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@ 56:272 Integer Programming & Network Flows @
@@@ Exam #2 - November 9, 1994 @@@

Answer any four of the five problems.

	possible	score	
1. Traveling Salesman Problem	25	_____	
2. Chinese Postman Problem	25	_____	
3. Median of Network	25	_____	
4. Center of Network	25	_____	Total
5. Primal Simplex Method for Networks	25	_____	_____

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1. Traveling Salesman Problem. Five products are to be manufactured weekly on the same machine. The table below gives the cost of switching the machine from one product to another product. (Assume that this is also the cost of switching to the last product of the week to the first product to be scheduled the following week!)

from: \ to:	A	B	C	D	E
A	-	6	5	7	6
B	3	-	2	8	3
C	4	2	-	7	3
D	1	3	1	-	5
E	5	3	2	6	-

a. The nearest neighbor heuristic, starting with product A, yields the product sequence ___________ with cost ____.

After row & column reduction of the above matrix to solve the associated assignment problem (with large number, M, inserted along the diagonal), we have:

from: \ to:	A	B	C	D	E
A	M	1	0	2	0
B	1	M	0	6	1
C	2	0	M	5	1
D	0	2	0	M	3
E	3	3	0	0	M

b. Is there a zero-cost assignment for the above reduced cost matrix? _______

c. If the answer to (b) is "no", perform additional reduction steps as necessary. What is the solution of this assignment problem? A-> ___ , B-> ___ , C-> ___ ,D-> ___ ,E-> ___

c. What is the cost of this assignment? _________

d. Is it a valid product sequence? _______

e. If yes, is it guaranteed to be optimal? _____

If not, why not? _________________

What bound (circle: upper / lower) on the optimal cost does this result provide? ________

What single constraint might be added to the assignment problem which would eliminate the solution which you have obtained (but not eliminate any valid sequence)?

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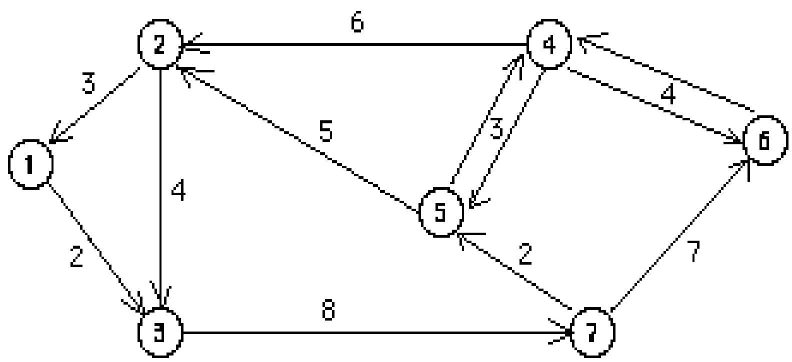
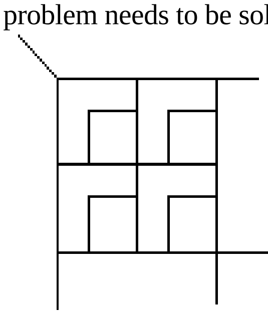
2. Postman Problem. Consider the street network given below, where the numbers alongside the streets are the lengths (in hundreds of feet). All but two of the streets are one-way, as indicated. The truck can pick up the garbage on both sides of the one-way streets on the same traversal of the street. On two-way streets, the truck must travel each direction once, picking up one side at a time.

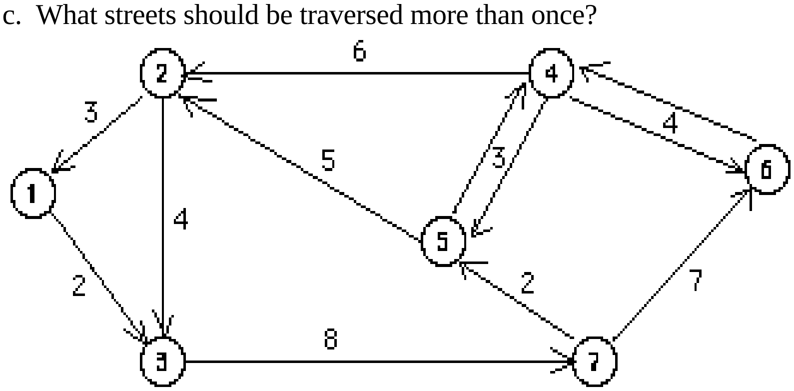


a. Why can't this be done without traversing some street(s) more than once?

b. What transportation problem needs to be solved to find the optimal truck route?



c. What streets should be traversed more than once?



d. Starting at intersection #1, list the intersections on an optimal route:
__->__->__->__->__ ->__->__->__->__->__-> __->__->__->__->__ etc.

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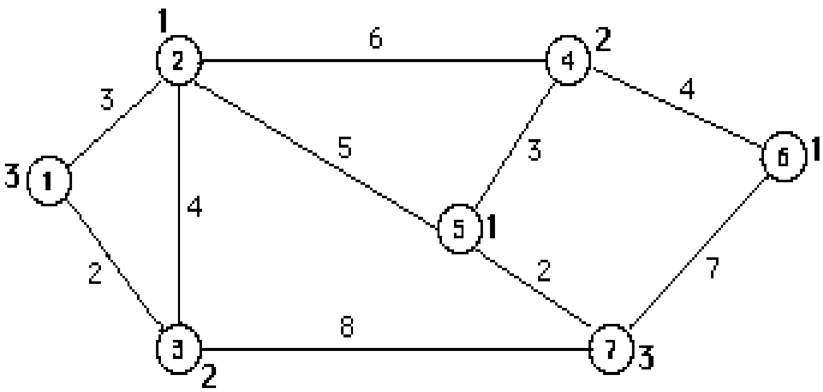
3. The Median Plant Location Problem: Consider the network below, where the bold numbers beside the nodes are the demands to be supplied.



Floyd's algorithm was applied to find the following matrix of shortest path lengths:

Shortest Path Lengths

from \ to	1	2	3	4	5	6	7
1	0	3	2	9	8	13	10
2	3	0	4	6	5	10	7
3	2	4	0	10	9	14	8
4	9	6	10	0	3	4	5
5	8	5	9	3	0	7	2
6	13	10	14	4	7	0	7
7	10	7	8	5	2	7	0

Weighted Shortest Path Lengths

from \ to	1	2	3	4	5	6	7
1	0	3	2	18	8	26	30
2	6	0	4	12	5	20	21
3	4	4	0	20	9	28	24
4	18	6	10	0	3	8	15
5	16	5	9	6	0	14	6
6	26	10	14	8	7	0	21
7	20	7	8	10	2	14	0

a. Formulate the 2-median problem as a binary integer LP.

The addition/substitution heuristic was applied to try to find the 2-median set, giving the output below:

K-median
Facility Location
Problem

<u>1-Median:</u>

Location 5
Cost = $\boxed{a}$

<u>2-Median:</u>

<u>Trial additions:</u>

Add	1	2	3	4	6	7
cost	31	36	34	44	42	49

Addition result: Locations 5 & $\boxed{b}$
Cost: $\boxed{c}$

Substitution Step

Cost	Locations
60	2 1
36	5 2
79	3 1
34	5 3
31	4 1
44	5 4
41	6 1
42	5 6
31	7 1
49	5 7

Substitution result: Locations $\boxed{d}$ & $\boxed{e}$
Cost: $\boxed{f}$

b. Six values are blanked in the output of the addition/substitution heuristic. What are these values?

- **a**. _________ (the cost of the 1-median set {5})
- **b**. _________ (the facility added to the 1-median set {5})
- **c**. _________ (the cost of the solution after the addition step)
- **d & e**. ________ (the pair of facilities resulting from the substitution step)
- **f.** _________ (the cost of the final solution)

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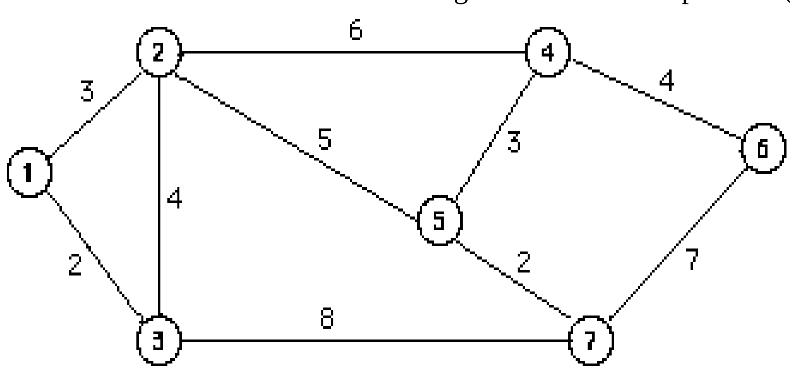
4. Center of Network. Consider again the network of problem (4):



Shortest Path Lengths

from \ to	1	2	3	4	5	6	7
1	0	3	2	9	8	13	10
2	3	0	4	6	5	10	7
3	2	4	0	10	9	14	8
4	9	6	10	0	3	4	5
5	8	5	9	3	0	7	2
6	13	10	14	4	7	0	7
7	10	7	8	5	2	7	0

a. Find the vertex **center** for the network. _______

b. Below is some output displaying a lower bound which may be computed for the center objective function on each edge. What is the missing value?

Lower Bounds on Local Edge-Centers

i	j	LB
1	2	10
1	3	12.5
2	3	10
2	4	7
2	5	7
3	7	8
4	5	
4	6	10
5	7	8.5
6	7	8.5

d. Which edges can be eliminated from consideration when searching for the absolute center?

e. Below is information about the center objective function on the edge (4,5). What are the three missing values?

Minimax Objective on Edge (4,5)

Monotonically increasing distance functions: d(x,k) where

k=	4	6
d(i,k)=	0	4
d(j,k)=	3	a

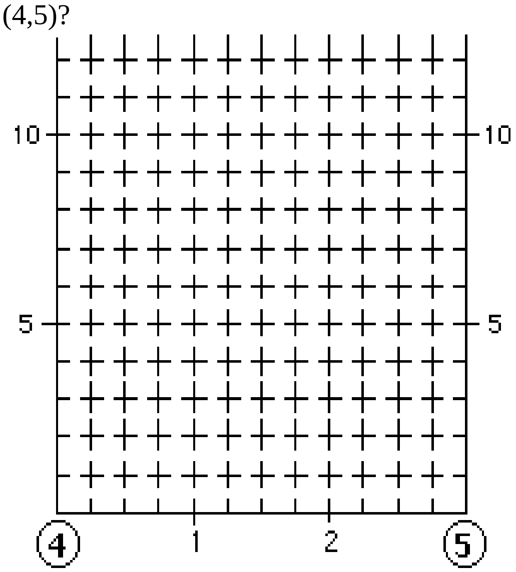
Monotonically decreasing distance functions: d(x,k) where

k=	5	7
d(i,k)=	3	5
d(j,k)=	0	2

Distance functions which increase to a peak at a point Δ units from i, then decrease: d(x,k) where

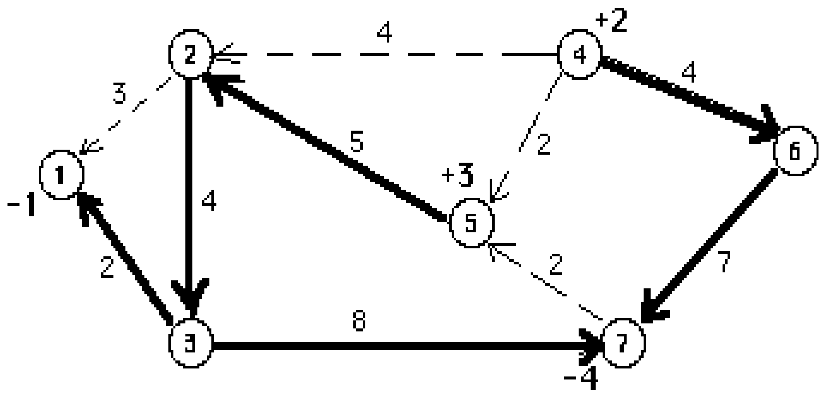
k=	1	2	3
d(i,k)=	9	6	b
d(j,k)=	8	5	9
Δ=	c	1	1

h. Sketch the center objective function on the edge (4,5). What is the edge center of the edge (4,5)?



5. Primal Simplex Algorithm for Networks.

Consider the network below, where the number alongside each node represents supply or demand, i.e., node #4 has a supply of 2 units of a commodity, node #5 has 3 unit, node #1 requires 1 unit, and node #7 requires 4 units. The numbers alongside the arcs represent unit shipping costs. The initial feasible solution is shown in bold.



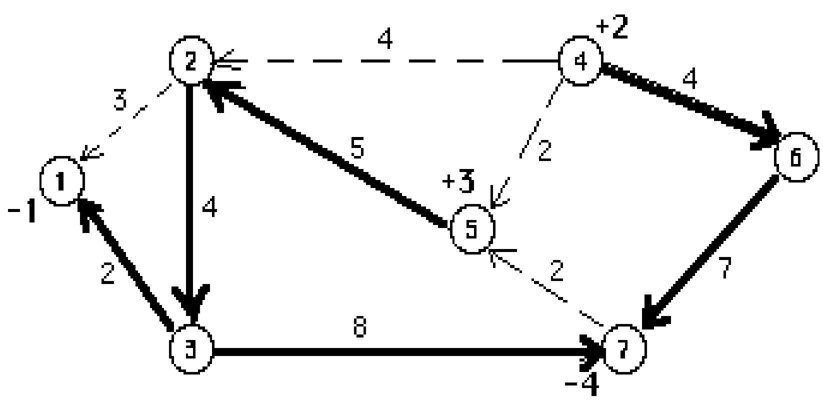
a. The node-arc incidence matrix will have ____ rows and _____ columns.

b. If an LP is formulated to find the minimum-cost flow, how many rows are there in the constraint matrix? _____ How many columns? _______

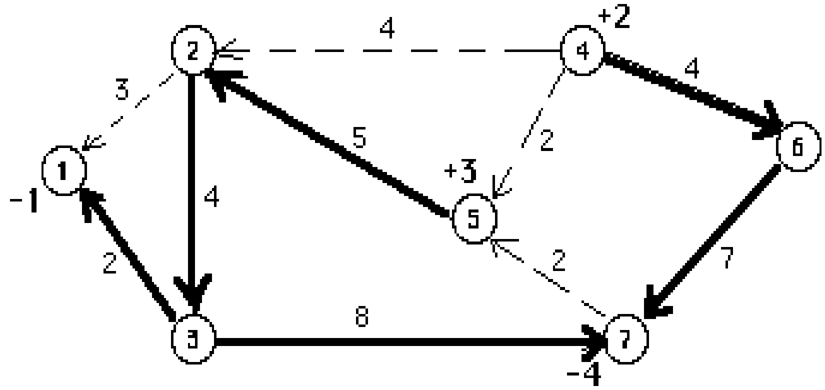
c. In order to obtain a complete basis of the LP, an "artificial" arc must be added. Indicate it above.

d. Write the node-arc incidence matrix of the subgraph representing the above basis of the LP.

e. Using the minimum spanning tree (plus artificial "root" arc) as an initial basis, compute the corresponding basic solution, i.e., flows. Indicate these flows below:



f. Using the same basis, compute the dual variables (simplex multipliers), and indicate below:



g. "Price" the arc (2,1), i.e., compute its reduced cost. Should this arc enter the basis or not?

h. Regardless of whether the arc (2,1) should enter the basis, enter the arc into the basis and indicate the new basis on the network below: