network Design.” Engineering Optimization **3**: 27-35.

Edwards, L. S. (1975). “Optimum Limit State Design of Highway Bridge Superstructures Using Geometric Programming.” Engineering Optimization **1**: 201-212.

Unklesbay, K. and D. L. Creighton (1978). “The Optimization of Multi-pass Machining Process.” Engineering Optimization **3**: 229-238.

Mine, H. and K. Ohno (1970). “Decomposition of Mathematical Programming Problems by Dynamic Programming and its Application to BLock-Diagonal Geometric Programs.” J. Jath. Anal. Appl. **32**: 370ff.

Yu, C. H., N. C. Dasgupta, et al. (1986). “Optimization of Prestressed Concrete Bridge Girders.” Engineering Optimization **10**(1): 13-24.

Kapur, K. C. (1978). Optimization in Probabilistic Design for Engineering Systems. Second International Symposium on Large Engineering Systems, Waterloo, Sandford Educational Press.

Wyman, F. P. (1978). “Use of Geometric Programming in the Design of an Algerian Water Conveyance System.” Interfaces **8**(3): 1-6.

Philipson, R. H. and A. Ravindran (1979). “Application of Mathematical Programming to Metal Cutting.” Mathematical Programming Study **11**: 116-134.
~~~~~~

Name ___________________________

Hivner, W. H. and R. P. Mehta (1977). "Optimizing Computer Performance with Geometric Programming." European Journal of Operational Research **1**(2):

Grange, F. E., G. A. Kochenberger, et al. (1992). Optimal Design of Multi-Pass Heat Exchangers with Geometric Programming.

Woolsey, R. E. D. An Analysis of a Model of the MPS MX Missile System Using Geometric Programming. Mathematics Dept., Colorado School of Mines.

Gopalakrishnan, B. and F. Al-Khayyal (1991). "Machine Parameter Selection for Turning based on Geometric Programming." International Journal of Production Research **29**: 1897-1908.

Smeers, Y. and D. Tyteca (1984). "A Geometric Programming Model for the Optimal Design of Wastewater Treatment Plants." Operations Research **32**(2): 314-342.

Questions:

a. Chose one reference from Part I of the list of GP applications.

b. Briefly summarize the application problem.

c. Write the mathematical model.

d. Identify the # of (primal) variables, # of constraints, total # of terms.

e. State whether posynomial or signomial in nature.

f. Describe the method used by the author(s) of the paper.

g. If sample data is given for a sample problem, or if you can guess at reasonable data, solve the GP problem.

~~~~~~

Part II: Selected References to Potential Geometric Programming Applications

Dinkel, J. J. and G. A. Kochenberger (1974). "On "A Cofferdam Design Optimization"." Mathematical Programming **6**(1): 114-116.

Neghabat, F. and R. M. Stark (1972). "A Cofferdam Design Optimization"." Mathematical Programming **3**: 263-275.

Feiring, B. R. (1990). "An Efficient Procedure for the N-city Traveling Salesman Problem." Mathematical and Computer Modelling **13**(3): 95-98.

Terry, W. R. and K. W. Cutright (1986). Computer Aided Design of a Broaching Process. Computers and Industrial Engineering. **11:** 576-580.

Ulusoy, A. G. and D. M. Miller (1979). Optimal Design of Pipeline Networks Carrying Homogeneous Coal Slurry. Mathematical Programming Study. North-Holland Publishing Company. 85-107.
~~~~~~

Name ____________________________

Cowton, C. J. and A. Wirth (1993). "On the Economics of Cutting Tools." International Journal of Production Research **31**(10): 2441-2446.

Quesada, I. and I. E. Grossmann (1992). A Global Optimization Algorithm for Heat Exchanger Networks. Engineering Design Research Center, Carnegie-Mellon University.

Questions:

a. Chose one reference from Part II of the list of potential GP applications.

b. Briefly summarize the application problem.

c. Write the mathematical model given by the author(s).

d. If possible, reformulate the model as a GP (either posynomial or signomial) problem.

e. Identify the # of (primal) variables, # of constraints, total # of terms.

f. Describe the method used by the author(s) of the paper.

g. If sample data is given for a sample problem, or if you can guess at reasonable data, solve the GP problem.