1. Network Simplex Method. Consider the minimum-cost network flow problem below:


Positive numbers at the nodes represent supplies, and negative numbers represent demands. Numbers on the edges represent unit shipping cost. Consider each undirected edge to be equivalent to a pair of directed edges, i.e., the shipments may be directed either way. (The arc directed to the right at node E is an artificial arc, with no flow allowed.)

We begin with the basis (spanning tree) shown, with the dual variable $\mathrm{W}_{\mathrm{E}}=0$.
Find:
Flow $\mathrm{X}_{\mathrm{EC}}=$ $\qquad$
Dual variables $\mathrm{W}_{\mathrm{B}}=$ $\qquad$ $\& \mathrm{~W}_{\mathrm{C}}=$ $\qquad$
The reduced costs $\bar{C}_{B C}=$ $\qquad$ $\& \bar{C}_{C B}=$ $\qquad$
Suppose that the $\operatorname{arc}(\mathrm{B}, \mathrm{C})$ is entered into the basis (i.e., the spanning tree).

- Which arc will be replaced? $\qquad$
- What will be the flow in arc (B,C)? $\qquad$
- What will be the value of the dual variable $\mathrm{W}_{\mathrm{C}}$ after the basis change (assuming we keep $\mathrm{W}_{\mathrm{E}}$ $=0$ )? $\qquad$
What is the node-arc incidence matrix of the original spanning tree shown above?

|  |  |  |  | $\mathbf{0}$ |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  | $\mathbf{0}$ |
|  |  |  |  | $\mathbf{0}$ |
|  |  |  |  | $\mathbf{0}$ |
|  |  |  |  | $\mathbf{1}$ |

(The last column corresponds to the "artificial arc" from node E.)

