56:270 LINEAR PROGRAMMING

Midterm Exam -- March 18, 1987

PROBLEM:	1	2	3	4	TOTAL
SCORE:					

1. Sensitivity Analysis: Consider the LP problem:

$$\text{Minimize } z = 2x_1 + x_2 + 2x_3 - 3x_4$$

$$\begin{aligned} \text{subject to} \quad 8x_1 - 4x_2 - x_3 + 3x_4 &\leq 10 \\ 2x_1 + 3x_2 + x_3 - x_4 &\leq 7 \\ -2x_2 - x_3 + 4x_4 &\leq 12 \\ X_j \geq 0, \ j=1,2,\ldots 4 & \end{aligned}$$

The optimal tableau is

-z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	rhs
1	2.4	0	1.4	0	0	0.2	0.8	-11
0	10	0	0.5	0	1	1	-0.5	11
0	0.8	1	0.3	0	0	0.4	0.1	4
0	0.4	0	-0.1	1	0	0.2	0.3	5

(a.) What is the current basis matrix?

(b.) What is the basis inverse matrix?

(c.) What is the current basic solution?

(d.) What are the values of the simplex multipliers?

(e.) FInd the range of the objective function coefficient of x_3 for which the above solution remains optimal.

(f.) Find the interval for the right-hand-side (currently 12) of the last constraint for which the current basis remains optimal.

(g.) How many basic (including infeasible) solutions exist for the above problem?

(2.) Write down the dual of the following LP problem:

$$\text{Minimize } z = 3x_1 + 2x_2 - 3x_3 + 4x_4$$

subject to
$$x_1 - 2x_2 + 3x_3 + 4x_4 \leq 3$$
$$x_2 + 3x_3 + 4x_4 \geq -5$$
$$2x_1 - 3x_2 - 7x_3 - 4x_4 = 2$$
$$x_1 \geq 0, \quad x_4 \leq 0$$

(3.) DUAL SIMPLEX METHOD: Consider the simplex tableau below:

-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	2	3	4	1	5	0	0	0	0	(MIN)
0	-1	-1	-1	1	1	0	1	0	-5	
0	1	-2	-1	1	1	1	0	0	1	
0	1	1	-2	2	-3	0	0	1	-2	

(a.) What is the current primal solution? Is it feasible? Does it satisfy the primal optimality conditions?

(b.) Circle every element in the tableau which the dual simplex method might select as a pivot element.

(4.) UPPER BOUNDING TECHNIQUE. Consider the following LP problem:

MINIMIZE $2X_1 + X_2 + X_3 - X_4$

subject to
$$2X_1 - X_2 + X_3 - 2X_4 \leq 6$$
$$-X_1 + X_2 - 2X_3 + 3X_4 \leq 9$$
$$3X_1 + X_2 - X_3 \geq 3$$
$$0 \leq X_1 \leq 5$$
$$2 \leq X_2 \leq 4$$
$$0 \leq X_3 \leq 9$$
$$1 \leq X_4 \leq 2$$

The APL output solving this problem using the Upper-Bounding Technique is attached. Please refer to it to answer the following questions.

(a.) Explain why in the initial basic solution, i.e.

X_1	X_2	X_3	X_4	X_5	X_6	X_7
5	2	0	1	0	9	14

the value of the basic variables (X_5, X_6, X_7) are NOT given by the quantity $(A^B)^{-1}b$, which is equal to (6, 9, -3).

(b.) What is the reduced cost of x1 in the first iteration? Which other variables, if any, might have been selected to enter the basis and improve the objective?

(c.) Explain why x_7 was selected to leave the basis in iteration #1.

(d.) In Iteration #2, explain how it is determined that, as the nonbasic variable X_4 is increased, the basic variable X_5 increases, while the basic variable X_6 decreases.

(e.) In iteration #2 again, explain how the "blocking values" 45.333 and 1.444 were computed.

(f.) Why does X_4 not enter the basis?

(g.) Explain why, if the basis does not change, the basic solution at the third iteration differs from that at the second iteration.

(h.) Explain why the algorithm terminates with an "optimal solution" if the reduced costs are not all non-negative.