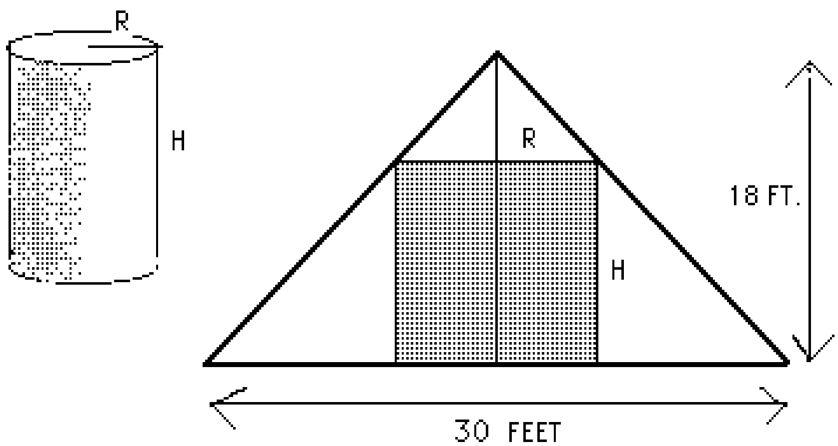
mm 56:271 Nonlinear Programming mm
Take-Home Final Exam
mm Due Friday, May 13, 1994 mm

1. Design a cylindrical tank, to hold at least 785 cubic feet of liquid, to fit in a shed with a sloping roof as shown:



The cost of the vertical surface of the tank is \$2/ft^2, while the cost of the top and bottom surfaces is \$3/ft^2.

(a.) Formulate a mathematical programming model for this problem, with the two design variables **R** and **H** (i.e. don't use the volume constraint in order to eliminate one of the variables!)

(b.) Sketch the feasible region for this problem.

(c.) Write down the Kuhn-Tucker optimality conditions.

(d.) Solve the problem, using the APL code in GPLIB for the geometric programming algorithm.

(e.) Indicate on your sketch the path taken by the algorithm (at least for the first few iterations).

(f.) Check that the Kuhn-Tucker conditions are satisfied by the solution found by the algorithm in part (d).

(g.) At the optimum, what fraction of the objective function is due to each term?

(h.) Given the dual variables (δ and λ, or equivalently, ρ and λ) appearing at the final iteration, together with the optimal objective function value, write the equations which would be solved to find **R** and **H**.

(i.) Suppose that the cost of the material for the top & bottom of the tank were to increase by 1% (from \$3.00 to \$3.03 per square foot). Show how to use the dual variables to estimate the increase in the optimal cost. Is this an under- or over-estimate of the actual increase?

2. Consider the nonlinear programming problem:

Minimize $F(X) = 2X_1 + X_1X_2 + 3X_2$

subject to:

$X_1^2 + X_2 \geq 3$, i.e., $G_1(X) = 3 - (X_1^2 + X_2) \leq 0$

$0.5\ X_1 + X_2 \geq 2$, i.e., $G_2(X) = 2 - (0.5\ X_1 + X_2) \leq 0$

(no sign restrictions on X_1 & X_2)

(a.) Sketch the feasible region of this problem, noting that the graph of $G_1(X) = 0$ is a parabola and that of $G_2(X) = 0$ is a line.

(b.) Sketch several contours of the objective function, at least one of which passes through the feasible region.

(c.) State the Kuhn-Tucker optimality conditions for this problem.

(d.) What are all the feasible solutions of the Kuhn-Tucker conditions?

(e.) Indicate on the sketch in (a.) the K-T solutions found in (d.)

(f.) For each K-T solution, sketch the steepest descent direction and the gradients of G_i for each tight constraint. Interpret the K-T conditions geometrically.

(g.) Discuss the difficulties one might have in solving this problem via the feasible direction or generalized reduced gradient (GRG) algorithm.

(h.) Use either the feasible directions algorithm or GRG (in the appropriate APL workspace) to try solving this problem, starting at the feasible point X=(2,2). Is it successful? Try one or two other starting points for the search, also.

(i.) State the Quadratic Programming Dual of this problem.

(j.) Discuss the difficulties which you might expect in solving this problem via Wolfe's method for quadratic programming.

(k.) Use Wolfe's algorithm (in the APL workspace) to try solving the problem. Is it successful?

(l.) Demonstrate why this problem cannot be formulated as a standard (posynomial) geometric programming problem. Show how it may be formulated as a signomial geometric programming problem. (There may be more than one way to do this, but you need to give only one signomial GP formulation!)

(m.) State the GP "pseudodual" problem corresponding to the formulation in (l.) What is its degree of difficulty?

(n.) Show how the signomial GP problem may be condensed into a posynomial GP problem, using the feasible point X=(2,2) again. (Show your computations.)

(o.) State the GP dual of the condensed posynomial GP problem. What is its degree of difficulty?

(p.) Sketch the feasible region of this condensed posynomial GP problem.

(q.) Is the feasible region of (p.) a subset of the feasible region of (a.), or vice versa? That is, is every feasible solution of the condensed posynomial GP problem also feasible in the original problem? If not, is every feasible solution of the original problem also feasible in the condensed GP problem?

(r.) Solve the condensed posynomial GP problem. Is the optimal solution feasible in the original problem?

(s.) Find the new condensation of the signomial GP at the point found in (r.), and sketch its feasible region together with the feasible region of the original problem.

(t.) Solve this second condensed GP problem. Is the optimal solution feasible in the original problem?

(u.) Using the SIGGP APL workspace, if you wish, perform several more iterations, until the optimal solutions of two successive condensed problems are "close". Has the procedure converged to a K-T point? to the optimal solution?

(v.) Using the SIGGP workspace again, but a different starting point for the condensation, perform the algorithm to determine if it will converge to the same final solution.

3. "Skim" through recent (≤ 3 years old) issues of journals (e.g., *Engineering Optimization*, *Operations Research*, *IEE Transactions*, *Computers & Operations Research*, *Computers & Industrial Engineering*, etc. which are all in the Engineering Library) to find an application of a nonlinear programming algorithm which you have studied in this course to a "real-world" problem of interest to you. If you have difficulty in finding such a journal paper, you might try looking for one in *International Abstracts in Operations Research*, which is also in the Engineering Library.) Write a summary of the article (one or more pages), describing the application, the NLP model, and the performance of the algorithm. *(If there is sample data for a small problem, you may wish to try to obtain the solution using the APL (or other) code for the algorithm.)*