56:270 LINEAR PROGRAMMING

Midterm Exam -- March 11, 1988

PROBLEM:	1	2	3	4	5	TOTAL
SCORE:						

Answer #1 and any three of the remaining four problems.

(1.) UPPER BOUNDING TECHNIQUE . Consider the following LP problem:

$$\begin{array}{ll} \text{MINIMIZE} & 2X_1 + X_2 + X_3 - X_4 \\ \text{subject to} & 2X_1 - X_2 + X_3 - 2X_4 \leq 12 \\ & -X_1 + X_2 - 2X_3 + 3X_4 \leq 9 \\ & 3X_1 + X_2 - X_3 \geq 3 \\ & 0 \leq X_1 \leq 5 \\ & 2 \leq X_2 \leq 4 \\ & 0 \leq X_3 \leq 9 \\ & 1 \leq X_4 \leq 2 \end{array}$$

The APL output solving this problem using the Upper-Bounding Technique is attached. Please refer to it to answer the following questions.

(a.) What is the reduced cost of x_2 in the first iteration? Why was x_2 selected to enter the basis? Which other variables, if any, might have been selected to enter the basis and improve the objective?

(b.) As x_2 enters the basis in iteration 1, which variables increase? which decrease?

(c.) Explain the computation of the "blocking values" 1.000 and 19.000 in iteration 1.

(d.) Explain why x_7 was selected to leave the basis in iteration #1.

(e.) In iteration #2, explain how the "blocking values" 3.4 and 0.333 were computed.

(f.) If the product form of the inverse were being used, what "eta vector" would be generated in iteration #2? What is the corresponding pivot matrix?

(g.) In iteration #3, why does X_4 not enter the basis?

(h.) Explain why, if the basis does not change, the basic solution at the third iteration differs from that at the second iteration.

(i.) Explain why the algorithm terminates in iteration #4 with an "optimal solution" even though the reduced costs are not all non-negative.

(2.) DUAL SIMPLEX METHOD: Consider the simplex tableau below:

-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	2	0	4	1	5	0	0	0	-10	(MIN)
0	-1	-1	-1	1	1	0	1	0	-5	
0	1	-2	-1	1	1	1	0	0	1	
0	1	1	-2	2	-3	0	0	1	-2	

(a.) What is the current primal solution? Is it feasible? Does it satisfy the primal optimality conditions?

(b.) Explain why this solution is "dual feasible".

(c.) Circle every element in the tableau which the dual simplex method might select as a pivot element.

(3.) Write down the dual of the following LP problem:

$$\begin{aligned}
\text{Maximize } \; & z = 3x_1 + 2x_2 - 3x_3 + 4x_4 \\
\text{subject to } \; & x_1 - 2x_2 + 3x_3 + 4x_4 \leq 3 \\
& x_2 + 3x_3 \geq -5 \\
& 2x_1 - 3x_2 - 7x_3 - 4x_4 = 12 \\
& x_1 \geq 0, \quad 2 \leq x_2 \leq 5, \quad x_3 \leq 0
\end{aligned}$$

(4.) At an intermediate step of the simplex algorithm, in which the objective is to be minimized, the tableau is:

	-z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
__________	1	0	0	0	-3	0	2	-10
	0	1	0	1	4	0	-1	3
	0	0	1	0	2	0	1	1
	0	-1	0	0	-2	1	3	0

(a.) What is the basis for this tableau?

(b.) What are the current values of z and x_1 through x_6 for this basic solution?

(c.) Would increasing x_4 increase or decrease the objective function?

(d.) Would increasing x_6 increase or decrease the objective function?

(e.) What is the substitution rate of x_4 for x_5? If x_4 is increased by 1 unit, how is x_5 changed?

(e.) Is the current solution optimal? If not, perform a pivot to improve the objective function.
(f.) What value(s) could be changed in the tableau above to make the solution unbounded?
(g.) Under what conditions can the simplex multipliers be determined from the tableau above?

(5.) Formulate the following problem as an LP:
A manufacturer must plan production of a certain item over the next 4 quarters. Each unit of the item requires 1 man-hour of labor. Labor costs are $10 per hour regularly, or $15 per hour for overtime. Overtime is limited to 50% of regular time available. If a unit of the item is available for sale during a quarter but is not sold, an inventory carrying cost of $2 per unit is charged. Other data are:

Quarter	Man-hrs Available	Demand	Profit($)
1	1000	1200	18
2	800	1100	16
3	900	1400	17
4	1000	1200	17

Demand limits the amount which can be sold, but need not be satisfied. Profit that is specified is based upon regular-time labor costs. If overtime is used, the profits decrease accordingly. How much should be produced each quarter? How much should be sold each quarter?
Clearly define your decision variables.