56:271 Nonlinear Programming
Take-home Final Exam
December 1987

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PART ONE

Consider the nonlinear programming problem:

Minimize $2X_1 + X_1X_2 + 3X_2$

subject to:

$X_1^2 + X_2 \geq 3$, or $G_1(X) = 3 - (X_1^2 + X_2) \leq 0$

$0.5 X_1 + X_2 \geq 2$, or $G_2(X) = 2 - (0.5 X_1 + X_2) \leq 0$

(no sign restrictions on X_1 & X_2)

(a.) Sketch the feasible region of this problem.
(b.) Sketch the several contours of the objective function.
(c.) Indicate the optimal solution.
(d.) State the Kuhn-Tucker optimality conditions for this problem.
(e.) What are all the feasible solutions of the Kuhn-Tucker conditions?
(f.) Indicate on the sketch in (a.) the K-T solutions found in (e.)
(g.) For each K-T solution, sketch the steepest descent direction and the gradients of G_i for each tight constraint. Interpret the K-T conditions geometrically.
(h.) Discuss the difficulties one might have in solving this problem via the feasible direction or generalized reduced gradient (GRG) algorithm.
(i.) Use either the feasible directions algorithm or GRG (in the appropriate APL workspace) to try solving this problem, starting at the feasible point X=(2,2). Is it successful? Try one or two other starting points for the search, also.
(j.) State the Quadratic Programming Dual of this problem.
(k.) Discuss the difficulties involved in solving this problem via Wolfe's method for quadratic programming.
(l.) Use Wolfe's algorithm (in the APL workspace) to try solving the problem. Is it successful?
(m.) Demonstrate why this problem cannot be formulated as a standard (posynomial) geometric programming problem. Show how it may be formulated as a signomial geometric programming problem. (There is more than one way to do this!)

(n.) State the GP dual of the formulation in (i.) What is its degree of difficulty?

(o.) Show how the signomial GP problem may be condensed into a posynomial GP problem, using the feasible point X=(2,2) again. (Show your computations.)

(p.) State the GP dual of the condensed posynomial GP problem. What is its degree of difficulty?

(q.) Sketch the feasible region of this condensed posynomial GP problem.

(r.) Is the feasible region of (q.) a subset of the feasible region of (a.), or vice versa? That is, is every feasible solution of the condensed posynomial GP problem also feasible in the original problem? If not, is every feasible solution of the original problem also feasible in the condensed GP problem?

(s.) Solve the condensed posynomial GP problem. Is the optimal solution feasible in the original problem?

(t.) Find the new condensation of the signomial GP at the point found in (s.), and sketch its feasible region together with the feasible region of the original problem.

(u.) Solve this second condensed GP problem. Is the optimal solution feasible in the original problem?

(v.) Using the SIGGP APL workspace, if you wish, perform several more iterations, until the optimal solutions of two successive condensed problems are "close". Has the procedure converged to a K-T point? to the optimal solution?

(w.) Using the SIGGP workspace again, but a different starting point for the condensation, perform the algorithm to determine if it will converge to the same final solution.

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PART TWO

"Skim" through recent issues of journals (e.g., Engineering Optimization, Operations Research, Management Science, IEE Transactions, Decision Science, European Journal of Operations Research, Computers & Operations Research, Computers & Industrial Engineering, Naval Logistics Research Quarterly, etc.) to find an application of a nonlinear programming algorithm which you have studied in this course to a "real-world" problem of interest to you. Write a summary of the article (one or more pages), describing the application, the NLP model, and the performance of the algorithm. (If there is sample data for a small problem, you may wish to try to obtain the solution using the APL (or other) code for the algorithm.)