56:270 Linear Programming
Final Exam - May 12, 1988

°°° PART ONE °°°

Select any THREE problems of Part One:

(1.) ANALYSIS OF MPSX OUTPUT: Please refer to your materials on the PURAIR OIL COMPANY (problem statement, formulation, and MPSX output).
Answer the following questions (if there is insufficient information in the MPSX output, simply answer NOT ENOUGH INFORMATION):

(a.) Refinery #2 is used at full capacity in the optimal solution. Suppose that it were to be shut down for a portion of a day, so that the capacity that day were only 19500 barrels/day, rather than 20000 barrels/day. How much profit would be lost? What changes would there be in the amount of crude oil from each field sent to each refinery? How would it effect the amount of gasoline purchased on the open market in area #6?(refer to the substitution rates)

(b.) HR2MO denotes the amount of heavy fuel oil produced by Refinery #2 and sold on the open market. Suppose that it must be sold in multiples of 100 barrels. How much profit would be lost if the optimal value were rounded up? if rounded down ? Should the variable be rounded up or down ? Which nonbasic variable must be adjusted (upward or downward) in order to accomplish this rounding?

(c.) HR2MO denotes the amount of heavy fuel oil produced by Refinery #2 and sold on the open market at a profit of $2.70/barrel. How low might this profit drop before the solution needs to be adjusted? If the profit reaches this value, how much less will be sold in this way? If there is a 10% increase in profit for this product, will the solution require changing?

(d.) C1M denotes the amount of crude oil from Field #1 sold on the open market at a profit of $3.50/barrel. In the optimal solution, no such sales are made. By how much must the profit increase before it would be optimal to make this sale? How much would then be sold? How would this change the amount of crude oil from each field to be sent each refinery? How would it change the amount of gasoline purchased on the open market in area #6?

(2.) DANTZIG-WOLFE DECOMPOSITION. We wish to use the decomposition technique to solve the following problem:

$$\begin{aligned}
\text{MINIMIZE} \quad & -X_1 - 2X_2 - 2X_3 - 3X_4 \\
\text{subject to} \quad & \\
& X_1 + 2X_2 + 2X_3 + X_4 \le 50 \\
& -X_1 + X_2 + X_3 + X_4 \le 25 \\
& X_1 + 3X_2 \le 40 \\
& 2X_1 + X_2 \le 35 \\
& X_3 \le 15 \\
& X_4 \le 10 \\
& 2X_3 - X_4 \le 18 \\
& X_3 + X_4 \le 20 \\
& X_1, X_2, X_3, \text{ and } X_4 \ge 0
\end{aligned}$$

It was decided to use two subproblems, one with variables (X_1,X_2) and the other with variables (X_3,X_4), writing the feasible region of each as a combination of its extreme points. (That is, there will be two sets of λ's and two "convexity" constraints.) The APL output solving this problem is attached. Since $X_1=X_2=X_3=X_4=0$ is feasible in this problem, we initially add $X_1=X_2=0$ to the master problem as proposal #1, and $X_3=X_4=0$ as proposal #2.

(a.) Write the first master problem tableau (after adding proposals 1 and 2).

(b.) Write the second master problem tableau (after adding proposals 3 and 4).

(c.) What are the values of X_1 through X_4 found by the second master problem? (Note: these values were blanked out in the output!)

(d.) What is the objective function for subproblem 2 at the next iteration (i.e. after the second master problem has been solved)?

(e.) Why does the algorithm terminate where shown? What is the optimal solution of the original problem?

(f.) Does the optimal solution consist of one extreme point from each subproblem?

(3.) STOCHASTIC PROGRAMMING. Consult the "Wool Blending Problem" on page C.1 of your notes. Consider now situations in which uncertainty may enter the problem, in that there is uncertainty in the availability of yarn from the primary supplier:

(a.) CHANCE CONSTRAINED PROGRAMMING: Suppose that the available quantities of each color of yarn given above are the expected values for normal distributions, with standard deviations of 100, 200, and 200, respectively. Reformulate the problem with the restriction that the availability constraint is infeasible with no more than 5 percent probability. (The Standard Normal CDF evaluated at 1.65 is approximately 0.95)

STOCHASTIC LP WITH RECOURSE: Now suppose that the availability of RED yarn is known with certainty to be 1000, but that the others have only two possible values each. There is a 10% probability that only 900 oz. of GREEN yarn will be received from the supplier (otherwise 1000 oz. of yarn is received), and a 20% probability that only 1000 oz. of YELLOW yarn is received (otherwise 1200 oz. of yarn is received.)

The production must be planned before learning the quantities to be received from the supplier. After learning the quantities received, you have several recourses available to you:

- Reduce scheduled production of Fabric #1 (at a cost of 1¢/yard in addition to losing the 5¢/yard profit)
- Increase scheduled production of Fabric #1 (but with only 3¢/yard profit because of use of overtime labor.)
- Order additional GREEN yarn from an alternate supplier, at a cost of 1¢/oz.
- Order additional YELLOW yarn from an alternate supplier, at a cost of 1¢/oz.

(b.) Formulate an LP model to compute the production plan which maximizes the expected profit, as well as the recourse to be used in the event of each possible outcome.

(c.) Explain how Dantzig-Wolfe decomposition might be used to solve this problem. (How many subproblems? What are the variables of each subproblem? etc.)

(4.) LP USING LINEAR COMPLEMENTARY SOLUTION TECHNIQUE. Consider the original LP model given in problem (3) for the Wool-Blending Problem.

(a.) Write down the primal and the dual problems, both using only equality constraints.

(b.) Write down the tableau containing both sets of equality constraints from part (a.)

(c.) Start with slack and surplus variables in the basis (negating rows with surplus variables as required). Is this solution feasible? Optimal? Does it satisfy complementary slackness?

(d.) Define a single artificial variable and insert its column into the tableau.

(e.) Where should you pivot to enter the artificial variable into the basis? (circle the entry) What variable leaves the basis? Will complementary slackness be satisfied after this pivot?

(f.) What variable or variables may now enter the basis at the next iteration?

(g.) How do you decide when to terminate, since you have no row of reduced costs?

° ° ° PART TWO ° ° °

Consider the LP: P: Min cx s.t. Ax=b, $x \geq 0$

where A is an mxn matrix. For each statement below, indicate by "+" the true statements and "O" the false statements:

a. ___ A basic solution of P is always feasible.

b. ___ The rank of A is mxn.

c. ___ If P has an optimal solution, then its dual does also.

d. ___ If P has a feasible solution, then its dual does also.

e. ___ If the feasible region of P is unbounded, then its dual LP is infeasible.

f. ___ If the feasible region of P is nonempty and bounded, then its dual LP cannot be unbounded.

g. ___ If you make a mistake in choosing the pivot row during the simplex method, the next basic solution will have one or more negative variables.

h. ___ The number of basic variables in P is m.

i. ___ When solving P using the simplex method, the value of the objective function decreases at each iteration, unless degeneracy is encountered.

j. ___ The number of basic solutions of P is n!/m!(n-m)!

k. ___ Cycling can occur only if P has degenerate basic solutions.

l. ___ The pivot column in the simplex method should never be an artificial variable's column.

m. ___ "Basic solution" and "extreme point solution" mean the same thing.

n. ___ A variable that becomes basic on one iteration of the simplex method can become nonbasic in the next iteration.

o. ___ In the two-phase simplex method, one first solves for the optimal dual variables in phase one, and then the optimal primal variables in phase two.

p. ___ If the system of linear equations Ax=b has a solution, it must have a non-negative solution.

q. ___ During phase one with an artificial variable for each row, in each iteration one artificial variable drops out of the basis.

r. ___ In solving P by the simplex method with product form of the inverse, if x_j is the entering variable during a pivot step, then the pivot matrix added to the product form differs from the identity matrix only in column j.

s. ___ During any iteration of the simplex method, if x_j is the variable entering the basis, its value after the pivot is the value of the minimum ratio.

t. ___ After obtaining the optimal solution and simplex multiplier vector π for P, if the value of each element of b is decreased and the problem needs to be solved again, the final basis may be used as initial basis for the dual simplex method.

u. ___ If P has an optimal solution, the value of the simplex multiplier vector π found in the final iteration of the simplex method is non-negative.

v. ___ If P has an unbounded optimum, then the value of π at each iteration includes some negative element.

w. ___ It is always possible to set up Phase One of the simplex method to solve P using only one artificial variable.

x. ___ When "cycling" occurs during the simplex method, the basis remains unchanged at each iteration.

y. ___ If P is nondegenerate during Phase Two of the simplex method, there will be an improvement at every iteration.

z. ___ If the rank of matrix A is m, there will be m positive variables in each basic feasible solution.

aa. ___ If the optimal solution of P is degenerate, then the dual of P will have alternate optima.

bb. ___ If the optimal solution of P is degenerate, it is possible that the simplex method fails to find the optimal solution unless special precautions are taken.

cc. ___ If P is nondegenerate during Phase Two of the simplex method, there will never be a tie in the minimum ratio test.

dd. ___ A feasible solution of P is always basic.

ee. ___ The dual of the LP in Phase One for P is always feasible.

ff. ___ Either P or its dual (or both) are feasible.

gg. ___ The reduced costs of the basic variables during each iteration of the simplex method for solving P will be the negative of the simplex multipliers.

hh. ___ If $c_j < 0$ for each j, then P will have an unbounded optimum.

ii. ___ If P has a feasible solution, then it must have a basic feasible solution.

jj. ___ The revised simplex method usually requires fewer iterations than the ordinary simplex method.

kk. ___ The revised simplex method is what is known as a "polynomial-time algorithm".

ll.___ LP problems belong to the class of NP problems, but not the class of NP-complete problems.

mm.___ The ellipsoid algorithm is a polynomial-time algorithm.
nn.___ Karmarkar's algorithm is a polynomial-time algorithm.
oo.___ The complementary pivoting algorithm is a polynomial-time algorithm.