@56:272 Integer Programming & Network Flows @
@@@ Exam #1 - September 28, 1994 @@@

Answer all of Part 1, plus any <u>three</u> of the remaining four parts.

	possible	score
1. Multiple Choice	10	_____
2. Transportation Problem	10	_____
3. Assignment Problem	10	_____
4. Knapsack Problem	10	_____
5. Miscellaneous	10	_____

Total _____ / 40

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1. Multiple Choice:

___ 1. Floyd's algorithm for a graph with n nodes ...
- a. finds the shortest paths from a single source node to each of the other nodes.
- b. requires exactly n iterations to be performed.
- c. is a specialized version of the LP simplex algorithm.
- d. none of the above.

___ 2. If the current solution of the transportation problem is degenerate...
- a. the next iteration will produce no improvement in the objective function.
- b. the reduced cost of at least one zero shipment is zero.
- c. the number of sources must be equal to the number of destinations.
- d. none of the above.

___ 3. A node-arc incidence matrix of an undirected graph with n nodes ...
- a. has n rows.
- b. is a square matrix.
- c. has n columns.
- d. none of the above.

___ 4. The adjacency matrix of an <u>directed</u> graph with n nodes ...
- a. is a square matrix.
- b. is a symmetric matrix.
- c. has +1, -1, and 0 as entries.
- d. none of the above.

___ 5. The LP formulation of the problem to find the shortest path in a network ...
- a. has two variables for each project activity.
- b. may require branch-and-bound if the solution is not integer.
- c. has a dual LP which finds the longest path in a network.
- d. none of the above.

___ 6. The LP model for an nxn assignment problem...
- a. has an integer optimal solution only if the costs are integer.
- b. has 2n basic variables.
- c. has only degenerate basic feasible solutions.
- d. none of the above

___ 7. The following is true of an n-item zero-one knapsack problem with integer weights, item values, and capacity...
- a. when solving by branch-&-bound, the # of terminal nodes in the complete search tree is 2^n
- b. in the DP model, the state variable has 2^n possible values
- c. in the DP model, the number of stages is 2^n
- d. none of the above

___ 8. When using the Hungarian method to solve assignment problems, if the number of lines drawn to cover the zeroes in the reduced matrix is smaller than the number of rows, ...
  a. a mistake has been made, and one should review previous steps.
  b. this indicates that no solution exists.
  c. this means that an optimal solution has been reached.
  d. none of the above.

___ 9. A minimum spanning tree of an undirected network with n nodes ...
  a. can never be given a strongly-connected orientation.
  b. contains no nodes of degree 2
  c. has 2n-1 edges.
  d. none of the above

___ 10. Vogel's Approximation Method...
  a. always yields a basic feasible solution of a transportation problem.
  b. cannot be applied to an assignment problem, because of degeneracy.
  c. will never result in a degenerate solution.
  d. none of the above

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2. Transportation Problem: Consider the transportation problem with the tableau below:

	A	B	C	D	E	supply
F	1	6	3	4	4	4
G	1	3	2	5	6	2
H	2	2	1	1	4	5
demand	3	2	2	3	1	

a. If the ordinary simplex tableau were to be written for this problem, it would have ___ rows, plus the objective row, and ____ columns (in addition to -z and the right-hand-side).

b. This problem will have ____ basic variables (not including -z).

c. Find an initial basic feasible solution using the "Northwest Corner Method" *(write the values of the variables in the tableau above.)*

d. What are the values of the dual variables for the solution in (c)? (Note that V_E has been assigned the value zero)? U_F = _____ , U_G = _____ , U_H = _____ ,
V_A = _____ , V_B = _____ , V_C = _____ , V_D = _____ , V_E = __0_ .

e. What is the reduced cost of the variable X_{GA}? ____________

f. Will increasing X_{GA} improve the objective function? ____________

g. Regardless of whether the answer to (f) is "yes" or "no", what variable must leave the basis if X_{GA} enters?__________

h. What will be the value of X_{GA} if it is entered into the solution as in (g)? ____________

i. Which variables (if any), if it were entered into the solution, would result in a degenerate solution? ________
Circle all that apply: none X_{GA} X_{HA} X_{HC} X_{GE} X_{FE}

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3. Assignment Problem: Consider the assignment problem with the following costs:

from: \ to:	1	2	3	4	5
A	2	3	5	7	6
B	3	5	1	8	2
C	4	1	2	8	3
D	1	3	1	4	5
E	5	3	2	6	2

a. After applying row and column reduction, as in the "Hungarian Method", we have:

from: \ to:	1	2	3	4	5
A	0	1	3	2	4
B	2	4	0	4	1
C	3	0	1	4	2
D	0	2	0	0	4
E	3	1	0	1	0

b. Test whether any further reductions are necessary, and if so, perform them and indicate the reduced matrix below:

from: \ to:	1	2	3	4	5
A	___	___	___	___	___
B	___	___	___	___	___
C	___	___	___	___	___
D	___	___	___	___	___
E	___	___	___	___	___

c. What are the optimal assignments? A--> ___ , B--> ___ , C-->___ , D-->___ , E-->___

d. What is the optimal cost? _________

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4. Knapsack Problem: A "knapsack" is to be filled to maximize the value of the contents, subject to a weight restriction:

```
Number of items: 4
Capacity of Knapsack: 10
Maximum units of any item to
   be included is 1
```

i	W	V
1	3	5
2	5	9
3	2	3
4	4	7

W = 'weight' of item
V = value of item

Sorted by Value per Unit Weight

i	W	V	V/W	r
2	5	9	1.8	1
4	4	7	1.75	2
1	3	5	1.66667	3
3	2	3	1.5	4

V/W = value per unit weight
r = rank by V/W

a. Formulate this problem as a 0-1 ILP problem:

b. Write the formulation of the LP relaxation of this problem.

Output of the branch-&-bound algorithm for this problem appears below:

```
→→→Subproblem # 1
Forced in:
Forced out:
Free:          1  2  3  4
Fractional solution: selected items = 2 4
                     plus ________ of item #___
                     value = 17.6667
Rounding down yields value 16
*** NEW INCUMBENT! ***
   →→→Subproblem # 2
   Forced in:    1
   Forced out:
   Free:         2  3  4
   Fractional solution: selected items = 1 2
                        plus 0.5 of item # 4
                        value = 17.5
   Rounding down yields value 14
      →→→Subproblem # 3
      Forced in:    1  4
      Forced out:
      Free:         2  3
      Fractional solution: selected items = 1 4
                           plus 0.6 of item # 2
                           value = 17.4
      Rounding down yields value 12
         →→→Subproblem # 4
         Forced in:    1  2  4
         Forced out:
         Free:         3
         Infeasible!
         ←←←Subproblem # 4 fathomed.
         →→→Subproblem # 5
         Forced in:    1  4
         Forced out:   2
         Free:         3
         Integer solution: selected items = 1 3 4
                              Value= 15
         ←←←Subproblem # 5 fathomed.
      ←←←Subproblem # 3 fathomed.
      →→→Subproblem # 6
      Forced in:    1
      Forced out:   4
      Free:         2  3
      Integer solution: selected items = 1 2 3
                           Value= 17
      *** NEW INCUMBENT! ***
      ←←←Subproblem # 6 fathomed.
   ←←←Subproblem # 2 fathomed.
   →→→Subproblem # 7
   Forced in:
   Forced out:   1
   Free:         2  3  4
   Fractional solution: selected items =__________
                        plus ____ of item #___
                        value =_____
   Rounding down yields value 16
   ←←←Subproblem # 7 fathomed.
←←←Subproblem # 1 fathomed.
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b. Complete the blanks in the output above.

c. On the back of this page, draw the search tree which was used, using the information in the above output to number the nodes, etc.

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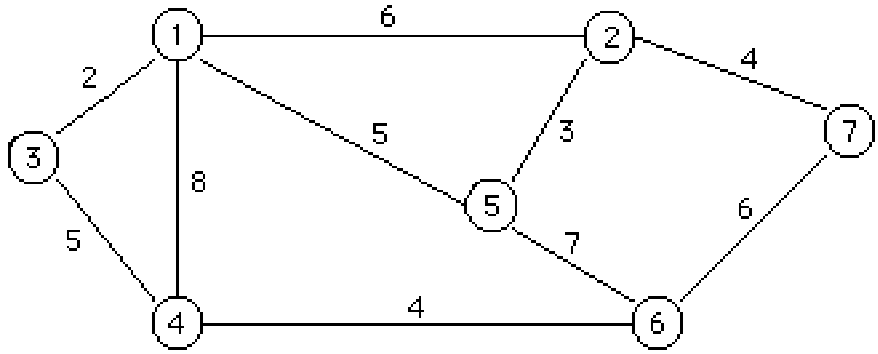
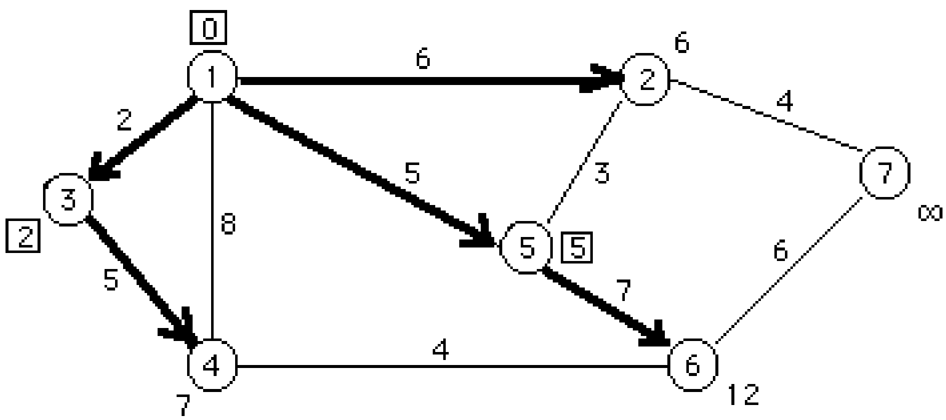
5. Miscellaneous. Consider the undirected network below, with the edge lengths indicated:



a. Find the minimum spanning tree of this network, and indicate it above.

Suppose that we wish to find the shortest paths from node #1 to each of the other nodes. After several iterations, we obtain the results below (where squares are drawn around permanent labels.)

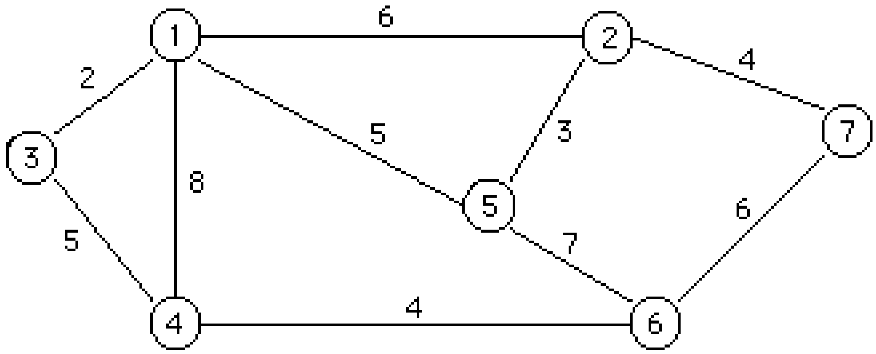
b. How many iterations have been performed to obtain the results below? ________



c. If we stop at this iteration, are we certain of having found the shortest path from node 1 to node 5? _____

If we stop at this iteration, are we certain of having found the shortest path from node 1 to node 7? _____

d. Complete (exactly!) two additional iterations, and indicate the results below:



e. If we stop at this iteration, are we certain of having found the shortest path from node 1 to node 5? _____

If we stop at this iteration, are we certain of having found the shortest path from node 1 to node 7? _____