Name ____________________________

56:272 Integer Programming & Network Flows
Quiz #11 – Fall 2003

___ 1. When applying Benders' method to the capacitated plant location problem, the "master" problem...
   a. evaluates the total cost if a specified set of plants are open
   b. selects the next trial set of plants to be open
   c. gives an upper bound on the cost of the optimal solution
   d. *none of the above.*

___ 2. The subproblem of Benders' decomposition algorithm applied to the capacitated plant location problem…
   a. finds solutions which, if feasible, must be optimal.
   b. produces a lower bound on the optimal value of the original problem.
   c. produces an upper bound on the optimal value of the original problem.
   d. *none of the above*

___ 3. Johnson's algorithm is to solve…
   a. assembly-line balancing problems
   b. flowshop scheduling problems
   c. traveling salesman problems
   d. *none of the above*

___ 4. The quadratic assignment problem...
   a. includes quadratic constraints.
   b. has the same constraints as the original assignment problem.
   c. includes X_{ij}^2 terms in the objective function.
   d. is a specialized form of the "generalized assignment problem" (GAP).
   e. *none of the above.*

True (+) or False (o)?

___ 5. Simulated annealing is a heuristic method which searches in a "neighborhood" of the current solution and may replace the current solution with a neighbor even if the neighbor has a higher cost (assuming a minimization problem).

___ 6. Simulated annealing is a randomized search algorithm, which may give different results each time it is applied.

___ 7. A "flowshop" differs from a "jobshop" in that all jobs in a flowshop follow the same sequence of machines, although each job's processing times will vary (and could be zero for some machines.)

Consider the following jobs which have arrived at a flowshop:

Job	A	B	C	D	E
Time, Machine 1	7	3	4	1	5
Time, Machine 2	4	5	2	3	6

8. In what sequence should the jobs be processed in order to complete all jobs in the shortest possible time? _____, _____, _____ , _____, _____
9. The quantity being minimized in the preceding question (8) is called the _____________ .

Benders' Decomposition of Capacitated Plant Location Problem: Consider the problem of determining which one or more of four possible plants should be built in order to serve 6 customers at minimum cost. (Four of the plant sites are adjacent to customer locations.) The data are:

	Customer 1	Customer 2	Customer 3	Customer 4	Customer 5	Customer 6	Plant Capacity	Fixed cost
Plant 1	0	17	77	43	93	52	10	8544
Plant 2	17	0	61	40	76	36	14	4050
Plant 3	77	61	0	60	30	39	15	1917
Plant 4	43	40	60	0	87	61	11	396
Demand	2	2	10	1	10	4		

(Total demand= 29)

Name ___________________________

Benders' decomposition is used to solve this problem, using the variation in which the master problem is not optimized-- instead a solution, if any, is found which is better than the incumbent).

<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><>

We begin by solving the subproblem with the trial set of plants {1,2,3,4}, i.e., build all four plants:

```
                           Solution of
                       Transportation Problem

Plants opened: # 1 2 3 4
Minimum transport cost                         =   674
Fixed cost of plants                           = 14907
Total              = 15581
*** New incumbent!
                              Optimal Shipments
                               |1 2  3 4 5 6  7
                              1|2 0  0 0 0 0  8
                              2|0 2  0 0 5 4  3
                              3|0 0 10 0 5 0  0
                              4|0 0  0 1 0 0 10

(Demand pt #7 is dummy demand for excess capacity.)

                             |  Dual Solution  |
                             |of Transportation|
                             |     Problem     |
Supply constraints
                              i=      1  2 3  4
                              U[i]=  46 46 0 46
Demand constraints
                         j=      1  2 3   4  5  6
                         V[j]=  ¯46 ¯46 0 ¯46 30 ¯10
Generated support is αY+β,
where β= ¯936, and
                                    i α[i]
                                    1 9004
                                    2 ____   ← missing value!
                                    3 1917
                                    4  902
This is support # 1
```

9. What is the missing value of α_2 in the first support that is generated? __________

<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><>

Next we solve the Master problem to get a new trial set of plants to be built:

```
                           Master Problem
Initial status vector: J= φ (empty)

(suboptimized, i.e., a solution Y such that v(Y) < incumbent.)

Trial set of plants: 2 3
with estimated cost 5675 < incumbent ( = 15581)

Current status vectors for Balas' additive algorithm: J = {3, 2, -1, -4}
```

<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><><>

Next the 2^{nd} subproblem is solved using the new trial set of plants (2 & 3):

```
Plants opened: # 2 3

Minimum transport cost                                   =  748
Fixed cost of plants                                     = 5967
 Total              = 6715
*** New incumbent! ***  (replaces 15581)

Optimal Shipments
                               |1 2  3 4 5 6 7
                              2|2 2  0 1 5 4 0
                              3|0 0 10 0 5 0 0

(Demand pt #7 is dummy demand for excess capacity.)

                             |  Dual Solution  |
                             |of Transportation|
                             |     Problem     |

Supply constraints
                                i=     1  2 3 4
                                U[i]=  0 46 0 0
Demand constraints
                           j=     1   2   3 4  5  6
                           V[j]=  0 ¯29 ¯46 0 ¯6 30

Generated support is αY+β,
where β= 104, and
                                     i α[i]
                                     1 8544
                                     2 4694
                                     3 1917
                                     4  396
This is support # 2
```

<><><><><><><><><><><><><><><><><><><><><><><><><><><><><><>

The master problem is solved again:

```
                              Master Problem

Initial status vector: J = {3, 2, -1, -4}

*** No solution with v(Y) less than incumbent! ***
(Current incumbent: 6715, with plants #2 3 open)
```

10. Using the two supports that have been generated, what cost $\underline{v}_2(Y)$ will be estimated for the solution Y= (1,0,1,0), i.e., building plants 1 & 3? ______________

11. This is *(check:)* ___ over-estimate ___ under-estimate ___ cannot be determined