A. Kusiak, *Engineering Design: Products, Processes, and Systems*, Academic Press, San Diego, CA, 1999.

CHAPTER 4

RELIABILITY OF PROCESS MODELS

1. INTRODUCTION

Reliability analysis of an IDEF3 model identifies critical activities in the model, improves its performance, and decreases operating cost of the process. Such an IDEF3 model can be complex as the model may include a large number of activities. This complexity warrants applications of the reliability evaluation techniques.

In the recent years, attempts have been made to apply the IDEF methodology for analysis of system reliability and project risk assessment. Ang and Gray (1993) examined the adequacy of IDEF methodology and suggested a number of modifications and enhancements in order to improve IDEF descriptive power for project risk assessment. Kusiak and Larson (1994) integrated techniques for analysis of system reliability with an IDEF model. Kusiak and Zakarian (1996a) developed a fault tree based methodology for reliability and risk assessment of parent activities in IDEF3 models. This chapter is based on Kusiak and Zakarian (1996) and it extends the system reliability evaluation techniques, such as, the reduction method, and minimum path and cut sets methods to reliability evaluation of IDEF3 models.

The relationship between activities in IDEF3 is modeled with three types of links: precedence, object flow, and relational. The precedence and object flow links expresses the simple temporal precedence between activities. The relational links highlight the existence of a relationship between activities. The logic of branching within a process is modeled using an *AND* (&), *OR* (O), and *exclusive OR* (X) junction boxes. To represent the reduction approach for reliability evaluation a serial and parallel system modeled with IDEF3 graphical syntax is discussed next. Assume the states of activities are statistically independent.

1) Parallel system modeled with an AND logical junction

The activities following an *AND* junction are performed in parallel. Therefore, the system fails if any of the parallel activities fails (see Figure 1).

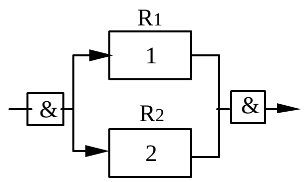


Figure 1. Parallel system with an *AND* logical junction

2) Serial system modeled with precedence, object flow, or relational links

For the activities arranged in a series, the system fails if any of the serial activities fails (see Figure 2(a), 2(b), and 2(c)).

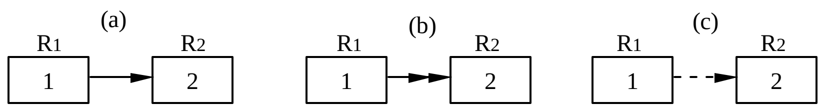


Figure 2. Serial system: (a) precedence link, (b) object flow link, and (c) relational link

The system reliability of parallel activities following an *AND* junction, and the system reliability of serial activities connected by precedence, object flow, or relational links is determined from (1)

$$R_s = \prod_{j=1}^{n} R_j \qquad (1)$$

where: R_s is the reliability of the system and R_j is the reliability of activity j.

The system reliability for each of the four cases in Figure 2 and 3 is

$$R_s = R_1 R_2$$

3) Parallel system modeled with an exclusive OR logical junction

In an *exclusive OR* junction box only one of the several parallel activities is carried out (see Figure 3). Each activity has a certain probability of occurrence.

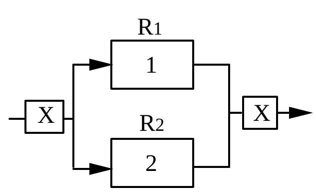


Figure 3. Parallel system an *exclusive OR* logical connector

The system reliability of parallel activities in Figure 3 is determined from (2)

$$R_s = \sum_{j=1}^{n} P_j R_j \qquad (2)$$

where: R_s is the system reliability, P_j is an occurrence probability of activity j, and $\sum_{j=1}^{n} P_j = 1$. For the system in Figure 4, $R_s = P_1R_1 + P_2R_2$, and $P_1 + P_2 = 1$.

3) Parallel system modeled with an OR junction box

In an *OR* junction box the activities are arranged in parallel and system functions if k out of n parallel activities in the system function. The value of k depends from the underlying process. To illustrate the reduction approach and minimum path and cut sets methods, assume that for an *OR* junction, the system functions if 1 out n parallel activities functions (see Figure 4).

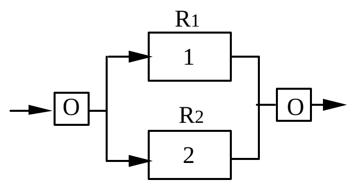


Figure 4. Parallel system modeled with an *OR* junction box

The system reliability for the parallel activities in Figure 4 is determined from (3)

$$R_s = P\left[\sum_{j=1}^{n} x_j \geq k\right] \qquad (3)$$

where, $x_j = \begin{cases} 1 & \text{if activity j functions} \\ 0 & \text{if activity j fails} \end{cases}$

For the process in Figure 4 the reliability is calculate as follows:

$R_s = P(x_1 = 1, x_2 = 1) + P(x_1 = 1, x_2 = 0) + P(x_1 = 0, x_2 = 1) = R_1 R_2 + R_1(1 - R_2) + R_2(1 - R_1) = R_1 + R_2 - R_1 R_2$

where: $P(x_j = 1) = R_j$ is the probability that activity j functions, and $P(x_j = 0) = (1 - R_j)$ is the probability that activity j fails.

In the next section, three different methods for reliability evaluation are discussed:

- Reduction method
- Minimum path set method
- Minimum cut set method

2. THE REDUCTION APPROACH

The basic idea behind the reduction approach applied to an IDEF3 model is to reduce its size by combining appropriate parallel *AND*, *OR*, and *exclusive OR* branches, and the activities connected by precedence, object flow, or relational links until a single equivalent activity is obtained. This equivalent activity represents the reliability of the original IDEF3 model. To illustrate the reduction approach, consider the IDEF3 model of the packaging process presented in Figure 5.

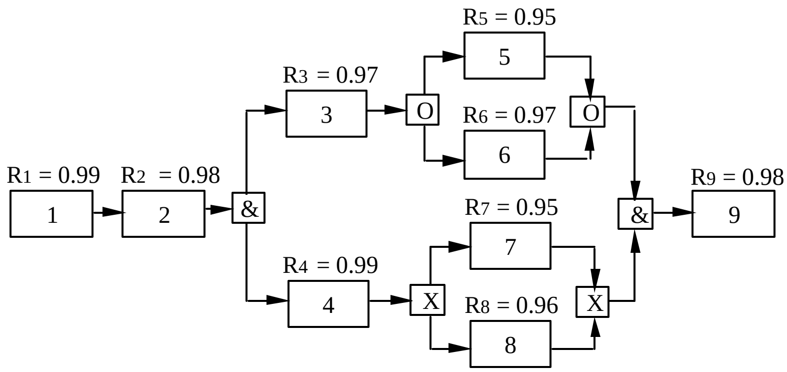


Figure 5. Packaging design process represented with an IDEF3 model

In the first reduction, activities 5 and 6 are combined in a single activity 56. Since activities 5 and 6 immediately follow an *OR* junction box, the reliability of the equivalent single activity 56 is determined as $R_{56} = R_5 + R_6 - R_5R_6 = 0.9985$

In the second reduction, one can combine activities 7 and 8 in an equivalent activity 78. The reliability of the equivalent activity 78 is $R_{78} = P_7R_7 + P_8R_8 = 0.957$ where: $P_7 = 0.3$ and $P_8 = 0.7$ is the occurrence probability for activity 7 and 8, respectively, and $P_7 + P_8 = 1$. Following the above logic, the reliability of the IDEF3 model in Figure 5 is $R_s = R_1R_2R_3R_4R_{56}R_{78}R_9 = 0.8725$.

3. THE MINIMUM PATH AND MINIMUM CUT SET METHODS

The reduction approach presented in the previous section can be used if the number of activities in an IDEF3 model is relatively small. When a process model is large, more efficient methods based on the notion of a *path set* and a *cut set* are used.

A minimum path set of an IDEF3 model is the minimum set of activities whose functioning ensures the functioning of the model. Consider the IDEF3 process model in Figure 5. There are four minimum path sets {1, 2, 3, 4, 5, 7, 9}, {1, 2, 3, 4, 5, 8, 9}, {1, 2, 3, 4, 6, 7, 9}, {1, 2, 3, 4, 6, 8, 9} exist in this system. One can see in Figure 5, that system will function if all the activities of at least one minimum path set are functioning. Therefore, the structure function of the system in Figure 5 is defined by (4) (Barlow and Proschan 1981).

$$\varnothing(x) = \max\{\min\{x_1, x_2, x_3, x_4, x_5, x_7, x_9\}, \min\{x_1, x_2, x_3, x_4, x_5, x_8, x_9\}, \min\{x_1, x_2, x_3, x_4, x_6, x_7, x_9\}, \min\{x_1, x_2, x_3, x_4, x_6, x_8, x_9\}\} \quad (4)$$

where, $\varnothing = \begin{cases} 1 & \text{if the IDEF 3 model functions} \\ 0 & \text{if the IDEF 3 model fails} \end{cases}$

$$x_j = \begin{cases} 1 & \text{if activity j functions} \\ 0 & \text{if activity j fails} \end{cases}$$

The structure function $\varnothing(x)$ for path sets of the model in Figure 5 is represented in Figure 6.

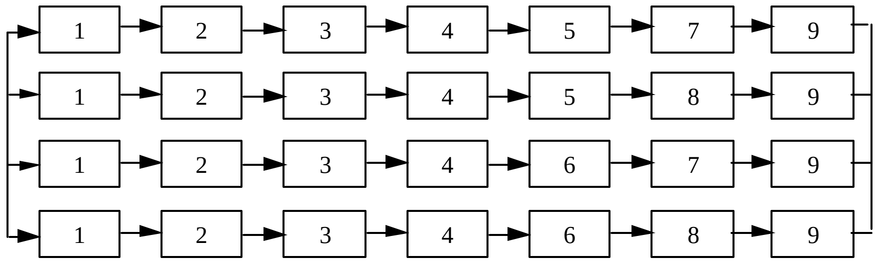


Figure 6. The representation of structure function ∅(x) for path sets

A minimum cut set of an IDEF3 model is the minimum set of activities, whose failure leads to the failure of the model. Therefore, the model fails if at least one minimum cut set is not functioning. There are seven minimum cut sets in the IDEF3 process model in Figure 5, namely {1}, {2}, {3}, {4}, {5, 6}, {7, 8}, {9}. Hence, the structure function ∅(x) of this IDEF3 model is defined in (5).

$$\varnothing(x) = \min\{\max\{x_1\}, \max\{x_2\}, \max\{x_3\}, \max\{x_4\} \max\{x_5, x_6\}, \max\{x_7, x_8\}, \max\{x_9\}\} \quad (5)$$

The representation of ∅(x) for cut sets of the model in Figure 5 is shown in Figure 7.

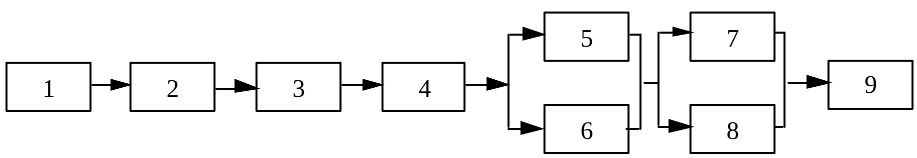


Figure 7. The representation of structure function ∅(x) for cut sets

The minimum path and minimum cut representations developed above provide means for systematically computing the reliability of the system R_s simply by taking the expectation of structure function (Barlow and Proschan 1981), that is

$$R_s = E(\varnothing(x)) \quad (6)$$

Assuming in equation (4) the probability of occurrence of paths {1, 2, 3, 4, 5, 7, 9} and {1, 2, 3, 4, 6, 7, 9} equals P_7, and the probability of occurrence of paths {1, 2, 3, 4, 5, 8, 9} and {1, 2, 3, 4, 6, 8, 9} is P_8, an upper bound on the reliability of IDEF3 process model can be obtained by setting $P_7 = P_8 = 1$. The latter is equivalent to replacing an *exclusive OR* junction with an *OR* junction box. Furthermore, one may see that in determining the cut sets of the IDEF3 model in Figure 5 the *exclusive OR* junction box is treated as an *OR* junction. Replacing it with an *AND* junction leads to a lower bound of the reliability of the IDEF3 process model. In this case the minimum cut sets are: {1}, {2}, {3}, {4}, {5, 6}, {7}, {8}, {9}.

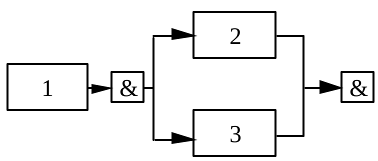
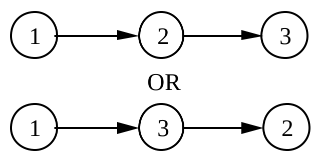
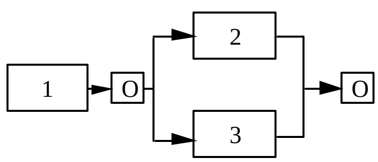
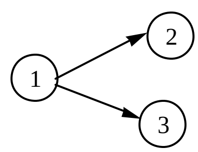
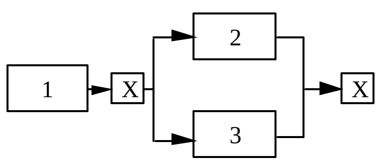
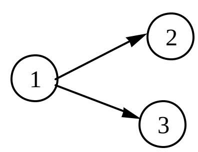
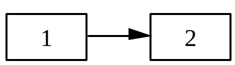
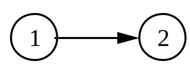
3.1. The Path Tree Algorithm

In this section, a path tree algorithm for deriving the minimum path and cut sets of an IDEF3 model is presented.

The reliability network of an IDEF3 process model corresponds to the original model except that the junction boxes are replaced with edges representing the corresponding logical boxes. Table 1 presents the elementary IDEF3 graphical components and the corresponding reliability network.

The two reliability networks for an *AND* junction presented in Table 1 are equivalent from the reliability point of view, however, they are different from the process flow prospective. Using the representation from Table 1, one can obtain the reliability network corresponding to the IDEF3 process model in Figure 5 (see Figure 8).

Table 1. IDEF3 graphical components and corresponding reliability network

IDEF3 Graphical Components	Equivalent Reliability Network
	
	
	
	

The terminology used by an algorithm to determine path sets is presented.

Terminology:

Start point — Node in the network that has only outgoing arrows (O point).

End point — Node in the network that has only incoming arrows (D point).

Minimum path — A path from the start point O to the end point D where no node is traversed more than ones.

Minimum cut set — A set with minimum number of activities whose failure ensures the failure of the end point D.

Tree — A representation that shows the relationship among all activities in the model from the start point O to the end point D.

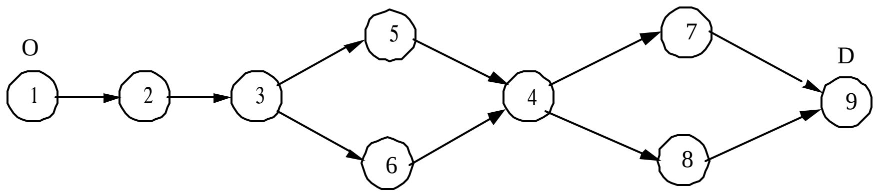


Figure 8. Reliability network of the IDEF3 model in Figure 5

The algorithm to determine the path sets in an IDEF3 model is presented next.

Algorithm 1

Step 1: Begin the start (origin) point O at the top of the tree (level k = 1).
Step 2: Place the activities connected to the start point O at the next level (level k = 2).
Step 3: Obtain level k + 1 activities connected to the activities at level k.
Step 4: If there is no more activities at the level k + 1, set k = k + 1 and repeat step 3.
Step 5: Repeat steps 3 and 4 until each activity at the last level is the end point D.

To illustrate Algorithm 1, consider the reliability network of the IDEF3 process model in Figure 5 shown in Figure 8.

In step 1 of the algorithm, the start point 1 is placed on the top of the tree (level 1). There is only one activity 2 connected to the start point 1. Therefore, this activity is placed at the next level (level 2). Following the steps of the algorithm, the tree in Figure 9 is obtained. It is seen in Figure 9 that each lowest level activity corresponds to the end point D = 9.

From the tree in Figure 9 one can easily obtain the minimum pat sets of the IDEF3 model, i.e., {1, 2, 3, 4, 5, 7, 9}, {1, 2, 3, 4, 5, 8, 9}, {1, 2, 3, 4, 6, 7, 9}, {1, 2, 3, 4, 6, 8, 9}. Those are the paths which connect the start point O = 1 to end point D = 9 in the tree.

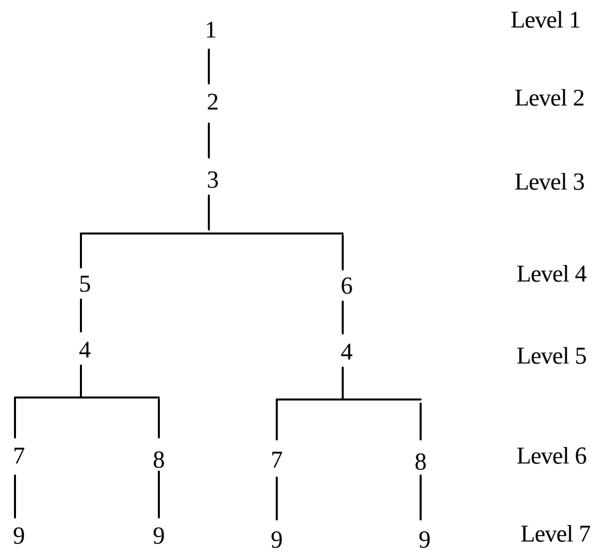


Figure 9. Tree structure for the network in Figure 8

Algorithm 1 presented above identifies the minimum path sets where no cycles exist in an IDEF3 model. An extension of Algorithm 1 for generation of minimum path sets of an IDEF3 model with cycles is presented next.

Algorithm 2

Step 1: Start with the start point O at the top of the tree (level k = 1).
Step 2: Place the activities connected to the start point O at the next level (level k = 2).
Step 3: Obtain level k +1 activities connected to activities at level k.
Step 4: If any activity obtained at level k + 1 is already included in a path, disregard this activity.
Step 5: If an activity obtained at level k + 1 is the same as any activity obtained at the higher level of any other path, fathom the path.
Step 6: If there is no more activities at the level k + 1, set k = k + 1 and repeat steps 3 - 5 until each activity at the last level is fathomed or corresponds to the end point D.

To illustrate Algorithm 2, consider the IDEF3 process model in Figure 10 and the corresponding reliability network in Figure 11.

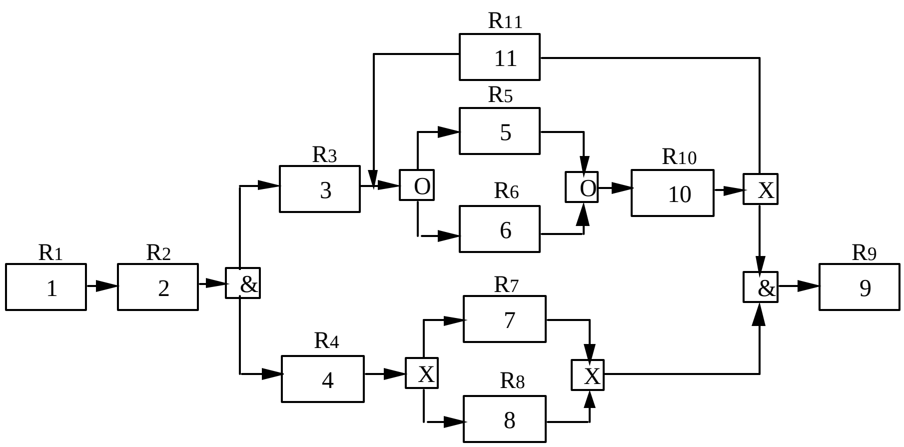


Figure 10. IDEF3 process model

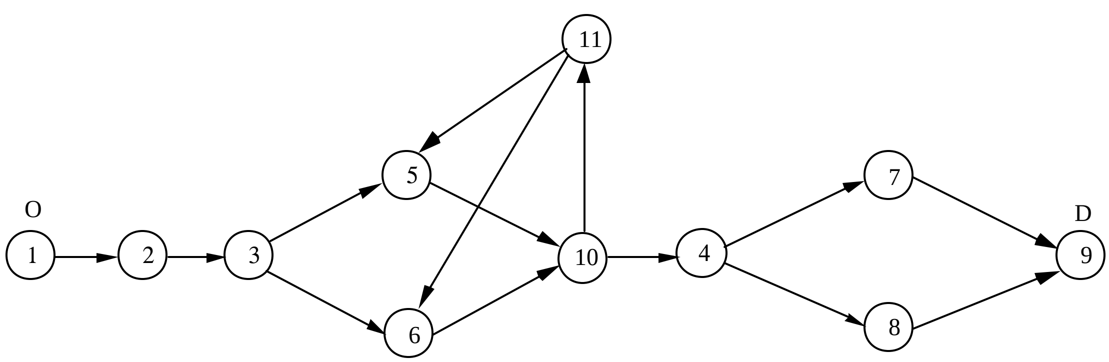


Figure 11. Reliability network of the IDEF3 model in Figure 10

Following the steps of Algorithm 2, the tree in Figure 12 is obtained. One can see from Figure 12 that each lowest level activity is either fathomed or corresponds to the end point D.

From tree in Figure 12 one can obtain the minimum pat sets of the IDEF3 model, i.e., {1, 2, 3, 5, 10, 4, 7, 9}, {1, 2, 3, 5, 10, 4, 8 9}, {1, 2, 3, 6, 10, 4, 7, 9}, {1, 2, 3, 6, 10, 4, 8, 9}, {1, 2, 3, 5, 10, 11, 4, 7, 9}, {1, 2, 3, 5, 10, 11, 4, 8, 9},{1, 2, 3, 5, 10, 11, 6, 4, 7, 9}, {1, 2, 3, 5, 10, 11, 6, 4, 8, 9}, {1, 2, 3, 6, 10, 11, 4, 7, 9}, {1, 2, 3, 6, 10, 11, 4, 8, 9}, {1, 2, 3, 6, 10, 11, 5, 4, 7, 9}, {1, 2, 3, 6, 10, 11, 5, 4, 8, 9}. Those are the paths that connect the start point O to end point D in the tree by expanding the fathomed activities whenever it is appropriate.

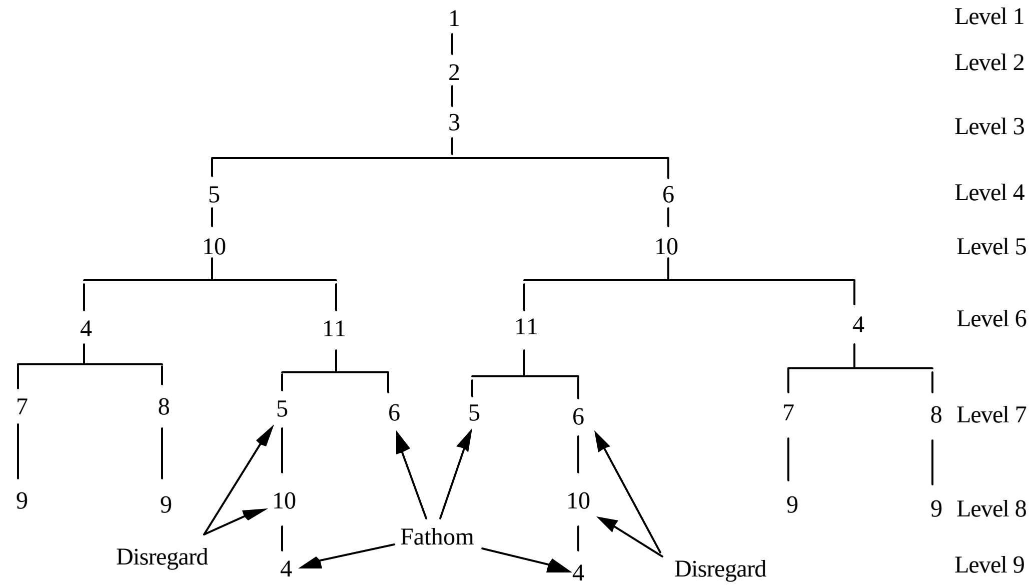


Figure 12. Tree structure for the network in Figure 11

Note that only the first repeating sequence, e.g., (5, 10) of the path set {1, 2, 3, 5, 10, 11, 4, 7, 9}, has been shown in the tree in Figure 12.

3.1.1. Film Deposition Case Study

Figure 13 shows an IDEF3 model of the film deposition process (Nguyen and Bachner 1987). Knowing the reliability of the activities involved in the process in Figure 13, the reduction approach presented earlier in this chapter determines the reliability of the process as $R_s = R_1R_{234567}R_8 = 0.884$, where $R_{234567} = (P_2R_2R_4) + (P_3R_3R_{567})$, $R_{567} = R_5 + R_6R_7 - R_5R_6R_7$, and $P_2 = 0.45$, $P_3 = 0.55$ is the occurrence probability for activity 2 and 3, respectively, $P_2 + P_3 = 1$. This result could not be obtained with the traditional reliability formulas due to logical connectors involved in the process model. In the system in Figure 2, a circuit is formed through the execution of a set of appropriate activities. For example, one such set may include activities {1, 2, 4, 8} (MOD film technology). Other possible subsets of activities guaranteeing the formation of a circuit are the sets {1, 3, 5, 8} and {1, 3, 6, 7, 8} (thin film technology). For a small size model, similar to the one presented in Figure 13, these sets could be possibly obtained by the inspection of the model. However, when the number of activities and logical junctions in the model is large, which is the case of full-scale industrial processes, formal approaches have to be used. Moreover, these sets are successfully used in evaluating the reliability of the system as well as in developing technique for tracking the functional state of a process under, given the state of activities. For example, assuming that in the process in Figure 13 the sputtering and electroplating operations have failed, Algorithm 1 presented in this section determines the only set of activities {1, 3, 6, 7, 8} that guarantees formation of a circuit with the corresponding process reliability $R_s = R_1R_3R_6R_7R_8 = 0.824$.

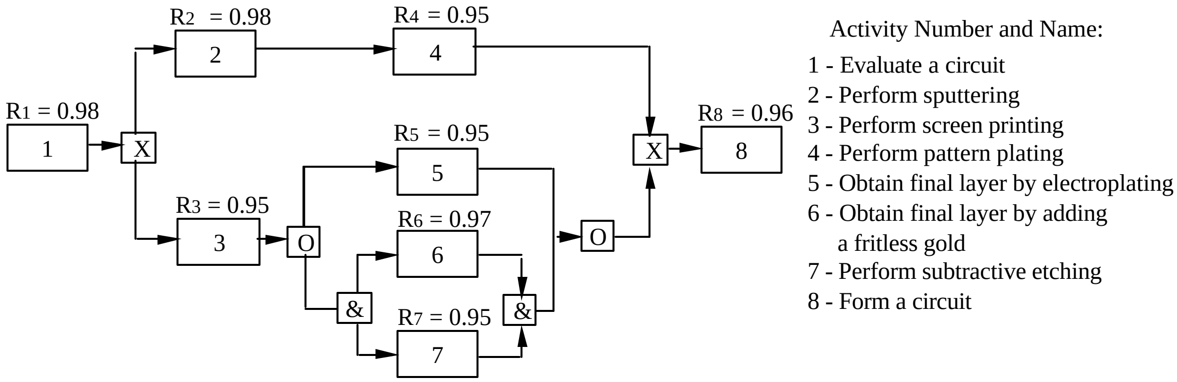


Figure 13. IDEF3 model of the film deposition process

3.2. Evaluation of Minimum Cut Sets

The earlier presented Algorithm 1 and Algorithm 2 generate minimum path sets of an IDEF3 model. The algorithm presented in this section determines the minimum cut sets provides that minimum path sets have been determined. In the literature, the minimum cut sets by combining nodes that break the minimum path sets (Rai and Aggarwal 1978, Rosenthal 1979). The algorithm described next follows the same logic.

Algorithm 3

Step 0: Initialize the current solution set $S_1 = \{Ø\}$, and two working sets $S_2 = \{Ø\}$ and $S_3 = \{Ø\}$.

Step 1: Place activities associated with the start point O and end point D in set S_1.

Step 2: Select a minimum path set. If any activity from the minimum path set selected occurs in the remaining minimum path sets, add this activity to set S_1.

Step 3: Place the activities of the minimum path set that are not in set S_1 in set S_2.

Step 4: Place all the activities of the model that are not in set S_1 in set S_3, and set the number of activities in a q-tuplet, q = 2.

Step 5: Find all possible q-tuplets of the activities in S_2 and S_3. Each q-tuplet is a cut set if each minimum path includes one of its activities. Add the cut sets obtained to the set S_1. If the maximum number of activities in the cut sets generated q < total number n of minimum path sets in the model, then repeat step 5 by creating (q + 1)-tuplets from S_2 and S_3; otherwise, stop.

To illustrate Algorithm 3, consider the IDEF3 reliability network in Figure 8. The minimum path sets obtained from the path tree algorithm are: {1, 2, 3, 4, 5,7, 9}, {1, 2, 3, 4, 5, 8, 9}, {1, 2, 3, 4, 6, 7, 9}, {1, 2, 3, 4, 6, 8, 9}.

Step 0: Initialize: $S_1 = \{Ø\}$, $S_2 = \{Ø\}$, $S_3 = \{Ø\}$.

Step 1: Activities 1 and 9 are added to set S_1. Therefore $S_1 = \{1, 9\}$.

Step 2: The path {1, 2, 3, 4, 5, 7, 9} is obtained.

Activities 2, 3, and 4 appear in each minimum path set.
Therefore, $S_1 = \{1, 2, 3, 4, 9\}$

Step 3: Activities 5 and 7 are included in $S_2 = \{5, 7\}$.

Step 4: The activities of the model that are not included in the set S_1 are: 6 and 8.
Therefore $S_3 = \{6, 8\}$.

Step 5: The possible 2-tuplets are: (5, 6), (5, 8), (7, 6), (7, 8).
Since minimum path set includes one element of the 2-tuplets (5, 6) and (7, 8), therefore $S_1 = \{1, 2, 3, 4, 9, (5, 6), (7, 8)\}$

The maximum number of activities in the cut sets generated $q = 2 <$ total number of minimum path sets $n = 4$. Create 3-tuplets from activities in S_2 and S_3. The 3-tuplets are: (5, 6, 8), (7, 6, 8). Note that the 3-tulets (5, 7, 6), (5, 7, 8) are not listed as they include (5, 7) that belong to the same minimum path set. Since $q = 3 < n$. Create 4-tuplets from the activities in S_2 and S_3. The 4-tuplet is (5, 6, 7, 8). $S_1 = \{1, 2, 3, 4, 9, 56, 78\}$ are the minimum cut sets of the IDEF3 model. One may see that combinations (5, 6, 8), (7, 6, 8), (7, 5, 6), (5, 7, 8), and (5, 6, 7, 8) are not considered, as they include the tuplets (5, 6) and (7, 8).

3.3. The Cut Set - Activity Incidence Matrix

For the cut sets of an IDEF3 model, a cut set - activity incidence matrix $[a_{ij}]$ can be constructed. The cut set - activity incidence matrix includes R_j entries, where R_j is the reliability of activity j in IDEF3 model and indicates that the activity j is included in the minimum cut set i. The cut set - activity incidence matrix allows one to check the functional state of the model under the current state of activities. As soon as the states of the activities are determined, one can obtain the reduced cut set - activity incidence matrix by removing the columns corresponding to all activities that have failed. If every row in the reduced matrix contains at least one positive entry R_j, then the IDEF3 model functions.

$[a_{ij}]$ =

Cut set \ Activity	1	2	3	4	5	6	7	8	9
1	R_1								
2		R_2							
3			R_3						
4				R_4					
5					R_5	R_6			
6							R_7	R_8	
7									R_9

Figure 14. Cut set - activity incidence matrix of IDEF3 model in Figure 5

To illustrate the above concept, consider the cut set - activity incidence matrix in Figure 14. Assume the activities 6 and 7 are not functioning. One can transform matrix $[a_{ij}]$ into $[a^1_{ij}]$ by removing the columns corresponding to non-functioning activities 6 and 7 (see Figure 15). Each row of the matrix $[a^1_{ij}]$ contains at least one positive entry R_j, therefore one may conclude that under the current state of the IDEF3 activities, the process is operational.

Activity

$[a^1_{ij}] =$

	1	2	3	4	5	8	9	
1	R_1							C
2		R_2						u
3			R_3					t
4				R_4				
5					R_5			s
6						R_8		e
7							R_9	t

Figure 15. Transformed matrix $[a^1_{ij}]$

3.4. The Path Set - Activity Incidence Matrix

Using the concept presented in the previous section, one can construct a path set - activity $[P_{ij}]$ incidence matrix for an IDEF3 model. The $[P_{ij}]$ matrix consists of entries R_j, where R_j is the reliability of activity j and indicates that this activity is included in the minimum path set i. The path set - activity incidence matrix allow one not only to check the functional state of the model, but also to calculate the reliability of IDEF3 model under the current state of activities. The algorithm for calculating reliability for an IDEF3 model from $[P_{ij}]$ matrix is presented next.

Algorithm 4

Step 1: Select the columns associated with activities that have failed in the model and draw a vertical lines v_j through each of these columns.

Step 2: For each R_j crossed by the vertical line v_j, draw a horizontal line h_i.

Step 3: Transform the incidence matrix $[P_{ij}]$ into $[P^1_{ij}]$ by removing rows and columns corresponding to all the vertical and horizontal lines drawn in Step 1 and 2.

Step 4: If $[P^1_{ij}]$ is empty, then stop. Model does not function.

Otherwise, calculate $r_i = \prod_j R_j$ for all j of matrix $[P^1_{ij}]$.

Step 5: Calculate the reliability of the new IDEF3 model as $R_s = 1 - \prod_i (1 - r_i)$, where i ranges over all minimum path sets in matrix $[P^1_{ij}]$.

To illustrate Algorithm 4, consider the path set - activity incidence matrix of IDEF3 model in Figure 16. Assume the activities 5 and 7 are not functioning.

Activity

$[P_{ij}] =$

	1	2	3	4	5	6	7	8	9	
1	R_1	R_2	R_3	R_4	R_5		R_7		R_9	Path
2	R_1	R_2	R_3	R_4	R_5			R_8	R_9	set
3	R_1	R_2	R_3	R_4		R_6	R_7		R_9	
4	R_1	R_2	R_3	R_4		R_6		R_8	R_9	

Figure 16. Path set - activity incidence matrix of the IDEF3 model in Figure 5

Steps of Algorithm 4

Step 1: Columns 5 and 7 of matrix $[P_{ij}]$ are selected and vertical lines v_5 and v_7 are drawn.

Step 2: Three horizontal lines h_1, h_2, and h_3 are drawn. The results of Step 1 and 2 are presented in matrix (9).

$$[P_{ij}] = \begin{array}{c} \text{Activity} \\ \begin{array}{c|ccccccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \\ \hline 1 & R_1 & R_2 & R_3 & R_4 & R_5 & & R_7 & & R_9 & h_1 \\ 2 & R_1 & R_2 & R_3 & R_4 & R_5 & & & R_8 & R_9 & h_2 \\ 3 & R_1 & R_2 & R_3 & R_4 & & R_6 & R_7 & & R_9 & h_3 \\ 4 & R_1 & R_2 & R_3 & R_4 & & R_6 & & R_8 & R_9 & \\ \hline & & & & & v_5 & & v_7 & & & \end{array} \end{array} \qquad (9)$$

Step 3: Matrix (9) is transformed into matrix (10).

$$[P^1_{ij}] = \begin{array}{c} \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 6 & 8 & 9 \end{array} \\ \begin{bmatrix} R_1 & R_2 & R_3 & R_4 & R_6 & R_8 & R_9 \end{bmatrix} \end{array} \qquad (10)$$

Step 4: Since matrix (10) is not empty, therefore

$$r_i = \prod_j R_j = R_1 R_2 R_3 R_4 R_6 R_8 R_9$$

Step 5: The reliability of the new IDEF3 model is

$$R_s = 1 - \prod_i (1 - r_i) = R_1 R_2 R_3 R_4 R_6 R_8 R_9 = 0.85$$

4. SUMMARY

Reliability analysis of IDEF3 models is of interest to practitioners and researchers for several reasons. It produces critical activities of the process, improves its performance, and decreases downtime and operating cost of the process. This paper extends the system reliability evaluation techniques, i.e., the system reduction approach and minimum path and cut sets method for reliability evaluation of IDEF3 models. Representation of IDEF3 models as reliability graphs, generation of the minimum path and cut sets of IDEF3 models with a path tree algorithm, and reliability analysis of IDEF3 models are the issues discussed in this chapter. An algorithm for computing reliability of an IDEF3 model from a path set - activity incidence matrix is also presented.

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QUESTIONS

1. Why traditional methods can not be used for reliability evaluation of IDEF3 process models?
2. Name the formulas for evaluation of reliability of process models.
3. What are the three main methods for the reliability evaluation?
4. Which reliability methods can be easily computerized and why?
5. What are the advantages and disadvantages of the reduction approach?
6. When a path set method should be used?
7. When the cut set method should be used?
8. Which method generates a lower bound of the process model reliability?
9. Which method generates an upper bound of the process model reliability?
10. Which algorithm can be used to compute reliability of a process model with cycles?

PROBLEMS

1. List all minimum path sets and cut sets of the IDEF3 model in Figure A1.

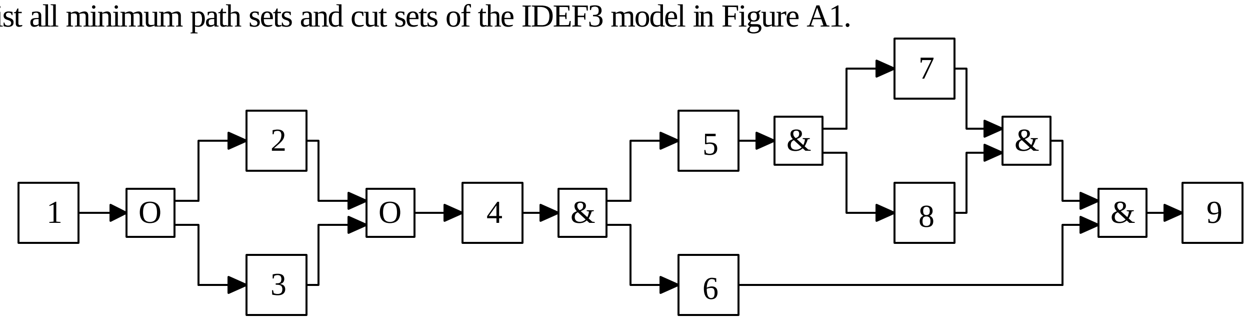

Figure A1. Process model

2. Consider the product realization process represented with the IDEF3 model in Figure A2. Determine the minimum path sets and cut sets. Calculate the overall reliability of the process model with the reduction approach and using the minimum path sets. The probabilities associated with the *exclusive OR* junction are $P_2 = .6$ and $P_3 = .4$.

Activity	Reliability
1	.95
2	.98
3	.90
4	.90
5	.96
6	.92
7	.94
8	.96

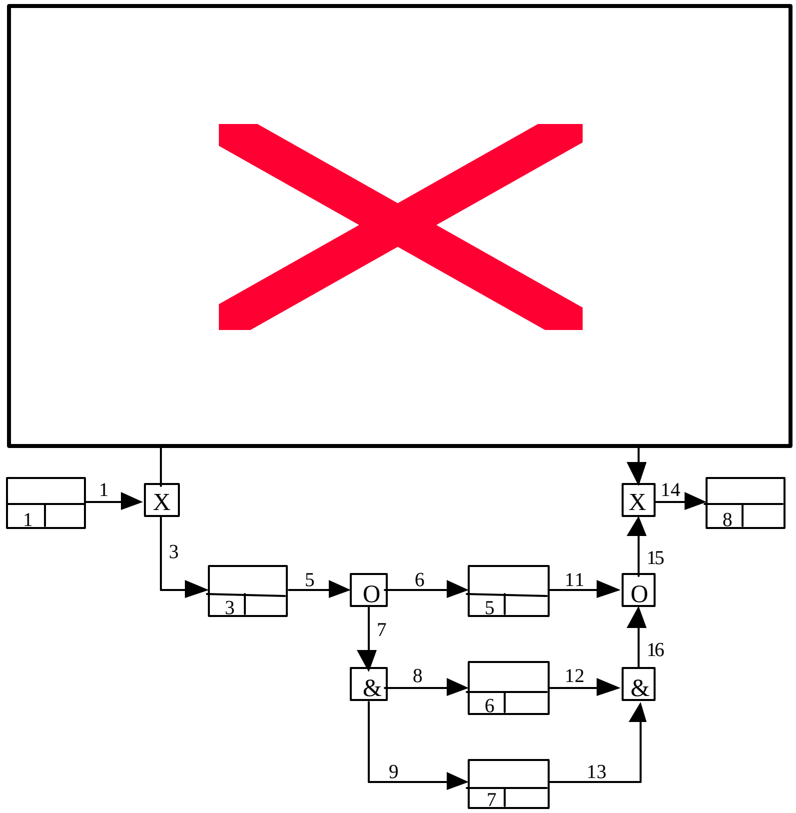

Figure A2. Process model

3. For the product development process represented with the model in Figure A3, determine the minimum path and cut sets. Provide interpretation of the reliability obtained with the two methods. Assume that the reliability of each activity $R_i = 0.95$, $i = 1, ..., 11$.

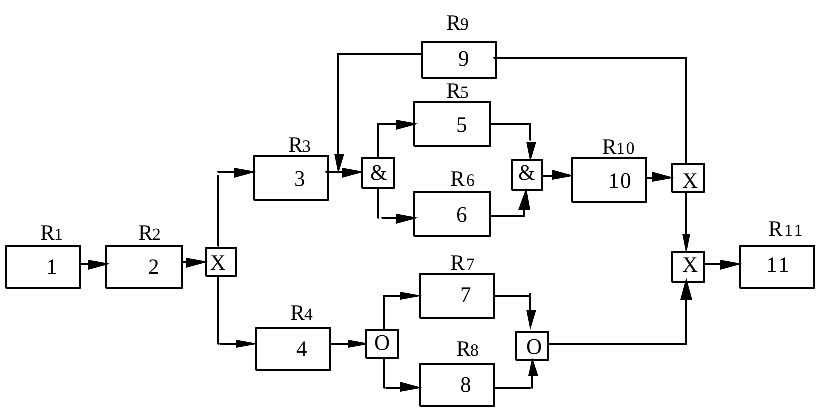

Figure A3. Process model

4. Find the overall reliability of the manufacturing process represented with the model in Figure A4. Use the reduction and minimal path methods. The probabilities associated with the *exclusive OR* junction are $P_{23} = .8$ and $P_{24} = .2$.

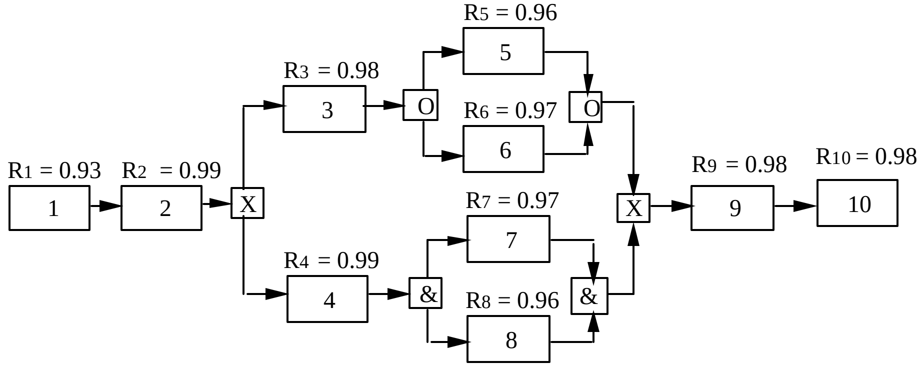

Figure A4. Process model

4. For the process model in Figure A5 and the data in Tables A1 and A2 determine:
(a) The model reliability using the reduction method,
(b) The minimum and maximum value of the process reliability,
(c) What action would you take to increase the value of the minimum reliability path by 5% ?

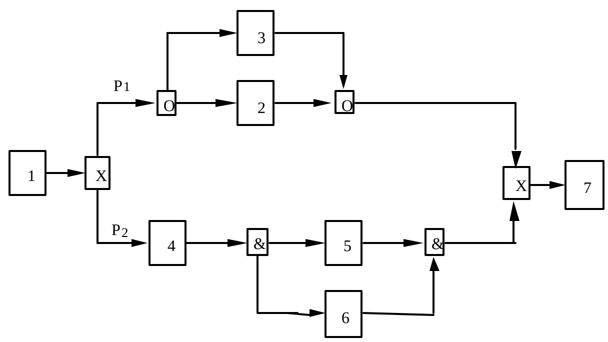

Figure A5. An IDEF3 process model

Table A1. Reliability data

Activity	Reliability
1	0.85
2	0.90
3	0.90
4	0.95
5	0.90
6	0.88
7	0.94

Table A2. Probability data

Probability	Value
P1	0.3
P2	0.7

6. Consider the product development process represented with the IDEF3 model in Figure A6. The risk of performing activities on time is measured with reliability.

The reliability of each activity is as follows:

Activity	Reliability
1	.95
2	.95
3	.90
4	.95
5	.96
6	.94
7	.96
7	.97

The probabilities associated with the exclusive OR junction are $P_2 = .2$ and $P_3 = .8$

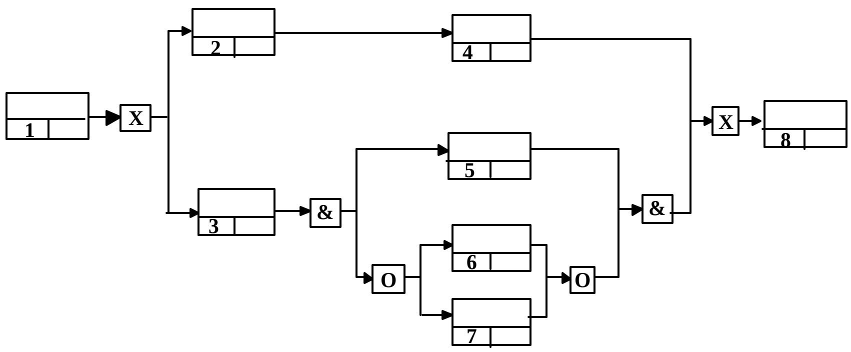

Figure A6. Process model

(a) Compute the minimum min cut set and maximum min path set.
(b) Compute the overall reliability of the model with the reduction method.