Setting Reorder Points

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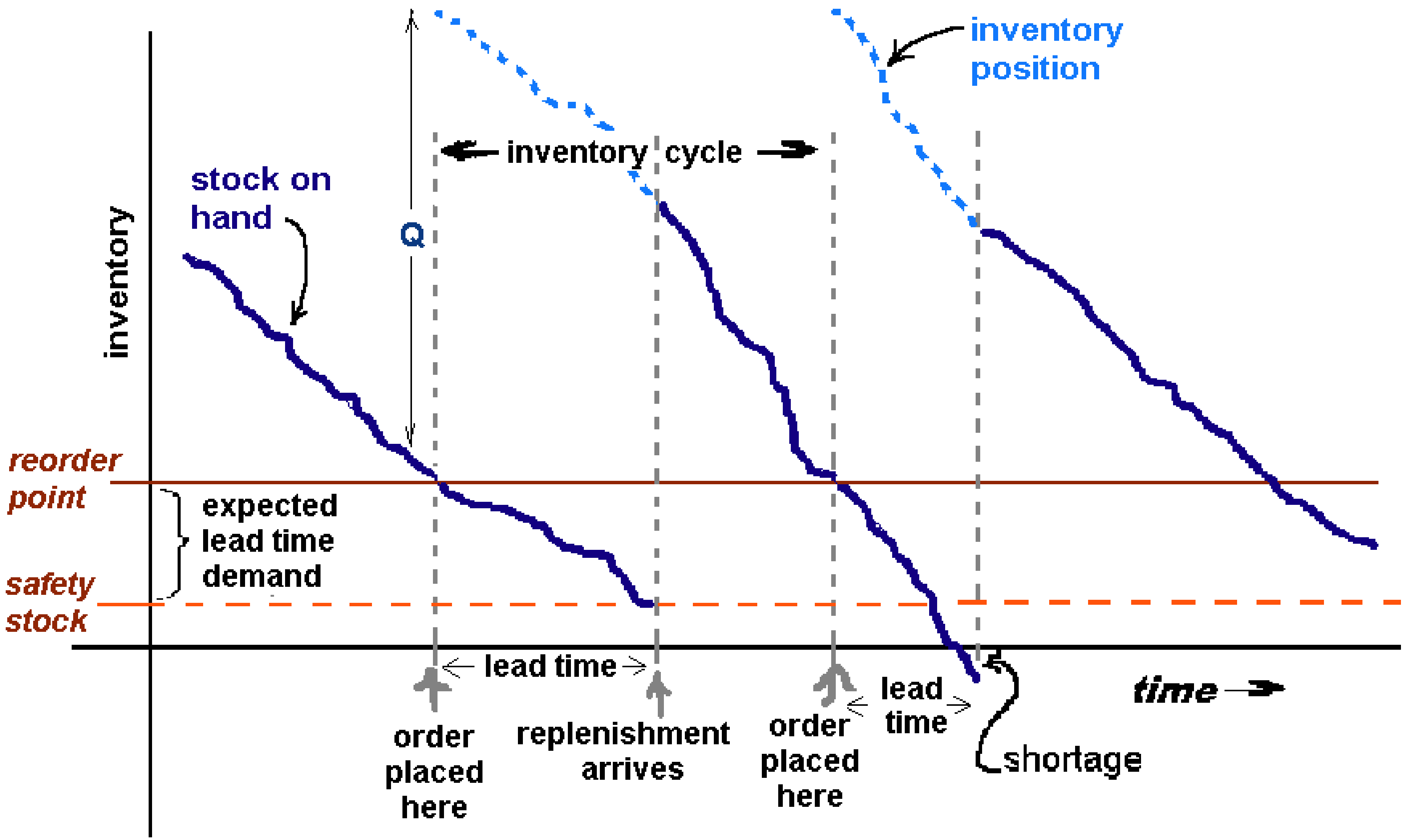
In an inventory control system for a single product with random demand, where the inventory level is continuously monitored, a company places an order for **Q** units of product every time the inventory position drops to a reorder point **R**.

Our goal is to find **"good"** values of **Q** and **R**, where "good" could mean *either*

- a high **level of service** to customer, *or*
- low **cost** of ordering and carrying inventory

- **Terminology:**
- **Stock-on-hand (SOH)**: # units of product physically available
- **Backorder**: product committed to fill a shortage when a replenishment arrives.
- **Inventory on-order**: replenishment ordered from supplier but not yet received
- **Inventory position**: #stock-on-hand + #inventory on-order – #backorders *(when SOH=0, can be either positive or negative!)*
- **Re-order point**: the inventory *position* which "triggers" the placing of a replenishment order.
- **Order lead time**: time interval between placing an order and receiving the product which was ordered.
- **Lead-time demand**: demand which occurs during the lead time of a replenishment order
- **Safety stock**: (reorder point) – (expected lead-time demand)



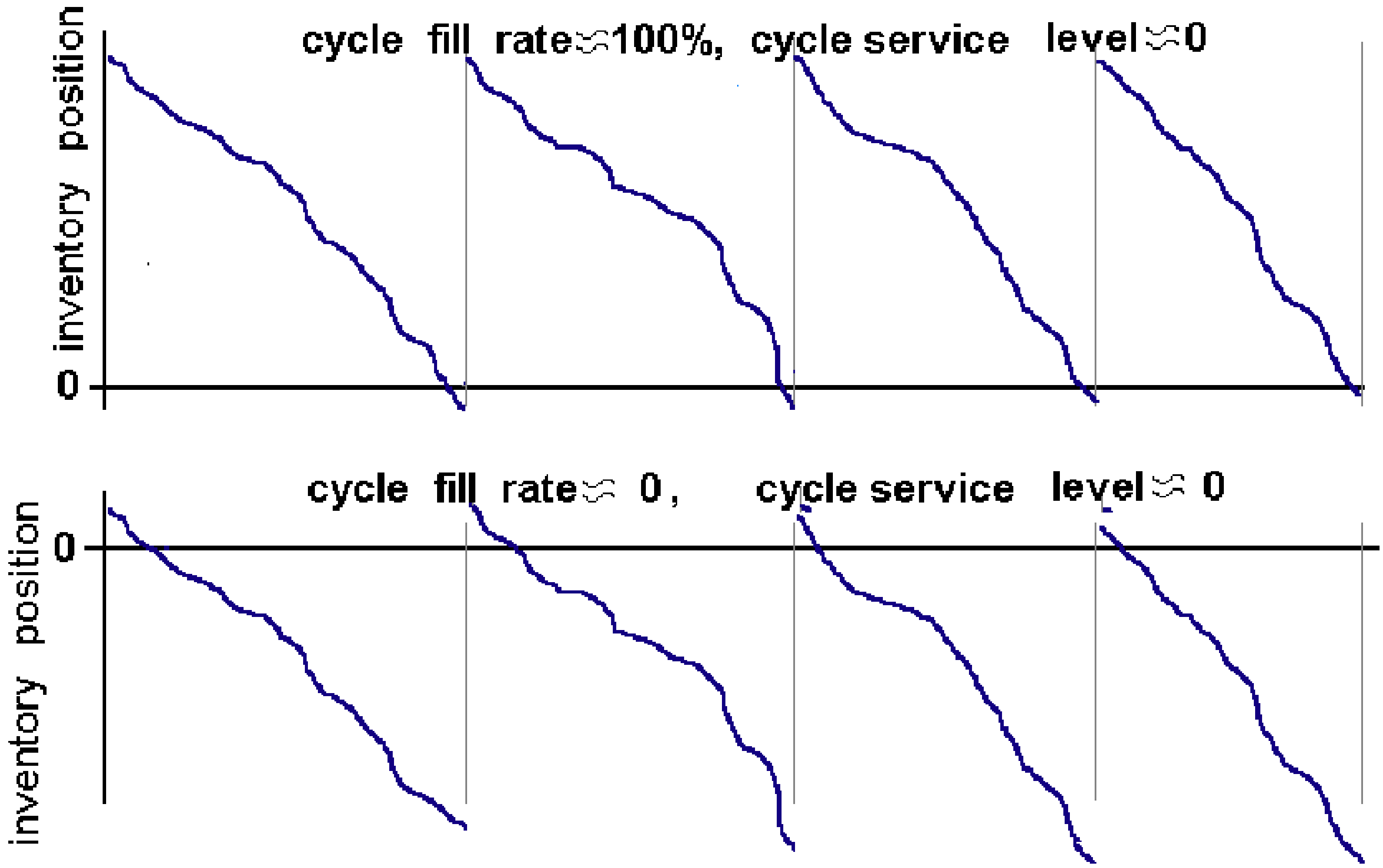
inventory
stock on hand
Q
inventory cycle
inventory position
reorder point
expected lead time demand
safety stock
lead time
order placed here
replenishment arrives
order placed here
lead time
shortage
time

Customer Service Measures *(two alternate ways to measure service)*

Cycle service level: percentage of cycles without a stockout

Product fill rate: percentage of product demand filled without backordering

These two measures can be quite different!



Both examples above have the *same cycle service* level, since a stockout occurs in every cycle, but the *fill rates* are much different!

Uncertainty in the lead-time demand can arise from

- variability in the **rate of demand**, *&/or*
- variability in the **length of the lead time**

*A **longer** lead time with **no** variability might require less safety stock, and therefore be more desirable than a **shorter** expected lead time with **high** variability!*

Distribution of Lead-time Demand

Let demand **d** during 1 time interval (e.g. day) have the normal distribution $N(\mu_d, \sigma_d)$.

Let the lead time **T** be random, with mean μ_t and standard deviation σ_t.

Then the expected demand during the lead time has mean

$$\mu_{LT} = E(T) \times E(d) = \mu_t \mu_d$$

and variance

$$E(T) \times \text{var}(d) + \left[E(d)\right]^2 \times \text{var}(T) = \mu_t \sigma_d^2 + \mu_d^2 \sigma_t^2$$

that is, the standard deviation of lead-time demand is

$$\sigma_{LT} = \sqrt{\mu_t \sigma_d^2 + \mu_d^2 \sigma_t^2}$$

N(0,1) distribution

z	Φ(z)	1-Φ(z)	L(z)
0.05	0.5199	0.4801	0.3744
0.10	0.5398	0.4602	0.3509
0.15	0.5596	0.4404	0.3284
0.20	0.5793	0.4207	0.3069
0.25	0.5987	0.4013	0.2863
0.30	0.6179	0.3821	0.2668
0.35	0.6368	0.3632	0.2481
0.40	0.6554	0.3446	0.2304
0.45	0.6736	0.3264	0.2137
0.50	0.6915	0.3085	0.1978
0.55	0.7088	0.2912	0.1828
0.60	0.7257	0.2743	0.1687
0.65	0.7422	0.2578	0.1554
0.70	0.7580	0.2420	0.1429
0.75	0.7734	0.2266	0.1312
0.80	0.7881	0.2119	0.1202
0.85	0.8023	0.1977	0.1100
0.90	0.8159	0.1841	0.1004
0.95	0.8289	0.1711	0.0916

z	Φ(z)	1-Φ(z)	L(z)
1.00	0.8413	0.1587	0.0833
1.10	0.8643	0.1357	0.0686
1.20	0.8849	0.1151	0.0561
1.30	0.9032	0.0968	0.0455
1.40	0.9192	0.0808	0.0367
1.50	0.9332	0.0668	0.0293
1.60	0.9452	0.0548	0.0232
1.70	0.9554	0.0446	0.0183
1.80	0.9641	0.0359	0.0143
1.90	0.9713	0.0287	0.0111
2.00	0.9772	0.0228	0.0085
2.10	0.9821	0.0179	0.0065
2.20	0.9861	0.0139	0.0049
2.30	0.9893	0.0107	0.0037
2.40	0.9918	0.0082	0.0027
2.50	0.9938	0.0062	0.0020
2.60	0.9953	0.0047	0.0015
2.70	0.9965	0.0035	0.0011
2.80	0.9974	0.0026	0.0008
2.90	0.9981	0.0019	0.0005
3.00	0.9987	0.0013	0.0004

Example:

A policy of providing a **cycle service level of 95%** has been selected. What is the amount of safety stock which will achieve this?

We want **R**, the re-order point, to have a value such that

$$P\{D_L \le R\} = P\left\{\frac{D_L - \mu}{\sigma} \le \frac{R-\mu}{\sigma}\right\} = \Phi\left(\frac{R-\mu}{\sigma}\right) = 0.95$$

where $\mathbf{D_L}$ is the demand (with distribution $N(\mu,\sigma)$) during the lead time.

According to the table, we should choose **R** such that

$$\frac{R-\mu}{\sigma}=1.65 \Rightarrow R=\mu+1.65\times\sigma$$

That is, the ***safety factor*** should be **1.65**, so that the **safety stock** would be 1.65×σ.

With this safety factor, the **expected shortage** will be

$$G(R)=\sigma L(1.65)=0.0263\times\sigma$$

Example:

Suppose that the inventory policy,

instead of having a **95% cycle service level** (i.e., probability of stockout during a cycle),

is to have a **99% fill rate**.

To determine the safety factor, we must assign a value to the **expected shortage**, i.e., 1% of the demand that occurs during the cycle,

and then choose **R** such that the value of **G(R)** is this expected shortage.

Two cases to consider:

- If shortages are *backordered*, then the demand during the order cycle is simply the size of the order, **Q**.
- If shortages are *lost sales*, then the demand during the order cycle is **Q+G(R)**, i.e., the demand before the stockout occurs plus the demand *after* the stockout occurs.

Suppose that shortages are **backordered**, and that

$\mathbf{Q} = 500$ units *(order quantity)*

$\mu = 150$ units *(expected lead-time demand)*

$\sigma = 50$ units *(std deviation of lead-time demand)*

Then the **expected shortage** should be $0.01 \times 500 = 5$.

We should choose **R** so that G(R) = 5, which means

$$G(R) = \sigma L\left(\frac{R-\mu}{\sigma}\right) = 50 \times L\left(\frac{R-150}{50}\right) = 5$$

$$\Rightarrow L\left(\frac{R-150}{50}\right) = \frac{5}{50} = 0.1$$

If we look at the table, we see that L(0.9)= 0.1

Therefore, we want $\frac{R-150}{50}=0.9$ or $R=150+0.9\times 50=195$.

That is, the ***safety factor*** should be 0.9.

We see also in the table that $\Phi(0.9)=0.816$

which means that the **cycle service level** would be only 81.6%.

(There would be a 18.4% probability of a stock-out in each cycle!)

Summary:

- The **re-order point** is the sum of the expected lead-time demand and the safety stock.
- The **safety stock** is the product of the **safety factor** and the standard deviation of lead-time demand.
- The safety stock could be selected so as to achieve either of the two measures of **customer service**:
  - **cycle service level**, i.e., probability that a shortage does not occur
  - **fill rate**, i.e., expected fraction of demand during a cycle which can be satisfied without backordering.