56:270 LINEAR PROGRAMMING

FINAL EXAMINATION - MAY 17, 1985

SELECT **TWO** PROBLEMS (OF A POSSIBLE FOUR) FROM PART ONE, AND **FOUR** PROBLEMS (OF A POSSIBLE FIVE) FROM PART TWO.

PART ONE: SCORE:	1	2	3	4		TOTAL	GRAND TOTAL:
PART TWO: SCORE:	1	2	3	4	5	TOTAL	

PART ONE

1. Consider the LP problem:

$$\begin{aligned} &\text{MINIMIZE } 2x_1 + 3x_2 \\ &\text{subject to } 3x_1 + x_2 \geq 10 \\ &5x_1 + 2x_2 \geq 25 \\ &x_1 - x_2 \geq 5 \\ &x_1 \geq 0,\ x_2 \geq 0 \end{aligned}$$

(a.) To begin the simplex algorithm, one must have a feasible basis. Write down the initial tableau, using three artificial variables and the "BIG-M" method.

(b.) An alternate to the "BIG-M" method is the Two-phase method. Define a Phase-One objective function for the same problem and write down the initial tableau.

(c.) Instead of using three artificial variables, a single artificial variable might be used. Write down an initial tableau for this problem, using a single artificial variable and the "BIG-M" method.

(d.) Now write down an initial tableau using a single artificial variable and the Two-Phase method.

(e.) What should one do if an artificial variable is basic and positive at the end of Phase-One?

(f.) What should one do if an artificial variable is basic but zero at the end of Phase-One?

(2.) Write down the dual of the following LP problem:

$$\begin{aligned} &\text{MAXIMIZE } -17X_2 + 83X_4 - 8X_5 \\ &\text{subject to: } -X_1 - 13X_2 + 45X_3 + 16X_5 - 7X_6 \geq 107 \\ &3X_3 - 18X_4 + 30X_7 \leq 81 \end{aligned}$$

2

$$\begin{array}{lllll} 4X_1 & -5X_3 & & +X_6 & = -13 \end{array}$$

$-10 \le X_1 \le -2$

$16 \le X_3$

$X_4 \le 0$

X_5 unrestricted in sign

X_2, X_6, and $X_7 \ge 0$

(3.) DUAL SIMPLEX METHOD: Consider the simplex tableau below:

-Z	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	
1	3	4	2	1	5	0	0	0	0	(MIN)
0	1	-2	-1	1	1	1	0	0	-3	
0	-1	-1	-1	1	1	0	1	0	-2	
0	1	1	-2	2	-3	0	0	1	4	

(a.) What is the current primal solution? Is it feasible? Does it satisfy the optimality conditions?

(b.) Circle every possible dual simplex pivot element.

(4.) UPPER BOUNDING TECHNIQUE. Consider the following LP problem:

$$\begin{array}{ll} \text{MINIMIZE} & 2X_1 + 2X_2 + 3X_3 - X_4 \\ \text{subject to} & X_1 - X_2 + X_3 - 2X_4 \le 6 \\ & -X_1 + X_2 - X_3 + X_4 \le 8 \\ & 2X_1 + X_2 - X_3 \ge 2 \\ & 0 \le X_1 \le 3 \\ & 1 \le X_2 \le 4 \\ & 0 \le X_3 \le 10 \\ & 2 \le X_4 \le 5 \end{array}$$

The APL output solving this problem using the Upper-Bounding Technique is attached. Please refer to it to answer the following questions.

(a.) Explain why in the initial basic solution, i.e.

X_1	X_2	X_3	X_4	X_5	X_6	X_7
0.5	1	0	2	10.5	5.5	0

the value of the basic variables (X_1, X_5, X_6) are **NOT** given by the quantity $(A^B)^{-1}b$, which is equal to (1, 5, 9).

(b.) Explain how it is determined that, as the nonbasic variable X_4 is increased, the basic variable X_5 increases, while the basic variable X_6 decreases.

(c.) Explain how the "blocking values" 44.75 and 5.5 were computed.

(d.) Why does X_4 not enter the basis?

(e.) Explain why, if the basis does not change, the basic solution at the second iteration differs from that at the first iteration.

(f.) Explain why the algorithm terminates with an "optimal solution" if the reduced costs are not all non-negative.

PART TWO

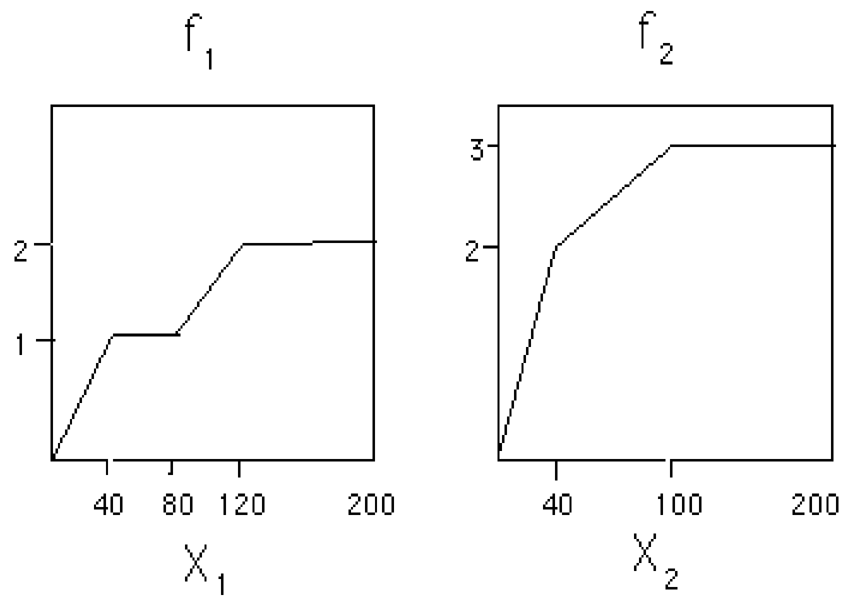
(1.) SEPARABLE PROGRAMMING: Consider the nonlinear programming problem

MINIMIZE $f_1(X_1) + f_2(X_2)$

subject to

$$X_1 + X_2 \geq 100$$
$$2X_1 + 3X_2 \geq 200$$
$$X_1 - 2X_2 \geq 50$$
$$0 \leq X_1 \leq 200, \quad 0 \leq X_2 \leq 200$$

where the functions f_1 and f_2 are piecewise linear as shown below.



The "lambda" formulation of the problem was first defined, with

$$X_1 = 0\lambda_{10} + 40\lambda_{11} + 80\lambda_{12} + 120\lambda_{13} + 200\lambda_{14}$$

$$X_2 = 0\lambda_{20} + 40\lambda_{21} + 100\lambda_{22} + 200\lambda_{23}$$

(a.) When the ordinary simplex method is used, the solution found is λ_{10}=0.58, μ_{14}= 0.42, λ_{20} = 0.92, λ_{23} = 0.08 (other variables being zero). What is wrong with this solution?

(b.) Below are reproduced the initial and the current tableaux, when a "restriced basis entry" rule is used. What is the corresponding value of X_1 and of X_2 for the current tableau?

(c.) Indicate (by circling) every possible pivot element in the current tableau which might improve the solution (if any).

(d.) Reformulate the problem, using the "delta" formulation method. Write X_1 and X_2 as expressions in the delta variables.

(e.) What are the values of the "delta" variables corresponding to the current solution in the "lambda" (i.e. λ) tableau?

(f.) If we are using the "upper bounding technique", which delta variables are in the basis at this iteration? Which delta variables are candidates for entry into the basis (i.e. variables for which the reduced cost must be computed)?

(2.) DANTZIG-WOLFE DECOMPOSITION. We wish to use the decomposition technique to solve the following problem:

$$
\begin{aligned}
&\text{MINIMIZE} \quad -X_1 - X_2 - 2X_3 - X_4 \\
&\text{subject to} \\
&\quad X_1 + 2X_2 + 2X_3 + X_4 \le 40 \\
&\quad -X_1 + X_2 + X_3 + X_4 \le 10 \\
&\quad X_1 + 3X_2 \le 30 \\
&\quad 2X_1 + X_2 \le 20 \\
&\quad X_3 \le 10 \\
&\quad X_4 \le 10 \\
&\quad X_3 + X_4 \le 15 \\
&\quad X_1,\ X_2,\ X_3, \text{ and } X_4 \ge 0
\end{aligned}
$$

It was decided to use two subproblems, one with variables (X_1,X_2) and the other with variables (X_3,X_4), writing the feasible region of each as a combination of its extreme points. The APL output solving this problem is attached.

(a.) Write the first master problem tableau (after adding proposals 1 and 2).

(b.) Write the second master problem tableau (after adding proposals 3 and 4).

(c.) What are the values of X_1 through X_4 found by the **second** master problem? (Note: these values were blacked out in the output!)

(d.) What is the objective function for subproblem 1 at the next iteration (i.e. after the second master problem has been solved)?

(3.) STOCHASTIC PROGRAMMING. The Eye-to-Eye Frozen Food Company has the following problem: Potatoes are processed into packages of French Fries, Hash Browns, and Flakes (for instant mashed potatoes). At the beginning of the manufacturing process, the raw potatoes are sorted by length and quality, and then allocated to the separate product lines.

The company can purchase potatoes from two sources, which differ in their yields of various sizes and quality. These yield characteristics are as follows: Source 1 potatoes yield 20 percent french fries, 20 percent hash browns, and 30 percent flakes (the remaining 30 percent being waste). Source 2 potatoes yield 30 percent, 10 percent, and 30 percent, respectively (with again 30 percent being waste). The profit per ton of potatoes processed from Source 1 and Source 2 are \$500 and \$600, respectively.

The limit on sales was first estimated to be 18 tons of French Fries, 12 tons of Hash Browns, and 24 tons of Flakes, and the problem was formulated as an LP as follows:

$$\begin{aligned} \text{MAXIMIZE } & 500X_1 + 600X_2 \\ \text{subject to } & 0.2X_1 + 0.3X_2 \le 18 \\ & 0.2X_1 + 0.1X_2 \le 12 \\ & 0.3X_1 + 0.3X_2 \le 24 \\ & X_1, X_2 \ge 0 \end{aligned}$$

In reality, however, the sales limits are random variables and not known with certainty.

(a.) CHANCE CONSTRAINED PROGRAMMING: Suppose that the sales limits given above are the expected values for normal distributions, with standard deviations of 4, 2, and 6, respectively. Reformulate the problem with the restriction that the sales limit be exceeded for each product no more than 5 percent of the time. (The Standard Normal CDF evaluated at 1.65 is approximately 0.95)

(b.) STOCHASTIC LP WITH RECOURSE: Now suppose that the sales limit for Flakes is known with certainty to be 24, but that the others have only two possible values each. The sales limit for French Fries is either 15 or 20 (with equal probability). The sales limit for Hash Browns is either 10 (with probability 40 percent) or 20 (with probability 60 percent). If the sales limit of either French Fries or Hash Browns are exceeded, then the surplus must be sold at a loss of \$100 per ton for French Fries, and \$200 per ton for Hash Browns. (If less than the sales limit is produced, then no recourse is necessary.)

The production **must be scheduled** before learning the sales limits. Formulate an LP model to compute the production schedule which maximizes the expected profit.

(4.) LP USING LINEAR COMPLEMENTARY SOLUTION TECHNIQUE. Consider the original LP model given in problem (3) for the Eye-to-Eye Frozen Food Company.

(a.) Write down the primal and the dual problems, both using only equality constraints.

(b.) Write down the tableau containing both sets of equality constraints from part (a.)

(c.) Start with slack and surplus variables in the basis (negating rows with surplus variables as required). Is this solution feasible? Optimal? Does it satisfy complementary slackness?

(d.) Define a **single** artificial variable and insert its column into the tableau.

(e.) Where should you pivot to enter the artificial variable into the basis? (circle the entry) What variable leaves the basis? Will complementary slackness be satisfied after this pivot?

(f.) What variable or variables may now enter the basis at the next iteration?

(g.) How do you decide when to terminate, since you have no row of reduced costs?

(5.) ANALYSIS OF **MPSX** OUTPUT: Please refer to your materials on the **PURAIR OIL COMPANY** (problem statement, formulation, and MPSX output).
Answer the following questions (if there is insufficient information in the MPSX output, simply answer NOT ENOUGH INFORMATION):

(a.) Pipeline A is used at full capacity in the optimal solution. Suppose that it were to be shut down for a portion of a day, so that the capacity that day were only 8000 barrels/day, rather than 9000 barrels/day. How much profit would be lost? How would the optimal solution be changed? (refer to the substitution rates)

(b.) GR1 denotes the amount of Gasoline produced by Refinery #1. Suppose that it must be produced in multiples of 100 barrels. Should the solution be rounded **up** or **down**? How much loss in profit would result? Which nonbasic variable must be adjusted (upward or downward) in order to accomplish this rounding?

(c.) C1R1 denotes the amount of Crude #1 sent (via tanker) to Refinery #1 at a cost of $0.30/barrel. How high might this shipping cost rise before the solution needs to be adjusted? If the cost reaches this value, how much less will be shipped?

(d.) GR2M3 denotes the amount of Gasoline shipped from Refinery #2 to Market #3, at a cost of $0.25/barrel. In the optimal solution, no such shipment is made. By how much must the shipping cost drop before it would be optimal to make this shipment? How much would then be shipped? What other changes would have to be made in the optimal solution?