EOQ
Economic Order Quantity Model

$$Q^* = \sqrt{\frac{2AD}{h}}$$

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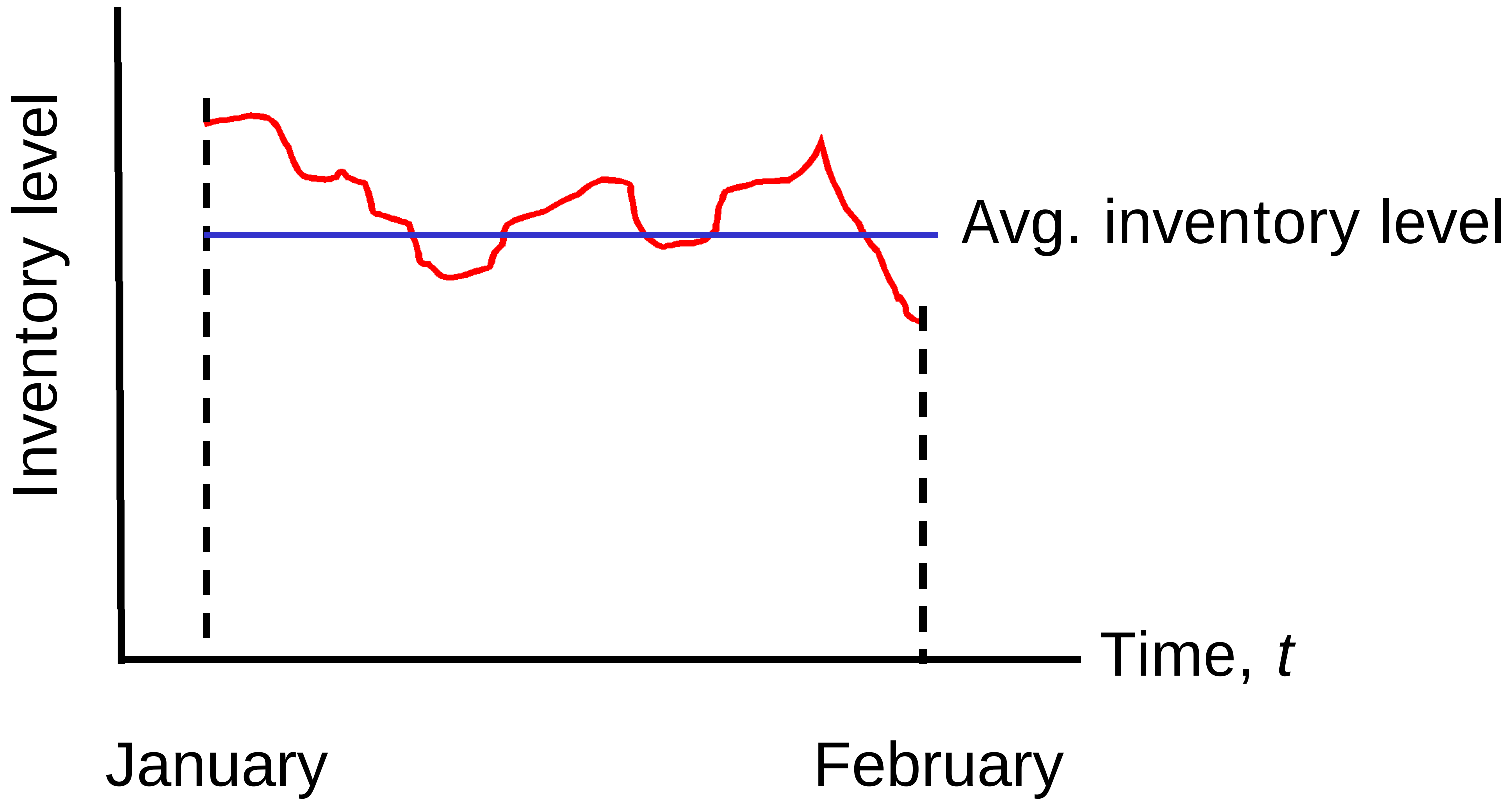
The “classical” or “basic” Economic Order Quantity (**EOQ**) model makes many simplifying assumptions, including:

- *demand rate* is constant & known
- *lead time* (time between placing order and receiving shipment) is known, not random
- replenishment is added *instantaneously* to inventory
- no *shortages* are allowed
- *cost of the product* does not depend upon order quantity, i.e., there are no quantity discounts

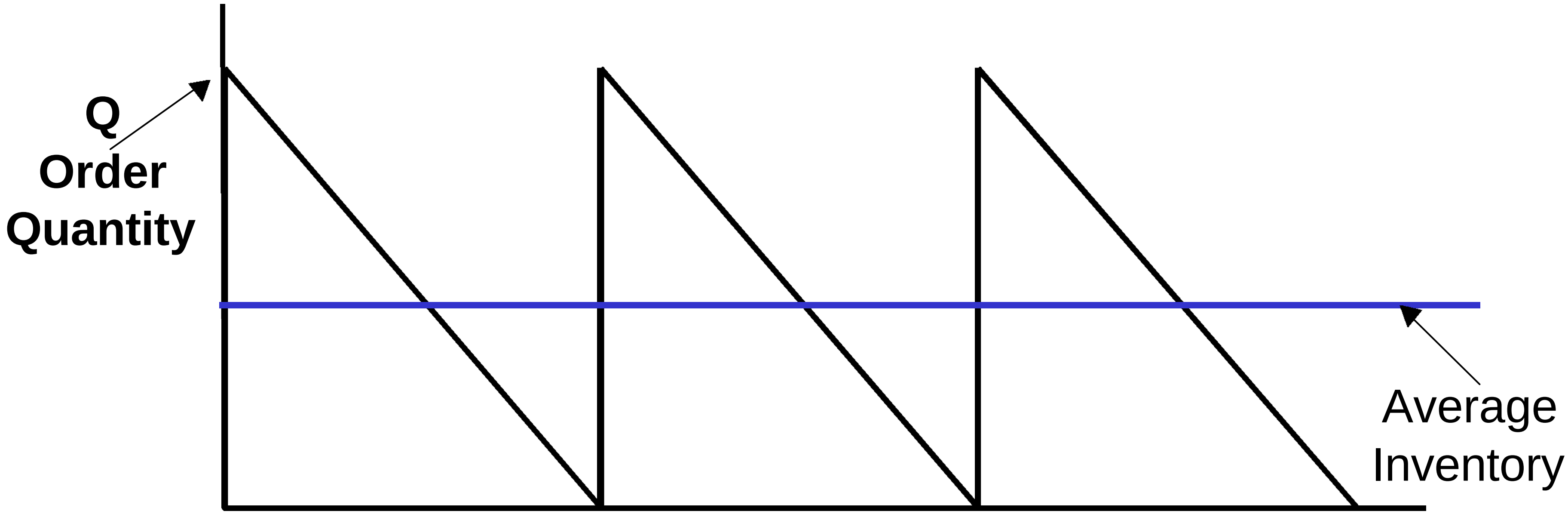
(The last assumption means that we can ignore the cost of the product in our optimization.)

Inventory holding cost is assumed to be proportional to *average* inventory level:



In the **EOQ** model, we assume that

- inventory is withdrawn at a **constant** & known rate
- replenishment arrives at exact time that inventory reaches zero (just-in-time, or **JIT**)

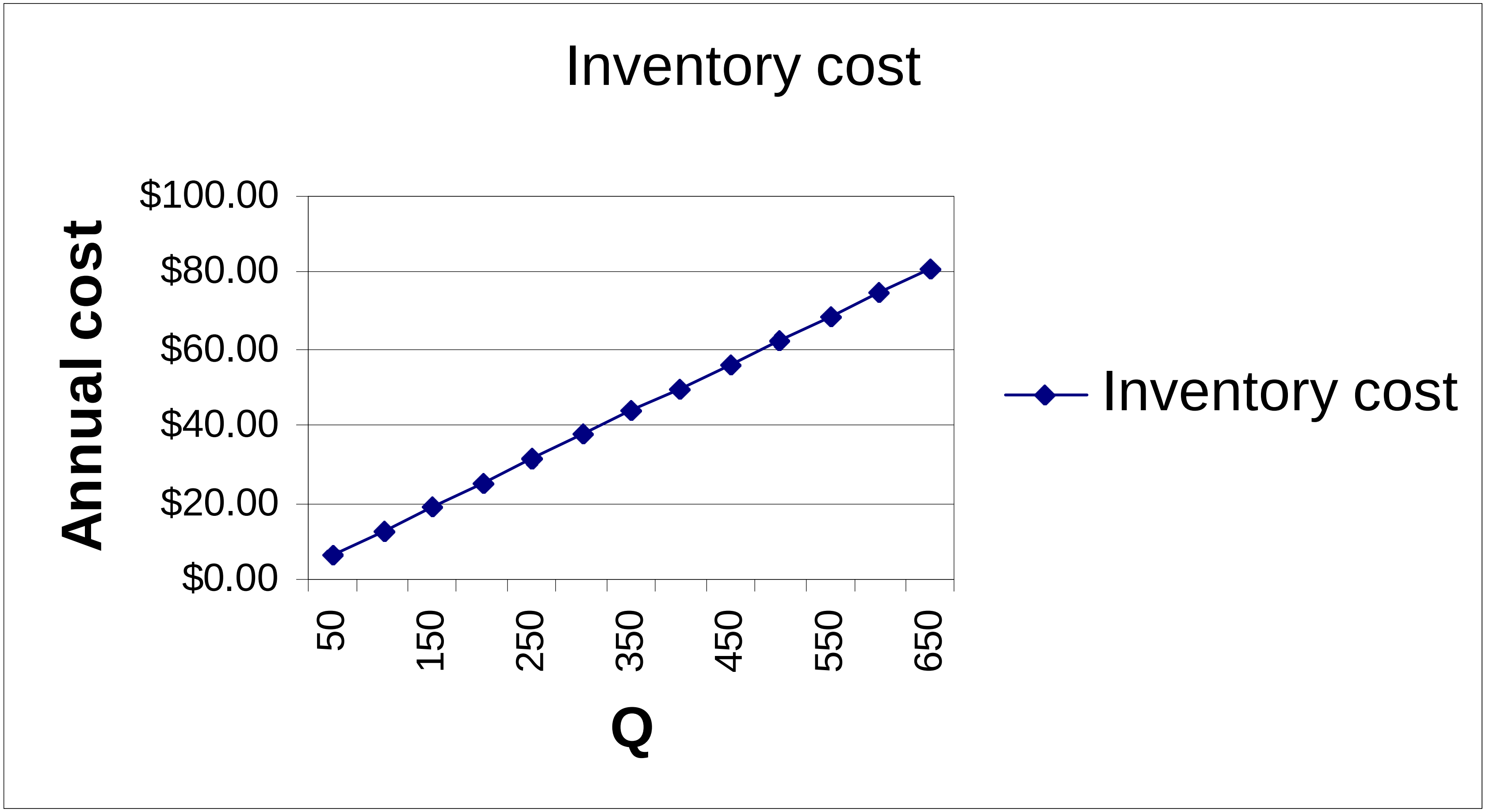


The **average inventory** is therefore $\frac{1}{2} \times Q$.

Example: holding cost is 25% of product value

Annual demand:	1000
Holding cost:	25%
Unit value:	$1.00

Order Quantity Q	Average Inventory	Annual Inventory cost
50	25	**$6.25**
100	50	**$12.50**
150	75	**$18.75**
200	100	**$25.00**
250	125	**$31.25**
300	150	**$37.50**
350	175	**$43.75**
400	200	**$50.00**
450	225	**$56.25**
500	250	**$62.50**
550	275	**$68.75**
600	300	**$75.00**
650	325	**$81.25**



Annual holding cost $\frac{1}{2}Qh$ increases linearly as Q increases, where h=0.25×$1.00.

Annual Ordering Cost

Number of orders per year = $\dfrac{D}{Q} = \dfrac{\text{annual demand}}{\text{order quantity}}$

For example, if D = 1000 units per year, and Q=200 units, then number orders per year is $^{1000}/_{200} = 5$.

In our example, each time we order a replenishment, the cost is $12.00.

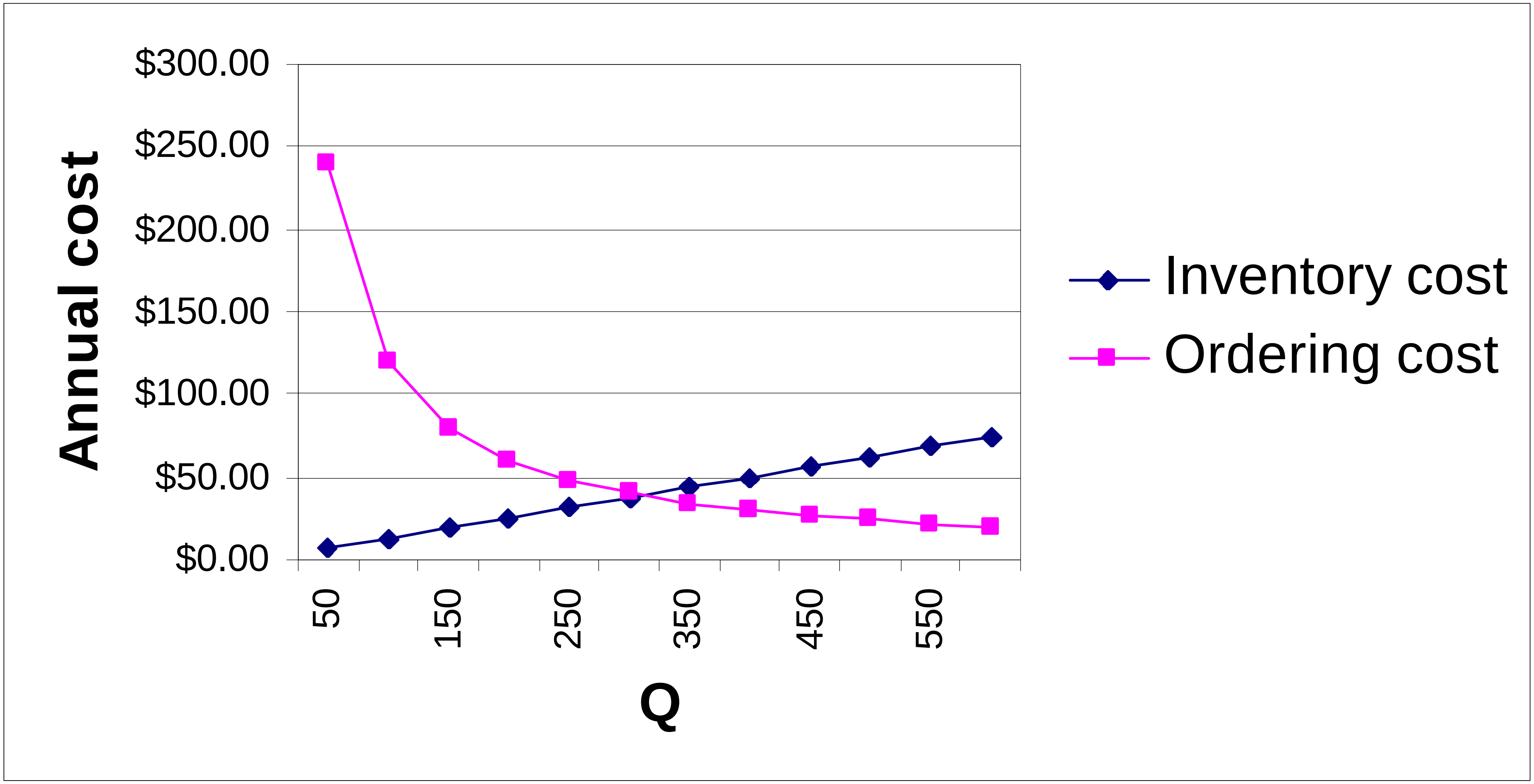
Annual ordering cost is therefore $A \times \dfrac{D}{Q}$, or $\dfrac{AD}{Q}$.

Note that this excludes the cost of the product itself, since it does not depend upon Q.

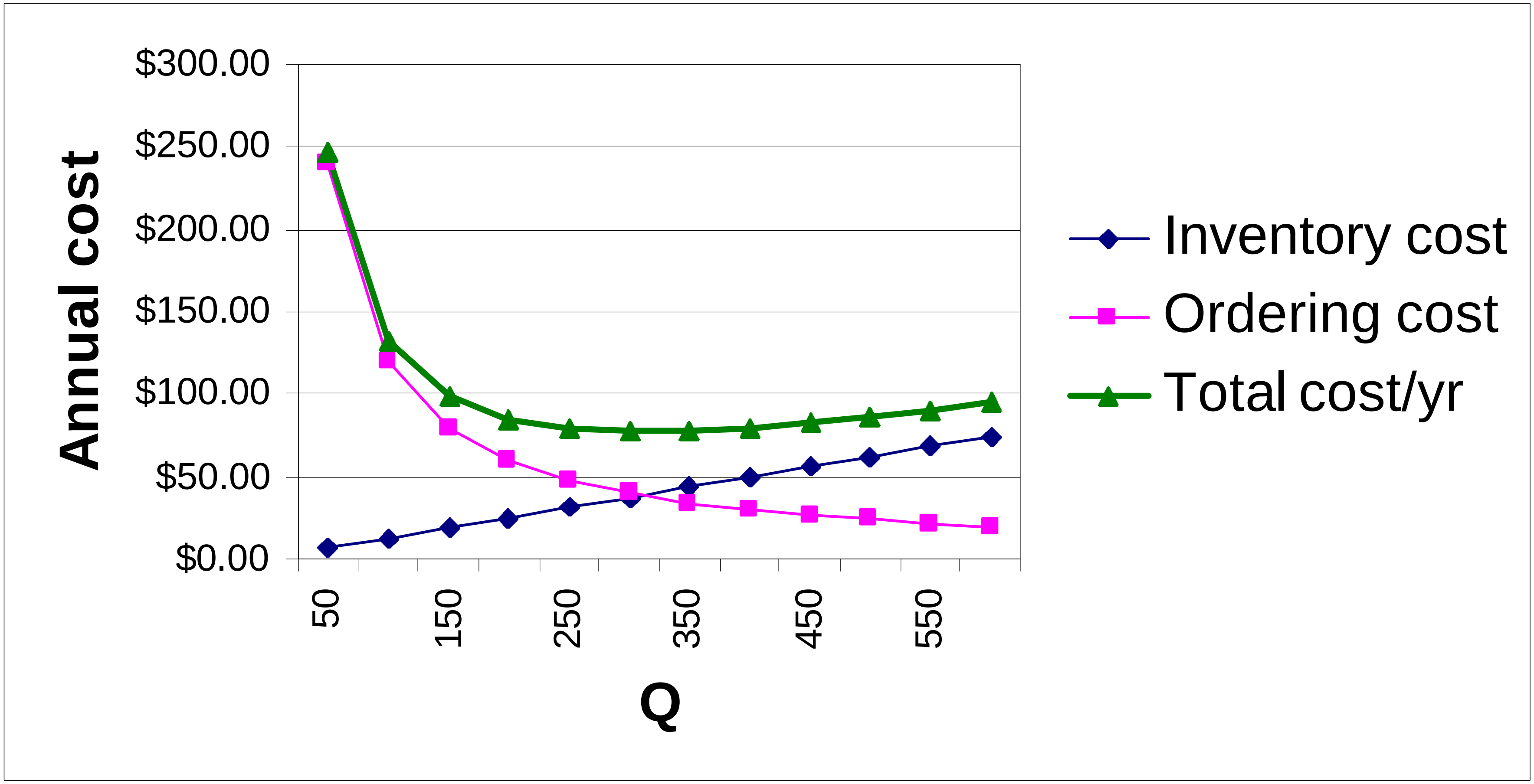
Example, continued:

Annual demand:	1000
Holding cost:	25%
Unit value:	$1.00
Fixed cost / order	$12.00

Order Quantity Q	Average Inventory	Annual Inventory cost	Average # orders year	Annual order cost
50	25	$6.25	**20.00**	**$240.00**
100	50	$12.50	**10.00**	**$120.00**
150	75	$18.75	**6.67**	**$80.00**
200	100	$25.00	**5.00**	**$60.00**
250	125	$31.25	**4.00**	**$48.00**
300	150	$37.50	**3.33**	**$40.00**
350	175	$43.75	**2.86**	**$34.29**
400	200	$50.00	**2.50**	**$30.00**
450	225	$56.25	**2.22**	**$26.67**
500	250	$62.50	**2.00**	**$24.00**
550	275	$68.75	**1.82**	**$21.82**
600	300	$75.00	**1.67**	**$20.00**
650	325	$81.25	**1.54**	**$18.46**

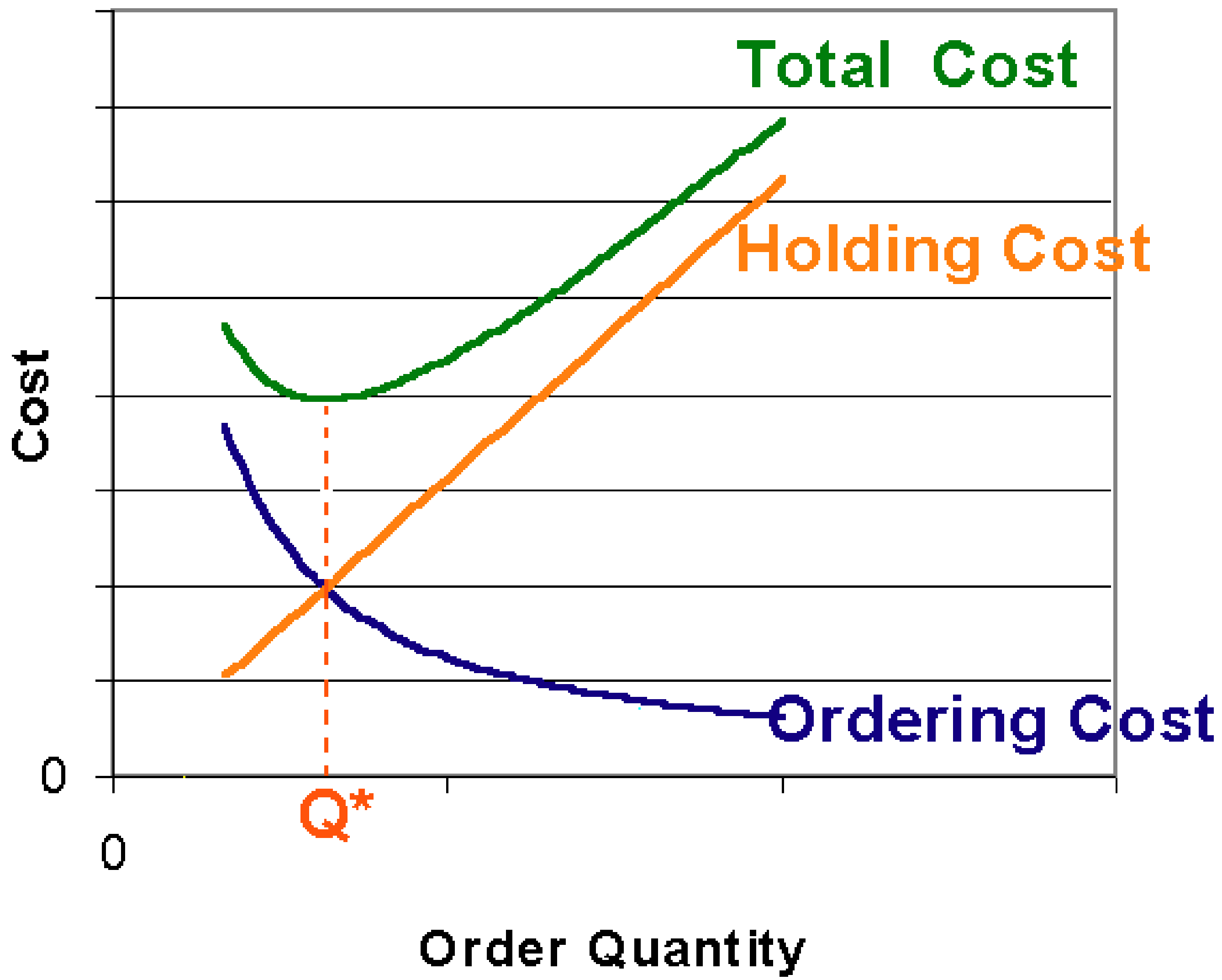


Annual ordering cost is inversely proportional to order quantity Q, i.e., as Q increases, frequency of orders decreases, and therefore ordering cost decreases.



The **Total Annual Cost** is a convex (bowl-shaped) function with a minimum value at

Q* = *economic order quantity* (**EOQ**)



(It happens, because of the specific form of the cost functions, that Q is found at the intersection of the two components of the total cost!)*

The value of Q* can be computed by finding the Q at which the total cost function

$$C(Q) = \frac{1}{2}Qh + A\frac{D}{Q}$$

where

h = $i \cdot v$ = holding cost rate × unit value of product

A = fixed cost per order

That is,

$$\frac{d}{dQ}C(Q) = \frac{h}{2} - \frac{AD}{Q^2} = 0$$

We can solve this equation for Q to obtain the formula

$$Q^* = \sqrt{\frac{2AD}{h}}$$

(Example, continued)

Recall that

Annual demand:	1000
Holding cost:	25%
Unit value:	$1.00
Fixed cost / order	$12.00

That is,

A = $12, D = 1000/year, h = 0.25×($1.00) = $0.25/year

$$\Rightarrow Q^* = \sqrt{\frac{2AD}{h}}$$

In our example, A=$12, D=1000, and h=0.25, so that

$$Q^* = \sqrt{\frac{2AD}{h}} = \sqrt{\frac{2\times 12\times 1000}{0.25}} = \sqrt{96000} = 309.8 \approx 310$$

Minimum Annual Cost

Evaluation of $C(Q) = \frac{1}{2}Qh + A\frac{D}{Q}$ for the EOQ, with

$$Q^* = \sqrt{\frac{2AD}{h}}$$

yields the formula

$$C(Q^*) = \sqrt{2ADh}$$

In our example,

$$C(Q^*) = \sqrt{2ADh} = \sqrt{2 \times 12 \times 1000 \times 0.25} = \sqrt{6000} = 77.46$$