56:270 Linear Programming
Midterm Exam - March 19, 1986

(1.) Below are several simplex tableaus. Assume that the objective in each case is to be **minimized**. Classify each tableau by writing to the right of the tableau a letter **A** through **G**, according to the descriptions below. Also answer the question accompanying each classification, if any.

(A) Nonoptimal, nondegenerate tableau. *Circle a pivot element which would improve the objective.*

(B) Nonoptimal, degenerate tableau. *Circle an appropriate pivot element. Would the objective improve with this pivot?*

(C) Unique optimum.

(D) Optimal tableau, with alternate optimum. *Circle a pivot element which would lead to another optimal basic solution.*

(E) Objective unbounded (below). *Specify a variable which, when going to infinity, will make the objective arbitrarily low.*

(F) Tableau with infeasible primal but feasible dual solution. *Circle an appropriate (dual simplex) pivot which would improve feasibility.*

(G) Tableau with both primal and dual solutions infeasible.

Warning: Some of these classifications might be used for several tableaus, while others might not be used at all!

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	-1	0	0	-2	0	-84	
0	0	0	-4	0	0	1	3	0	13	________
0	4	1	2	-5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	15	

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	1	0	0	2	12	-84	
0	0	0	-4	0	0	1	3	0	13	________
0	4	1	2	-5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	15	

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	-1	0	0	-2	0	-84	
0	0	0	-4	0	0	1	3	0	13	________
0	4	1	2	5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	15	

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	3	0	0	2	0	-84	
0	0	0	-4	0	0	1	3	0	13	________
0	4	1	2	-5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	-15	

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	-1	0	0	-2	0	-84	
0	0	0	-4	0	0	1	3	0	13	________
0	4	1	2	-5	0	0	2	1	0	
0	-6	0	3	2	1	0	-4	3	8	

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	0	0	0	-2	0	-84	
0	0	0	-4	0	0	1	3	0	-3	________
0	4	1	2	-5	0	0	2	1	0	
0	-6	0	3	-2	1	0	-4	3	15	

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	3	0	0	0	0	-84	
0	0	0	-4	0	0	1	3	0	13	________
0	4	1	2	-5	0	0	2	1	8	
0	-6	0	3	-2	1	0	-4	3	15	

-z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	0	1	3	0	0	2	0	-84	
0	0	0	-4	0	0	1	3	0	13	________
0	-4	1	2	-5	0	0	-2	1	-2	
0	-6	0	3	-2	1	0	-4	3	0	

(2.) Refer to the **MPSX** output for the **Depot Maintenance** problem in your MPSX handbook to answer the following questions:

> (a.) How many iterations of the *Revised Simplex Method* were performed in Phase 1? in Phase 2?
>
> (b.) How sensitive is the solution to the number of units of product 3 in the repairable items inventory? How would the cost and the other variables change if this number were to increase by 50?
>
> (c.) Suppose that resource #1 at Depot A is man-minutes of labor. If overtime is available at the additional cost of $6 per hour, would you recommend its use?
>
> (d.) If in part (c) an additional 600 man-minutes of labor were available at Depot A, would the operating plans of the other depots be changed? For which products at Depot A would there be increased maintenance?

(3.) **UPPER BOUNDING TECHNIQUE. Consider the following LP problem:**

$$\begin{aligned}
\textbf{MINIMIZE}\quad & X_1 + 2X_2 + 3X_3 - X_4 \\
\textbf{subject to}\quad & X_1 - X_2 + X_3 - 2X_4 \le 6 \\
& -X_1 + X_2 - X_3 + X_4 \le 8 \\
& 2X_1 + X_2 - X_3 \ge 2 \\
& 0 \le X_1 \le 3 \\
& 1 \le X_2 \le 4 \\
& 0 \le X_3 \le 10 \\
& 2 \le X_4 \le 5
\end{aligned}$$

The APL output solving this problem using the Upper-Bounding Technique is attached. Please refer to it to answer the following questions.

```
       MAT
 1 ¯1  1 ¯2  1  0  0
¯1  1 ¯1  1  0  1  0
 2  1 ¯1  0  0  0 ¯1

       RHS
6 8 2

       COST
1 2 3 ¯1 0 0 0

       LOWER
0 1 0 2 0 0 0

       UPPER
3 4 10 5 100 100 100
```

(a.) Explain why in the initial basic solution, i.e.

X_1	X_2	X_3	X_4	X_5	X_6	X_7
0.5	1	0	2	10.5	5.5	0

the value of the basic variables, (X_1, X_5, X_6) are NOT given by the quantity $(A^B)^{-1} b$, which is equal to (1, 5, 9).

(b.) Explain how it is determined that, as the nonbasic variable X_4 is increased, the basic variable X_5 increases, while the basic variable X_6 decreases.

(c.) Explain how the "blocking values" 44.75 and 5.5 were computed.

(d.) Why does X_4 not enter the basis?

(e.) Explain why, if the basis does not change, the basic solution at the second iteration differs from that at the first iteration.

(f.) Explain why the algorithm terminates with an "optimal solution" if the reduced costs are not all non-negative.