**Course Syllabus**

1. **Color**
2. **Camera models, camera calibration**
3. **Advanced image pre-processing**
   - **Line detection**
   - **Corner detection**
   - **Maximally stable extremal regions**
4. **Mathematical Morphology**
   - **binary**
   - **gray-scale**
   - **skeletonization**
   - **granulometry**
   - **morphological segmentation**
   - **Scale in image processing**
5. **Wavelet theory in image processing**
6. **Image Compression**
7. **Texture**
8. **Image Registration**
   - **rigid**
   - **non-rigid**
   - **RANSAC**

- Thinning and thickening often used sequentially
- Let $B = \{B_{(1)}, B_{(2)}, B_{(3)}, \dots, B_{(n)}\}$ denote a sequence of composite structuring
- elements $B_{(i)} = (B_{i_1}, B_{i_2})$
- **Sequential thinning** – sequence of *n* structuring elements

$$X \oslash B = \left(\left(\left(X \oslash B_{(1)}\right) \oslash B_{(1)}\right) \dots \oslash B_{(n)}\right)$$

- several sequences of structuring elements $\{B_{(i)}\}$ are useful in practice
- These sequences are called the **Golay alphabet**
- composite structuring element – expressed by a single matrix
- “one” means that this element belongs to *B*1 (it is a subset of objects in the
- hit-or-miss transformation)
- “zero” belongs to *B*2 and is a subset of the background
- $*$ ... element not used in matching process = its value is not significant
- Thinning sequential transformations converge to some image — the number of iterations needed depends on the objects in the image and the structuring element used
- if two successive images in the sequence are identical, the thinning (or thickening) is stopped

## Sequential thinning by structuring element *L*

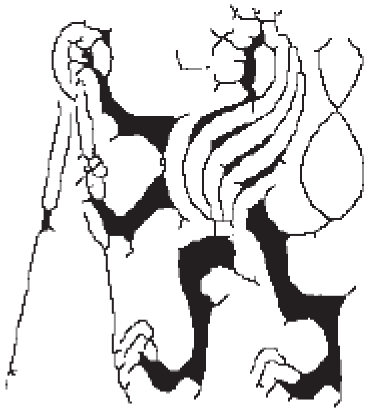
- thinning by *L* serves as homotopic substitute of the skeleton;
- final thinned image consists only of lines of width one and isolated points

- structuring element *L* from the Golay alphabet is given by

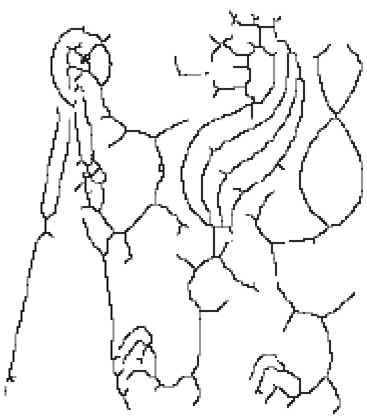
$$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ d & 1 & d \\ 1 & 1 & 1 \end{bmatrix}, \qquad L_1 = \begin{bmatrix} d & 0 & 0 \\ 1 & 1 & 0 \\ d & 1 & d \end{bmatrix}$$

- (The other six elements are given by rotation).

Original



after 5 iteration



final result

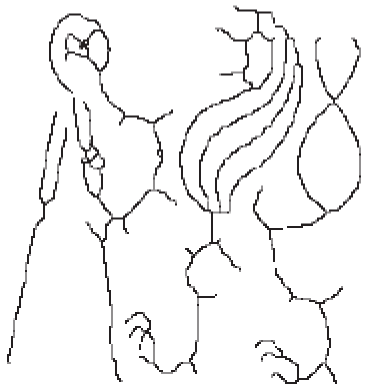
**Sequential thinning by structuring element *E***

- structuring element *E* from the Golay alphabet is given by

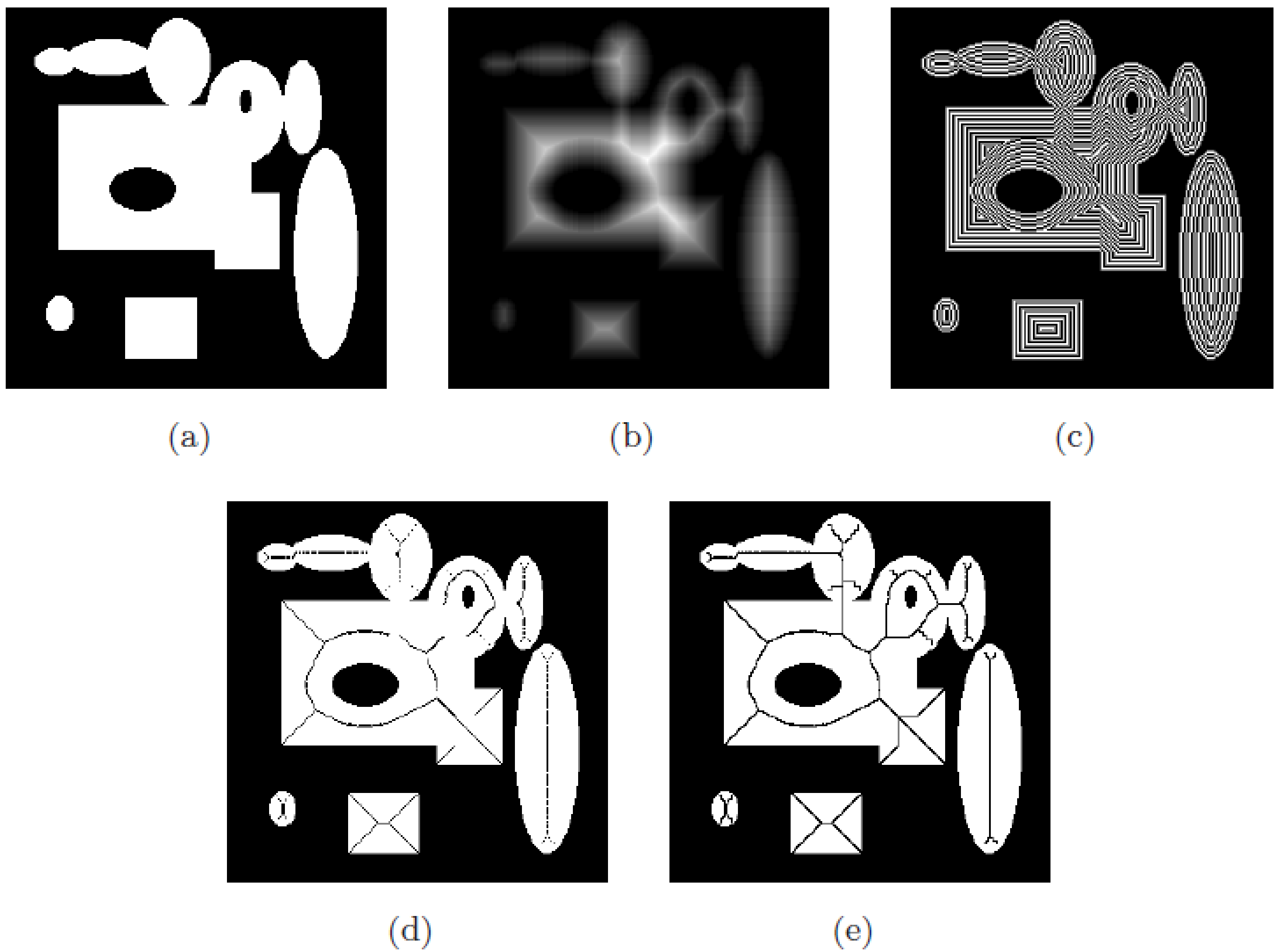
$$E_1 = \begin{bmatrix} d & 1 & d \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad L_1 = \begin{bmatrix} 0 & d & d \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Less jagged skeletons

Original



skeleton



(a) (b) (c)

(d) (e)

# Distance Transform

Binary image: $(O, \mathbb{Z}^n)$; $O$: set of object pixels/voxels (points)

Distance transform = an images or a function $DT$: $O \longrightarrow \mathbb{Z}^n$ where $DT(p)$ is the distance between $p$ and the background $\bar{O}$

Distance transform is a measure of "local depth"

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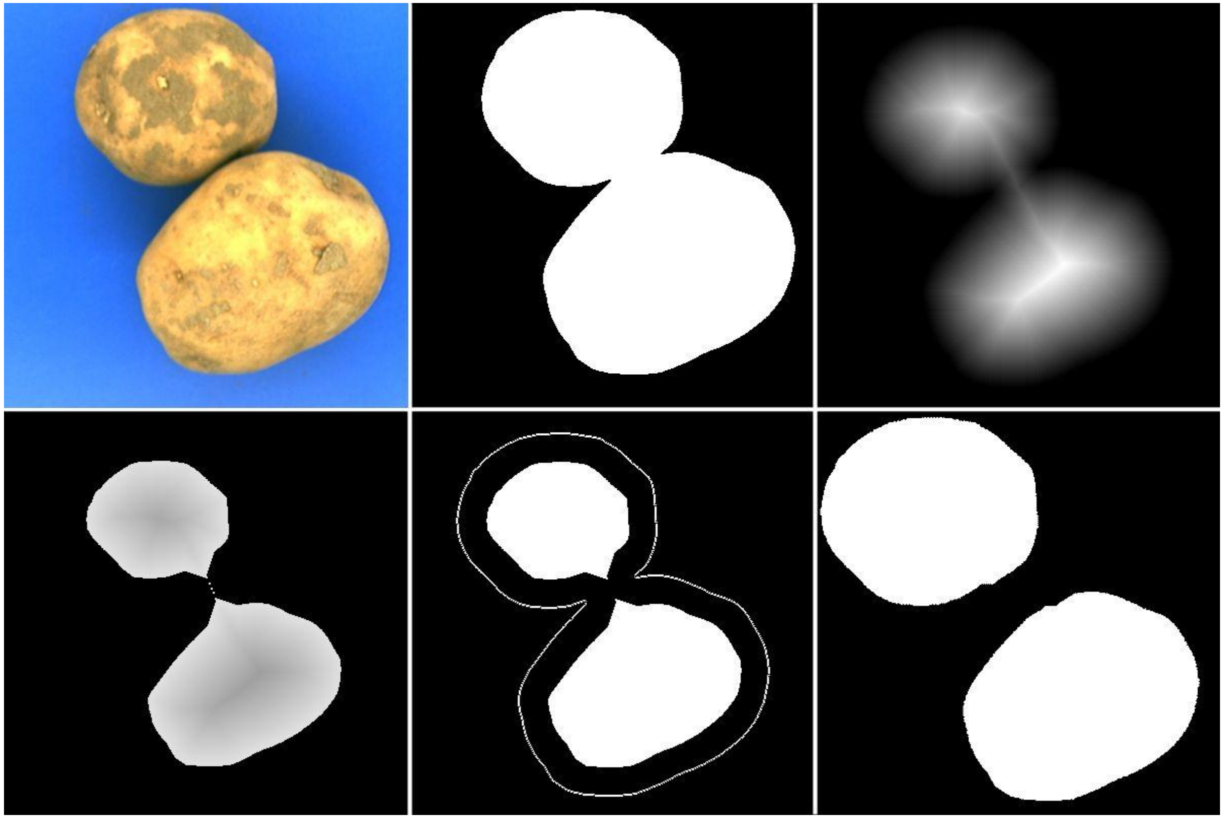

# Distance Transform

Another example

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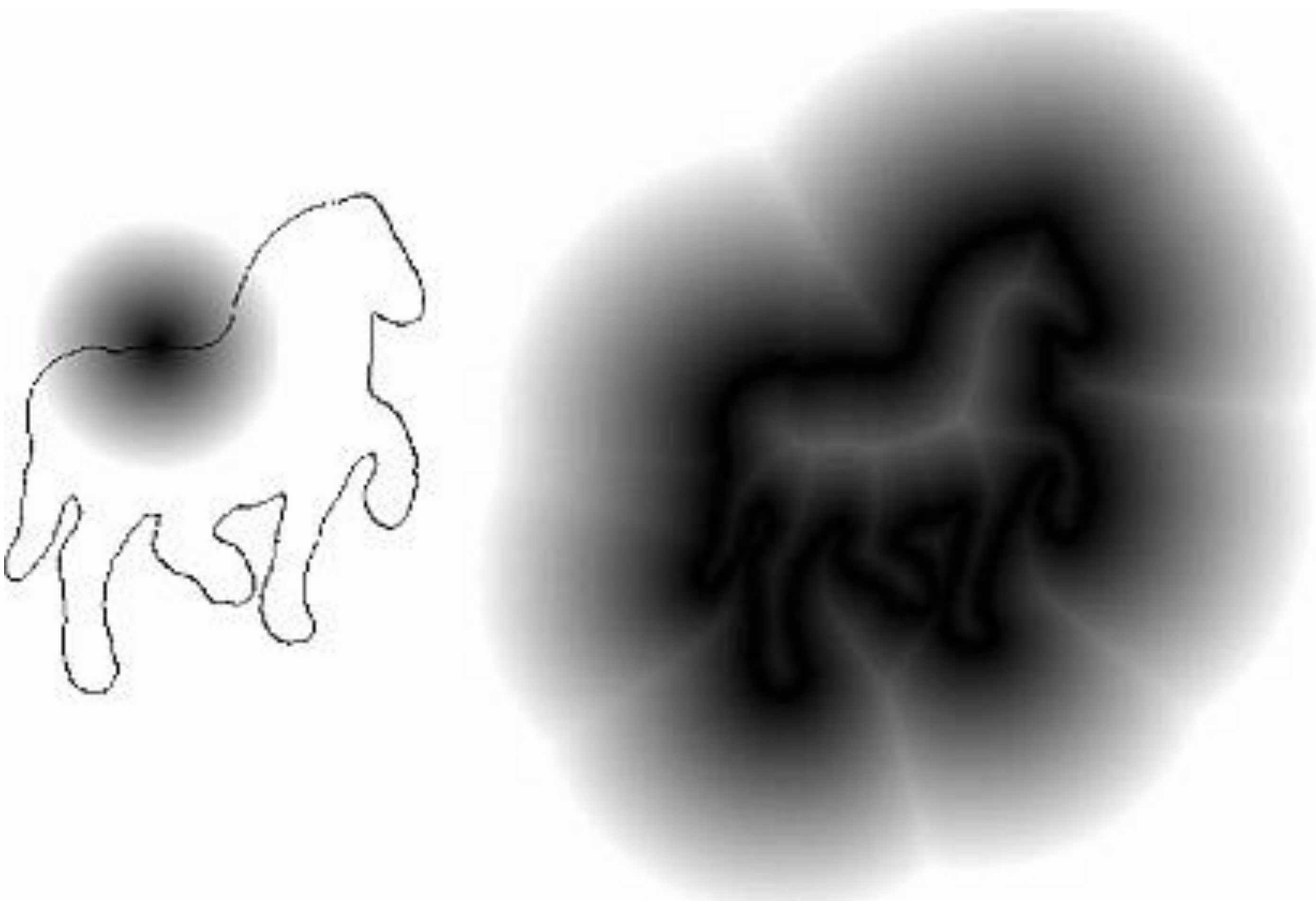

**Distance Transform**

Algorithm

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for all object points $p$

assign $DT(p) = MAX$;

for all background points $p$

assign $DT(p) = 0$;

for iteration = 1 and 2

scan the image in slice$^+$ row$^+$ column$^+$ direction and for each $p$ do

if $DT(p) < \min_{q \in N(p)} DT(q) + dist(p,q)$

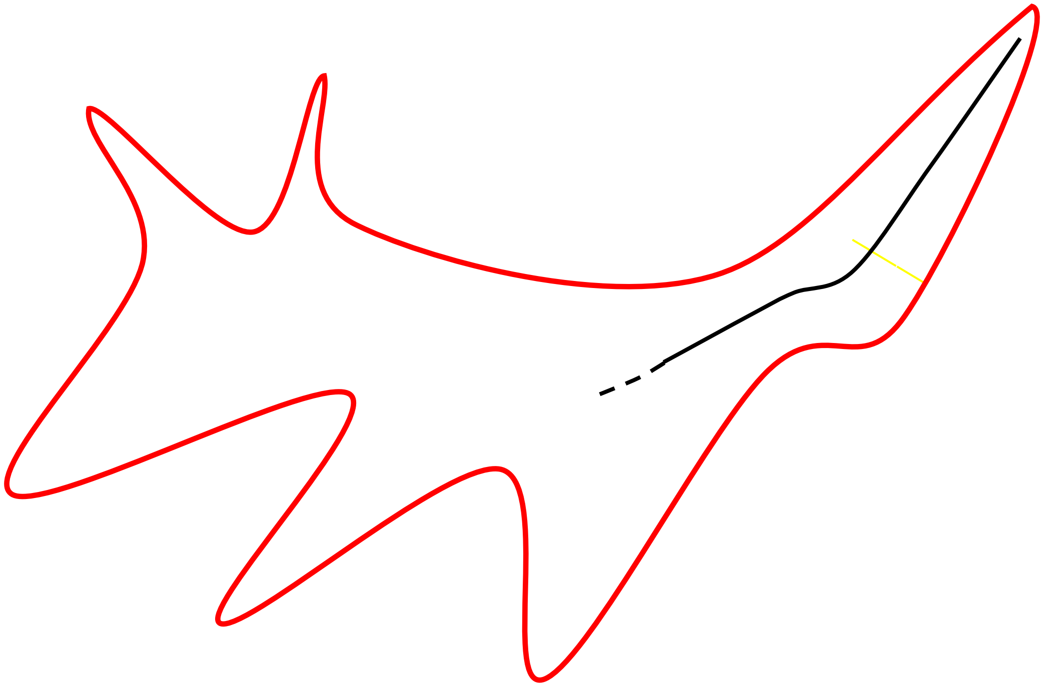
assign $DT(p) = \min_{q \in N(p)} DT(q) + dist(p,q)$

scan the image in slice$^-$ row$^-$ column$^-$ direction and for each $p$ do

if $DT(p) < \min_{q \in N(p)} DT(q) + dist(p,q)$

assign $DT(p) = \min_{q \in N(p)} DT(q) + dist(p,q)$

**Axial line detection using Distance transform**

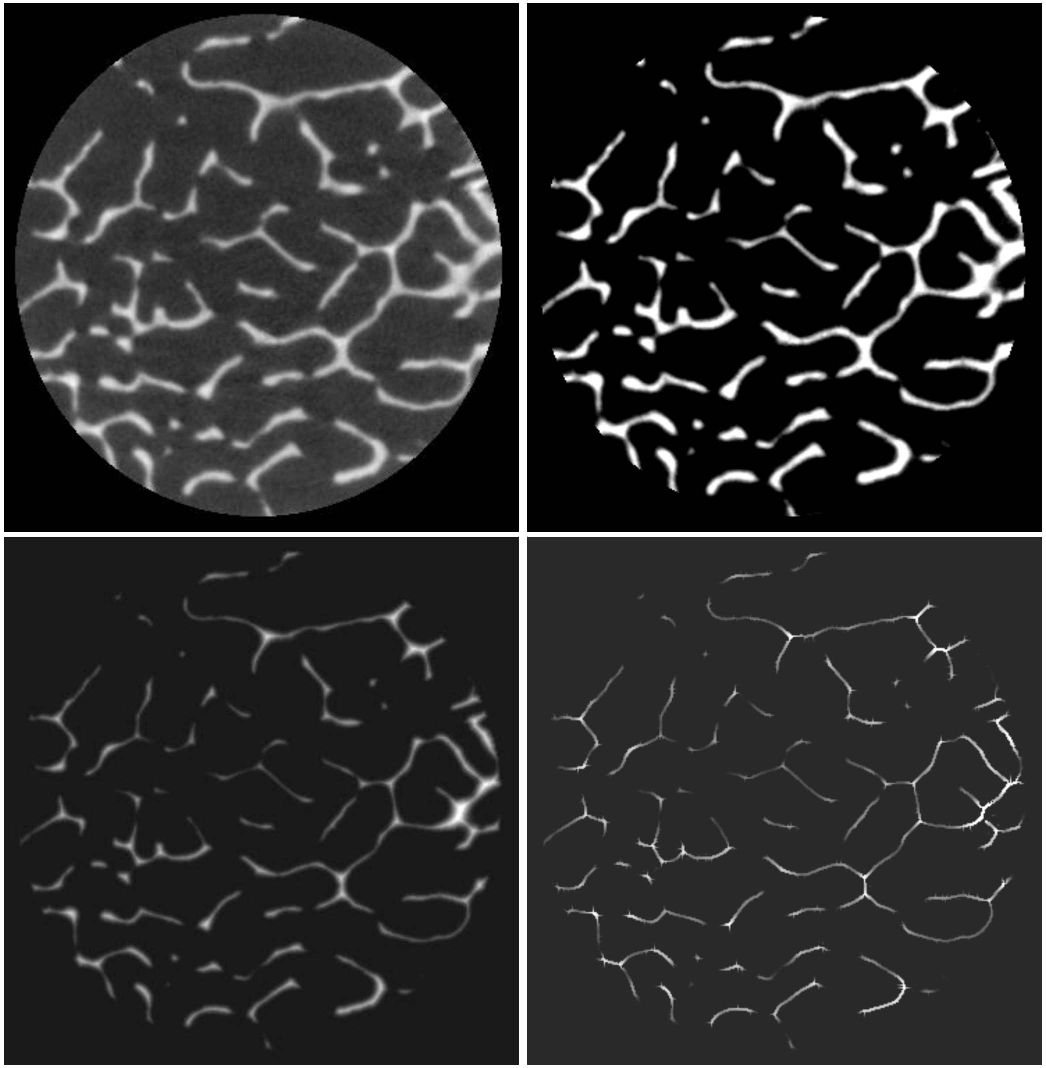


a point $p$ is an axial point if there is no point $p'$ such that a shortest path from $p'$ to the boundary passes through $p$.

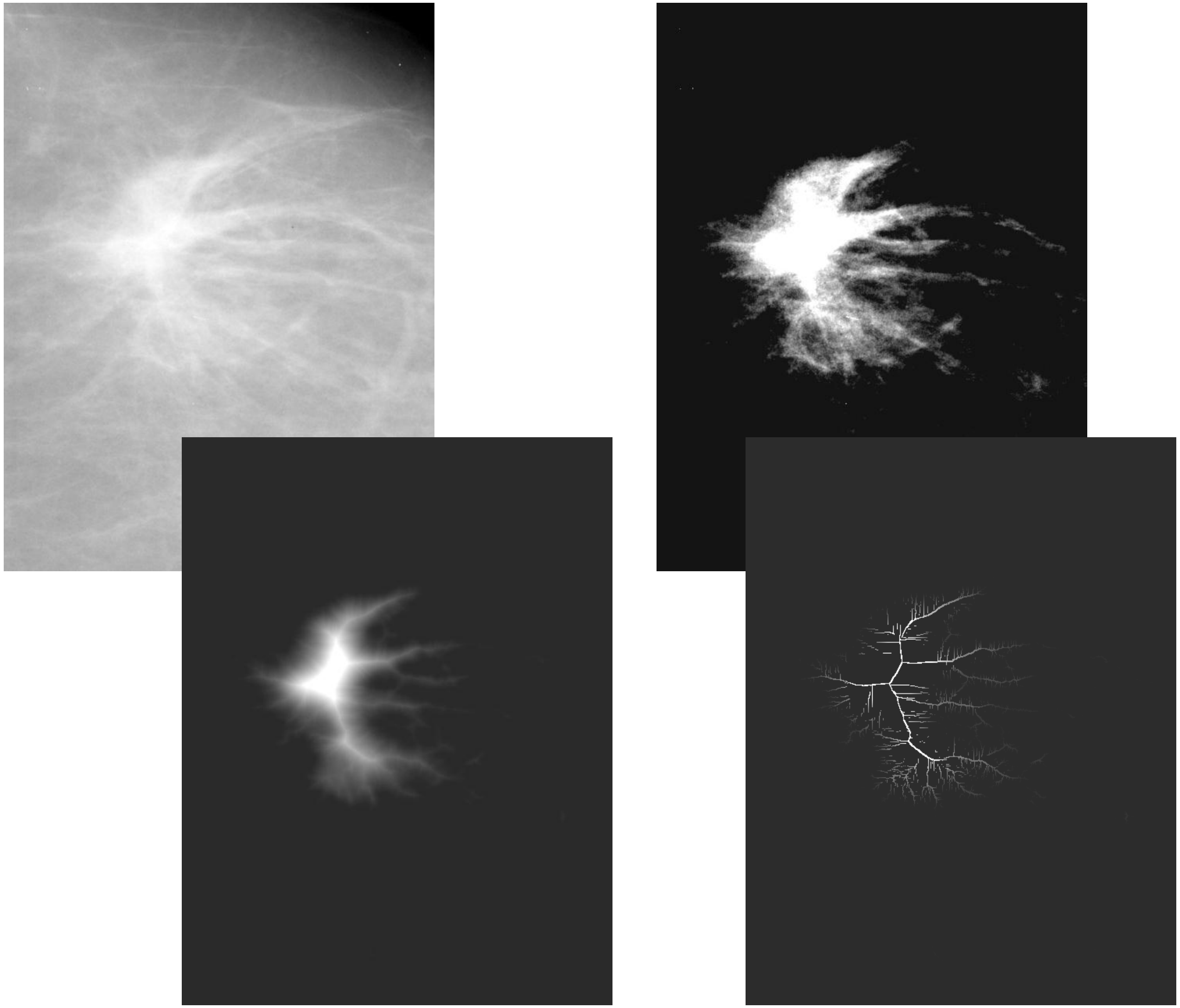
a point $p$ is an axial point if there is no point $q$ in the neighborhood of p such that

$$DT(q) = DT(p) + |p - q|$$

# Axial line detection using Distance transform



# Axial line detection using Distance transform



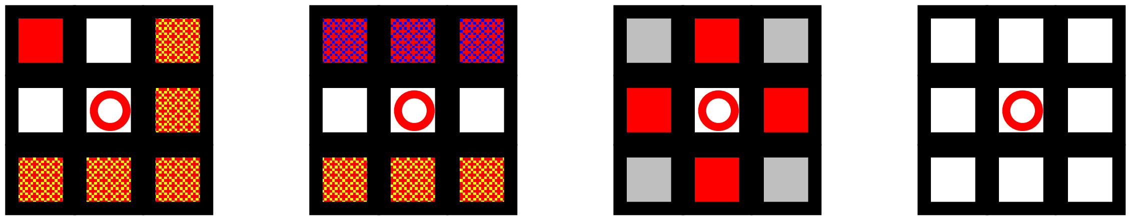
# Axial line detection using Distance transform

# Simple Points

- A points (not necessarily an object point) in a 2D or 3D digital image is a simple point if and only it its binary transformation doesn't change local image topology

2D simple point characterization is straight forward

- A point that doesn't match with any of the following windows or variations produced by their mirror reflection and/or rotation, is a 2D simple point in (8,4) connectivity



: object : background : origin : don't care

, : At least one in each group is object

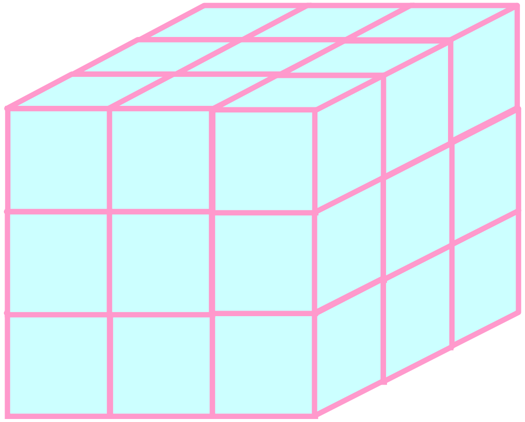
# 3D Simple Points

- Included neighborhood

  $N(p)$ = set of all 26-adjacent points including $p$

- Excluded neighborhood

  $N^*(p)=N(p) - \{p\}$



**Theorem 1.** $N(p) \cap B$ is always a topological ball.

Therefore, $p$ is a simple points if and only if $N^*(p)$ is a topological ball.

# Local Topological Parameters

$\xi(p)$: number of components in $N^*(p) \cap B$

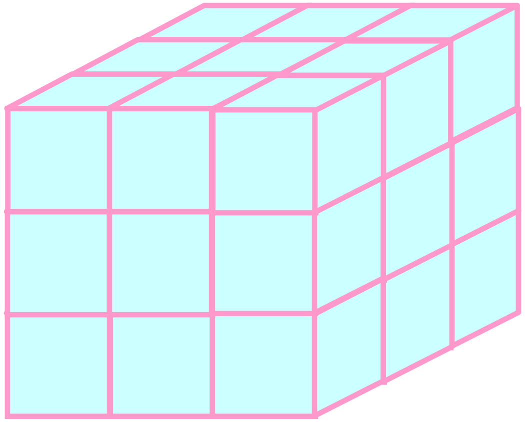
$\eta(p)$: number of tunnels in $N^*(p) \cap B$

$\delta(p)$: number of cavities in $N^*(p) \cap B$

# Cavities in excluded neighborhood

**Theorem 2.** $\delta(p)$ is one if and only if all the 6-neighbors of $p$ are in $B$.

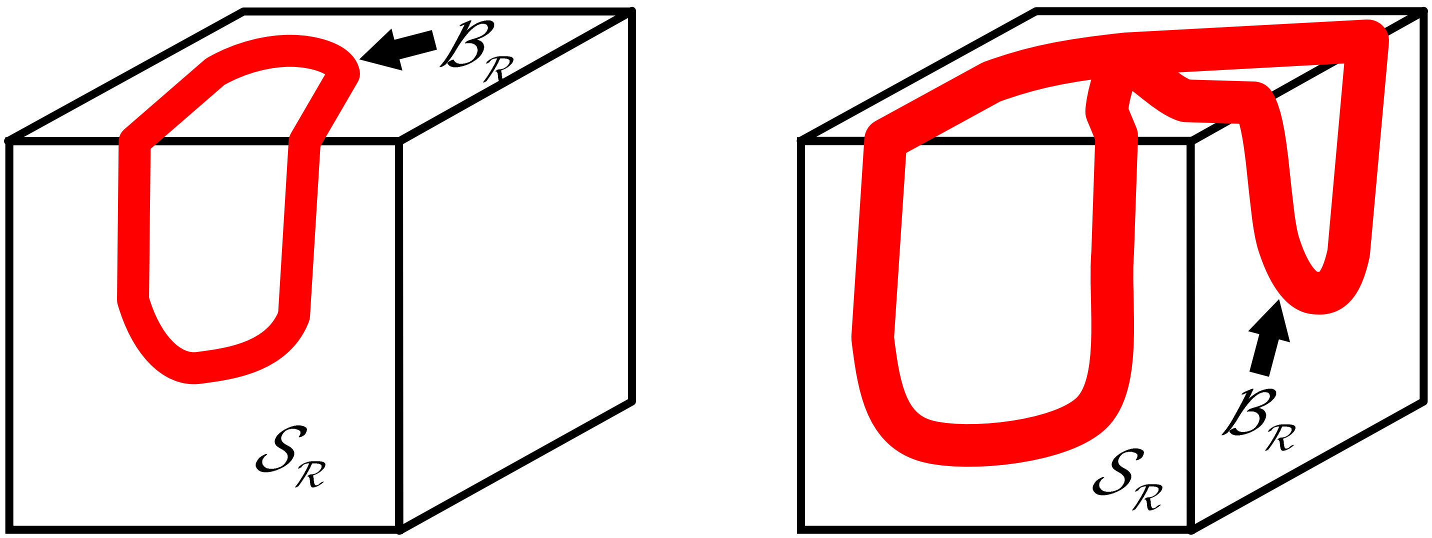
# Tunnels in the excluded neighborhood



**Key Observations**

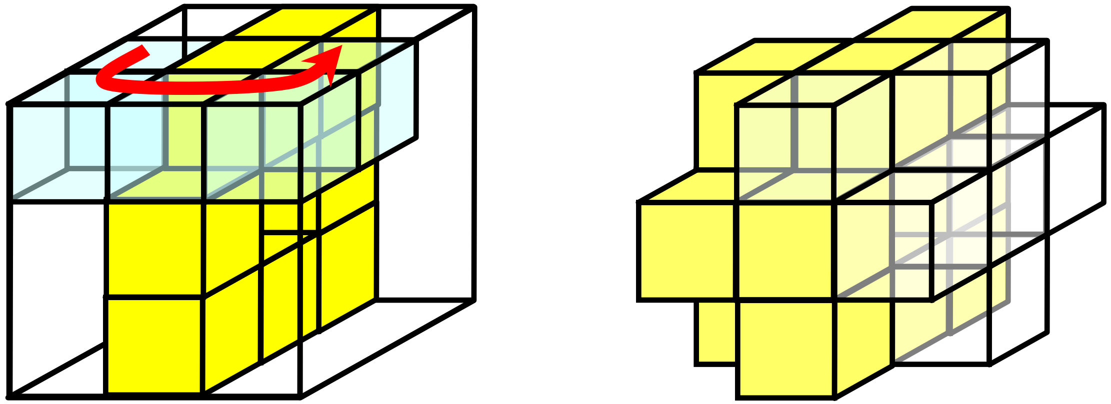
- $N^*(p)$ is a digital topological sphere
- All points in $N^*(p) \cap B$ lie on a digital topological sphere

# The Continuous Case




If $\mathcal{S}_{\mathcal{R}} - \mathcal{B}_{\mathcal{R}}$ is nonempty, the number of tunnels in $\mathcal{B}_{\mathcal{R}}$ is one less than the number of components in $\mathcal{S}_{\mathcal{R}} - \mathcal{B}_{\mathcal{R}}$, and it is zero when $\mathcal{S}_{\mathcal{R}} - \mathcal{B}_{\mathcal{R}}$ is empty.

# The Digital Case



- A path of $N^*(p)$ – B crosses a closed loop of $N^*(p) \cap$ B
- A pair of disconnected components of $N^*(p) - B$ contributes to a tunnel only when both are adjacent to the interior point $p$.

# Tunnels in excluded neighborhood

$X(p)$: the set of 6-neighbors of $p$ in $N^*(p) - B$

$Y(p)$: the set of 18-neighbors of $p$ in $N^*(p) - B$

Theorem 3. If $X(p)$ is non-empty, $\eta(p)$ is one less than the number of marrow objects in the intersection of $X(p)$ and $Y(p)$, and $\eta(p)$ is zero when $X(p)$ is empty.

# 3D Simple Point Characterization

For any 3D digital image $(\mathbb{Z}^3, 26{,}6, B)$, any point $p \in B$ is a simple point if and only if it satisfies all the four conditions

*1)* $\xi(p) = 0$,

*2)* $\eta(p) = 0$

*3)* $\delta(p) = 0$

# 3D Simple Point Characterization

For any 3D digital image $(\mathbb{Z}^3, 26, 6, B)$, any point $p \in B$ is a simple point if and only if it satisfies all the four conditions

1. $N^*(p) \cap B$ is non-empty,
2. $N^*(p) \cap B$ is 26-connected,
3. $X(p)$ is non-empty, and
4. $X(p)$ is 6-connected in $Y(p)$.

$X(p)$: the set of 6-neighbors of $p$ in $N^*(p) - B$

$Y(p)$: the set of 18-neighbors of $p$ in $N^*(p) - B$