Name ________________________________

56:272 Integer Programming & Network Flows
Quiz #4 – September 24, 2003

Consider the following ***zero-one knapsack problem***, with a capacity of 8 units of weight:

item #	Weight	Value
1	5	9
2	3	5
3	2	3
4	4	7

The knapsack problem is solved by ***Branch-&-Bound***, with the LP relaxation providing the upper bound. First the items are sorted by $ per unit weight:

i	W	V	V/W	r
1	5	9	1.8	1
4	4	7	1.75	2
2	3	5	1.667	3
3	2	3	1.5	4

The results of the branch-and-bound search are:

```
 ->Subproblem # 1
Forced in:
Forced out:
Free:         1  2  3  4
Fractional solution: selected items = __A__
                    plus __B__ of item #__C__
                    value = 14.25
                    Rounding down yields value 9
   ->Subproblem # 2
   Forced in:    4
   Forced out:
   Free:         1  2  3
   Fractional solution: selected items = 4
                       plus 0.8 of item # 1
                       value = 14.2
                       Rounding down yields value 7
      ->Subproblem # 3
      Forced in:    1  4
      Forced out:
      Free:         2  3
      Infeasible!
      <-Subproblem # 3 fathomed.
      ->Subproblem # 4
      Forced in:    4
      Forced out:   1
      Free:         2  3
      Fractional solution: selected items = 2 4
                          plus 0.5 of item # 3
                          value = 13.5
                          Rounding down yields value 12
         ->Subproblem # 5
         Forced in:    3  4
         Forced out:   1
         Free:         2
         Fractional solution: selected items = 3 4
```

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                          plus 0.6667 of item # 2
                          value = 13.33
                          Rounding down yields value 10
               ->Subproblem # 6
               Forced in:     2  3  4
               Forced out:    1
               Free:
               Infeasible!
               <-Subproblem # 6 fathomed.
               ->Subproblem # 7
               Forced in:     3  4
               Forced out:    1  2
               Free:
               Integer solution: selected items = 3 4
                                   Value= 10
               <-Subproblem # 7 fathomed.
            <-Subproblem # 5 fathomed.
            ->Subproblem # 8
            Forced in:     4
            Forced out:    1  3
            Free:          2
            Integer solution: selected items = 2 4
                                Value= 12
            <-Subproblem # 8 fathomed.
         <-Subproblem # 4 fathomed.
      <-Subproblem # 2 fathomed.

->Subproblem # 9
   Forced in:
   Forced out:   4
   Free:         1  2  3
   Integer solution: selected items = 1 2
                       Value= 14
   <-Subproblem # 9 fathomed.
<-Subproblem # 1 fathomed.
Done. # subproblems = 9
```

At the top of the tree, i.e., subproblem #1, the solution of the ***LP relaxation*** is:
X_1 = ____ , X_2 = ____ , X_3 = ____ , X_4 = ____ , that is, include all of item(s) A = ____ and the fraction B = ______ of item C = ____. At this time, the upper bound is ____ and the lower bound is _____ . The intial ***incumbent*** value is _____ .
When beginning to consider subproblem #5, the current incumbent value is _____ , found in subproblem #____ .

Which subproblems are fathomed because the upper bound is no better than the incumbent? __________

Which subproblems are fathomed because the LP relaxation is infeasible? _____________

Which subproblems are fathomed because the LP relaxation has an integer solution? _________