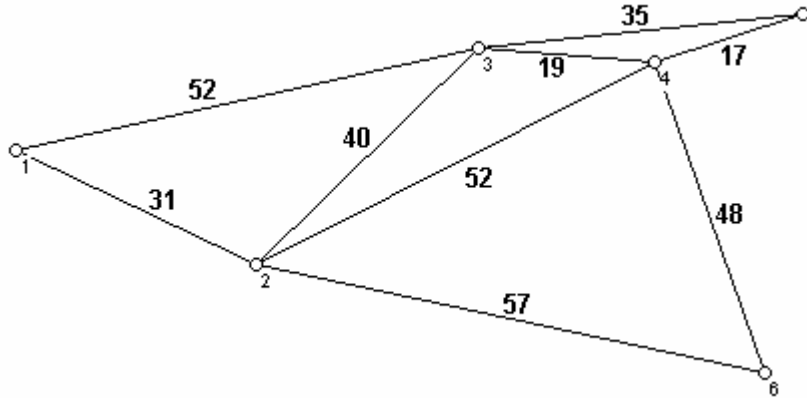


56:272 Integer Programming & Network Flows  
 Quiz #9 – Fall 2003

1. . **Location in a network:** Consider the network,



where the numbers on the edges are distances. The demand at the nodes are all equal, which we may consider to be 1 unit. The table of shortest path lengths found by Floyd's algorithm is:

	1	2	3	4	5	6
1	0	31	52	71	87	88
2	31	0	40	52	69	57
3	52	40	0	19	35	67
4	71	52	19	0	17	48
5	87	69	35	17	0	65
6	88	57	67	48	65	0

The following information is available about the function  $\sigma(x)$  (the objective function for the **center** problem) on the edge (2,6):

**Monotonically increasing** distance functions:  $d(x,k)$  where

k=	1	2
$d(i,k)$	31	0
$d(j,k)$	88	57

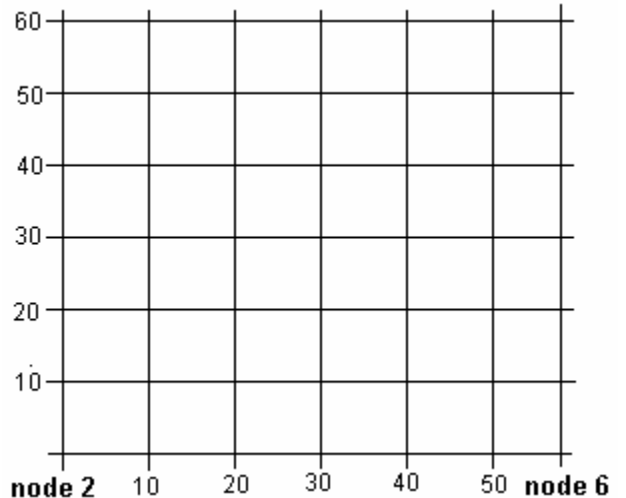
**Monotonically decreasing** distance functions:  $d(x,k)$  where

k=	6
$d(i,k)$	57
$d(j,k)$	0

Distance functions which increase to a **peak** at a point  $\Delta$  units from  $i$ , then decrease:  $d(x,k)$  where

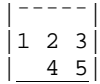
k=	3	4	5
$d(i,k)$	40	52	69
$d(j,k)$	67	48	65
$\Delta$	42	26.5	26.5

Plot the distance function from location  $x$  (on edge (2,6)) to node #4:



An office manager has five available rooms (numbered 1,...5) to which he wants to assign five departments (labeled A,...E, also referred to as “facilities”). He has data which indicates the number of persons per unit time traveling between the departments (the “interfacility flow matrix” below-- since these are round trips, the matrix is symmetric.) Distances are in units equal to the length of the side of the square rooms, measured between centers of the rooms. He wishes to assign the departments to the rooms so as to minimize the weighted sum of the trips (i.e., weighted by the distances traveled).

Room Layout



(5 rooms are equal in area)

Interfacility Flow Matrix

Distance Matrix

o	1	2	3	4	5
1	0	1	2	2	3
2	1	0	1	1	2
3	2	1	0	2	1
4	2	1	2	0	1
5	3	2	1	1	0

o	A	B	C	D	E
A	0	0	2	2	4
B	0	0	4	3	4
C	2	4	0	5	3
D	2	3	5	0	2
E	4	4	3	2	0

Density = 90.00%

- Complete the blanks:** The QAP problem will have  binary variables, while the equivalent integer LP will have an additional  binary variables, at total of .
- Define one binary variable of the *quadratic* assignment problem. (Insert some subscripts and state the meaning of the variable.)  
 $X_{\underline{\quad}} = 1$  if.... (complete definition)  
 0 otherwise
- Write one of the equality constraints for the quadratic programming model to solve this problem:
- Define one of the binary variables in the integer LP for solving this problem:  
 $Y_{\underline{\quad}} = 1$  if.... (complete definition)  
 0 otherwise
- Write one of the inequality constraints for the integer LP for solving this problem:
- In the solution shown below, what is the cost of the flow between departments B and C?

***Simulated annealing result:***

Solution: Random QAP (seed= 933504)

Facility Location

Facility	Room#
A	5
B	1
C	2
D	4
E	3



Costs: 80 =sum of weighted distances

