CONSERVATION OF MASS, LINEAR MOMENTUM, AND ENERGY IN A SLUICE GATE FLOW

Purpose

To measure the total piezometric pressure at various locations along a vertical sluice gate, to calculate the horizontal force on the gate, and to compare the experimental value of the force (including its uncertainty) with the value computed using the conservation of mass, linear momentum, and energy.

Design of the Experiment

The flow through a channel in which a gate partially obstructs the flow will be used for this measurement of total force. This obstruction is called a sluice gate (see Figure 1). The flow is from left to right and enters at a velocity V_o. The fluid in the upstream section builds up against the gate to a level y_0, and exits the upstream section under the gate of height b. The fluid attains a higher velocity V_1 as it passes under the gate and a shallower free surface height y_1 downstream.

Three assumptions will be made in this derivation of the equation for horizontal force on a sluice gate:

1) The viscous force at the bottom of the channel and the energy dissipation at the gate are negligible.
2) The flow is steady and has a uniform velocity distribution at the inlet and outlet sections.
3) Flow at upstream and downstream sections is uniform and the effect of the side-walls is negligible.

Using the Reynolds Transport Theorem to derive the equation for the force on the vertical gate,

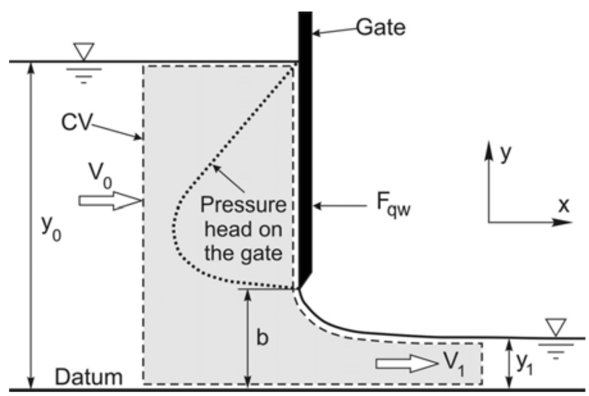

Figure 1. Flow under a vertical sluice gate.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{cv} \beta\,\rho\,dV + \int_{cs} \beta\,\rho\,\overline{V}\cdot dA \tag{1}$$

For Momentum transport let B = momentum and β = momentum per unit mass or v (velocity referenced to an inertial frame) in equation (1).

$$\frac{d\,(momentum)_{sys}}{dt} = \frac{d}{dt}\int_{cv} \overline{v}\rho\,dV + \int_{cs} \overline{v}\rho\overline{V}\cdot dA \tag{2}$$

According to Newton's second law, the summation of all the external forces on a system is equal to the rate of change of momentum of that system.

$$\sum \overline{F} = \frac{d\,(momentum)_{sys}}{dt} \tag{3}$$

Substituting into the momentum transport equation (2), yields:

$$\sum F_{surface} + \sum F_{body} = \frac{d}{dt}\int_{cv} \overline{v}\rho\;dV + \int_{cs} \overline{v}\rho\;\overline{V}\cdot dA \tag{4}$$

For uniform velocity in the streams crossing the control surface:

$$\int_{cs} \bar{v}\,\rho \bar{V} \cdot d\bar{A} = \sum_{cs} \bar{v}\rho\bar{V} \cdot \bar{A} \qquad (5)$$

For steady flow

$$\frac{d}{dt}\int_{cv} \bar{v}\rho \; dV = 0 \qquad (6)$$

Therefore equation (4) becomes:

$$\sum F_{surface} + \sum F_{body} = \sum_{cs} \bar{v}\rho\bar{V} \cdot \bar{A} \qquad (7)$$

The total surface and body forces on the gate (acting in the *x*-direction) are as follows:

$$\sum F_{surface(x)} = F_{GW} \qquad \sum F_{body(x)} = \frac{1}{2}\gamma y_0^2 w - \frac{1}{2}\gamma y_1^2 w$$

By applying the conservation of momentum in the *x*-direction yields:

$$\sum_{cs(x)} \bar{v}\rho\bar{V} \cdot \bar{A} = V_0\rho(-V_0 y_0 w) + V_1\rho(V_1 y_1 w) = -\rho\, y_0 w V_0^2 + \rho\, y_1 w V_1^2$$

where F_{GW} is the force of the gate acting on the water in the control volume.
Substituting the above expressions into equation (7) yields:

$$\frac{1}{2}\gamma y_0^2 w - \frac{1}{2}\gamma y_1^2 w - F_{gate} = -\rho\, y_0 w V_0^2 + \rho\, y_1 w V_1^2 \qquad (8)$$

Finally, the force on the gate can be expressed as follows:

$$F_{gate} = \rho\, w(y_0 V_0^2 - y_1 V_1^2) + \frac{1}{2}\gamma w(y_0^2 - y_1^2) \qquad (9)$$

Substitute in for V_0 and V_1 expressions derived using the conservation of mass and conservation of energy.
Conservation of mass throughout system yields:

$$\dot{m}_0 = \dot{m}_1 \qquad (10)$$

$$\rho V_0 A_0 = \rho V_1 A_1$$

As the density and width of the channel remain constant throughout the system, and the cross sectional area, $A = wy$, allows to express V_0 in Equation (10) in terms of V_1 as

$$V_0 = V_1 \frac{y_1}{y_0} \qquad (11)$$

The application of the energy equation (first law of thermodynamics) for the steady state control volume shown in Figure 1 leads to:

$$\frac{dE}{dt} = \dot{m}_0(h_0 + \frac{V_0^2}{2} + gz_0) - \dot{m}_1(h_1 + \frac{V_1^2}{2} + gz_1) = 0 \qquad (12)$$

Assuming the same datum for the upstream and downstream sections, $h_0 = h_1$, equation (12) becomes:

$$\frac{V_0^2}{2} + gy_0 = \frac{V_1^2}{2} + gy_1 \qquad (13)$$

Substituting (11) into (13) yields:

$$V_0 = \left[\frac{2gy_1^2}{y_0 + y_1} \right]^{1/2} \tag{14}$$

Substituting Equation (14) into (9) yields the data reduction equation for the force on the gate:

$$F_{gate} = \frac{1}{2}\gamma w (y_0^2 \; - \; y_1^2) + 2\rho g w y_0 y_1 \frac{y_1 - y_0}{y_0 + y_1} = \frac{\gamma w (y_0 - y_1)^3}{2(y_0 + y_1)} \tag{15}$$

Measurement Systems

The sluice gate is located in the open-channel flume of the Fluids Laboratory. The glass-walled flume is 2 ft wide and 30 ft long (see Figure 2.a). Water is circulated in the channel using a pump. The discharge is measured using an orifice-meter mounted on the water supply line. The orifice meter on the return pump has the following equation $Q = 2.04\sqrt{\Delta z}$ (cfs) with z (ft) being the difference indicated by the differential manometer connected to the orifice meter. A gate at the downstream end of the flume controls the flow. The other needed geometrical characteristics are measured with a measure taps. The water depth is measured using a point gage attached to the instrument carriage mounted on top of the flume. The sluice gate is provided with piezometric taps, displaced as shown in Figure 2.b. The taps are connected to a panel of piezometers set on the flume frame.

a)

b)

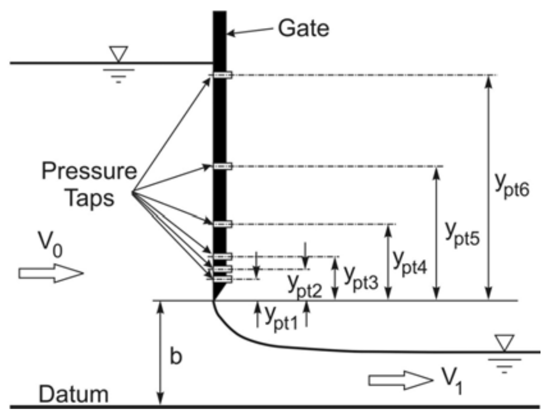

Figure 2. Experimental setup: a) the experimental flume; b) positioning of the pressure taps onto the sluice gate

Measurement Procedures

1. After the flume has running water in the channel, adjust the inflow to the flow rate indicated by the TA . Use the readings of the differential manometer and the orifice-meter calibration equation to select the desired discharge.
2. Adjust the gate height so the water level upstream of the gate, y_0, is just above the sixth pressure tap on the gate.
3. Measure the upstream, y_0, and downstream, y_1 depths using the point gage (make sure that the gage is zeroed for the flume bottom).
4. Measure the height from the bottom of the gate to the base of the flume, *b*, and the flume width, *w*.
5. Measure successively the pressure at the six piezometric taps (h_i, i = 1,...,6) and record the values in the measurements table. Note that the pressure at each location along the gate is provided by $\gamma(y_0 - b - y_{pti})$, where h_i is the depth where each of the individual taps are located.
6. Change the inflow to an intermediate and a higher value. Repeat steps 1- 5 for each established flow.

Use the sheets provided in Table 1 to record the measurements made during the conduct of the experiment

Data Analysis

The force on a sluice gate in an open channel is determined by two methods:

1. Using Equation (15) based on conservation of mass, momentum, and energy equations determine the force acting on the sluice gate.
2. Using the actual pressure distribution over the gate. Once all the pressures have been measured at the pressure taps, the trapezoidal rule is applied to calculate the force on the gate by integrating the pressure distribution

Compare the force exerted on the gate determined by the two methods for each of the three trials. Assuming the bias errors determine the total errors for both measurements, determine if the two methods "agree". Explain the reasoning behind your determination.

Uncertainty Assessment

Figure 3 illustrates the block diagram of the measurement systems, the data reduction equations for the results, and illustrates the propagation of the elemental error sources to the final results.

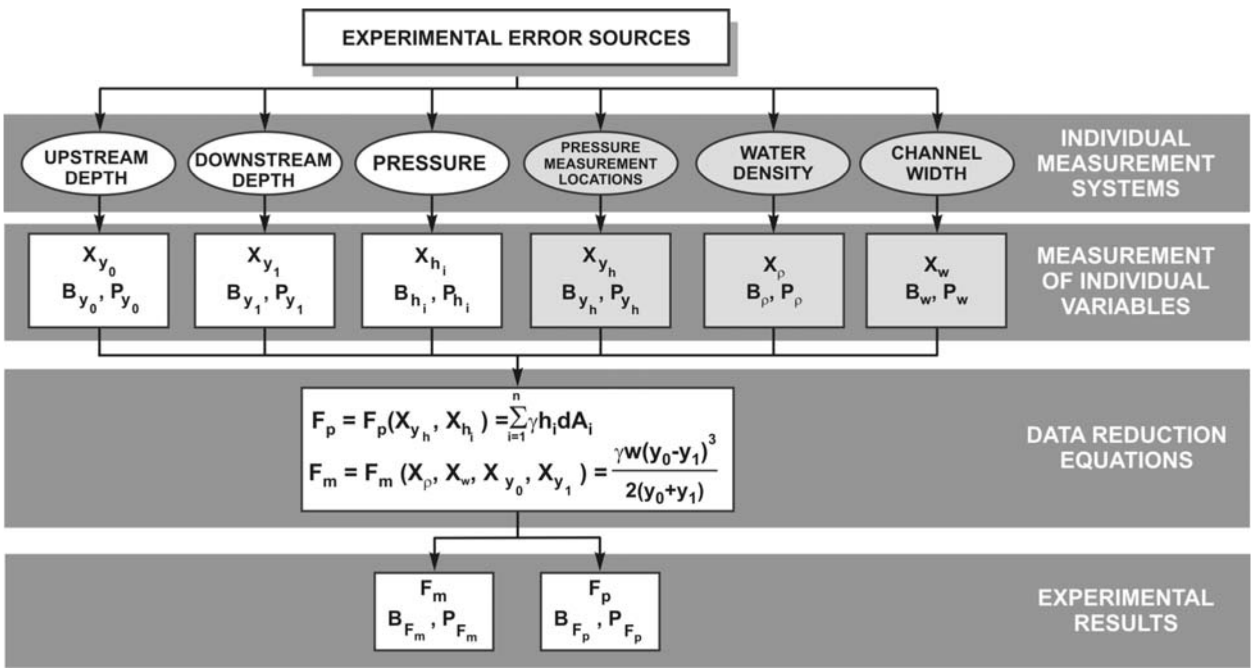

Figure 3. Block Diagram of the sluice gate experiment including measurement systems / data reduction equations

In the interest of saving time, we will not make repeated measurements in this experiment. We will examine closer the sources of bias errors, as in this experiment they are the dominant source of errors. Estimate the elemental errors for each of the independent variables and enter them in Table 2 as appropriate.

Using the AIAA (1995) methodology, one can write the bias limit as:

$$B_F^2 = \sum_{i=1}^{j} \Theta_i^2 B_i^2 = \theta_{y_0}^2 B_{y_0}^2 + \theta_{y_1}^2 B_{y_1}^2 + \Theta_b^2 B_b^2$$

Using the data reduction equation for the force exerted by the gate as determined using conservation of momentum in the x direction, derive the sensitivity coefficients, Θ_{y0}, Θ_{y1}, and Θ_w. Propagate the bias errors to find the bias limit for the force exerted by the gate. For one of the three trials, make several measurements of y_1 and estimate the precision error associated with the measurement of y_1. Justify the number of trials that you select to make this estimation.

Discussion

1. Why is the pressure distribution on the gate not triangular (hydrostatic)?
2. If you are asked to calculate the force on the gate of a 10 foot wide flume, with a discharge of 240 cfs, and a gate opening, *b*, of 1 ft, using the laboratory flume, what discharge and gate opening would you use in the model? Explain your results.
3. Using your repeated measurements of y_1, consider the assumption that bias errors contribute the majority of the uncertainty in this experiment. Is this assumption warranted?

References

Robertson, J.A. and Crowe, C.T. (1993). *Engineering Fluid Mechanics,* 5th edition, Houghton Mifflin, Boston, MA.

White, F.M. (1994). *Fluid Mechanics,* 3rd edition, McGraw-Hill, Inc., New York, N.Y.

Stern F., Muste M., Beninati M-L., and Eichinger W.E. (1999). "Summary of Experimental Uncertainty Assessment Methodology with Example," IIHR *Report* No. 406, Iowa Institute of Hydraulic Research, The University of Iowa, Iowa City, IA.

Table 1. Data Collection and Analysis

Determination of the Gate Force from Conservation of Momentum

Case	w (width)	z	Q_{orifice}	b	y_0	y_1	V_0	V_1	Q_{meas}	$Q_{\text{meas}}/Q_{\text{orifice}}$	Force - Conservation momentum
							$\left[\frac{2gy_1^2}{y_0+y_1}\right]^{1/2}$	$\left[\frac{2gy_0^2}{y_0+y_1}\right]^{1/2}$	$V_0 y_0 w$		$\frac{\gamma w(y_0-y_1)^3}{2(y_0+y_1)}$
1											
2											
3											

Determination of the Gate Force from Measurements

Tap	y_{tpi} (ft)	y_{01}	y_{02}	y_{03}	P_{1i}	P_{2i}	P_{3i}	Δy_i	F_{1i}	F_{2i}	F_{3i}
					$\rho g\left[y_{01}-\left(y_{pti}+b\right)\right]$	$\rho g\left[y_{02}-\left(y_{pti}+b\right)\right]$	$\rho g\left[y_{03}-\left(y_{pti}+b\right)\right]$				
1	0.025										
2	0.05										
3	0.10										
4	0.20										
5	0.45										
6	0.90										
								Total Force			
								% Difference (conservation of momentum/Measurements)			

Table 2: Assessment of Bias Errors of the Independent Variables

Variable	**Bias Limit**	**Comments**	**B_i**
w - width	½ instrument resolution		
y_0	½ instrument resolution		
	Total Bias Error		
y_1	½ instrument resolution		
	Total Bias Error		
P_i	½ instrument resolution		
	Total Bias Error		