Course Syllabus

1. Color
2. Camera models, camera calibration
3. Advanced image pre-processing
    - Line detection
    - Corner detection
    - Maximally stable extremal regions
4. Mathematical Morphology
    - binary
    - gray-scale
    - skeletonization
    - granulometry
    - morphological segmentation
    - Scale in image processing
5. Wavelet theory in image processing
6. Image Compression
7. Texture
8. Image Registration
    - rigid
    - non-rigid
    - RANSAC

Quiz

Object = black

original

?

?

13.5 Skeletons and object marking

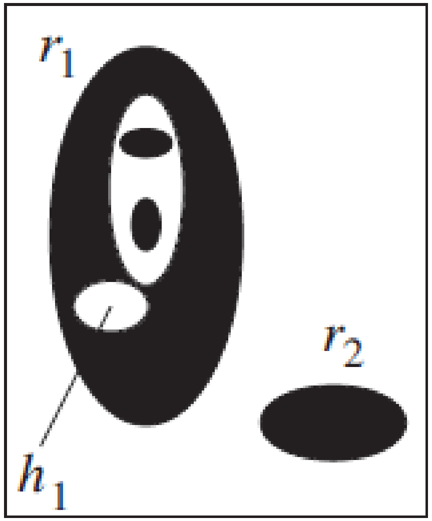
13.5.1 Homotopic transformations

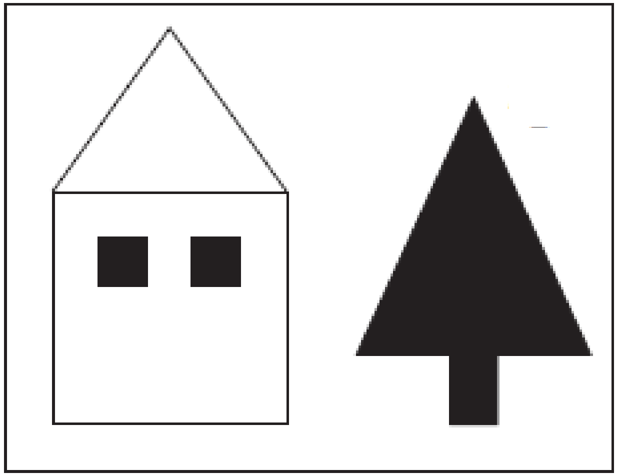
- transformation is homotopic if it does not change the continuity relation between regions and holes in the image.

- this relation is expressed by homotopic tree
  - its root ... image background
  - first-level branches ... objects (regions)
  - second-level branches ... holes
  - etc.

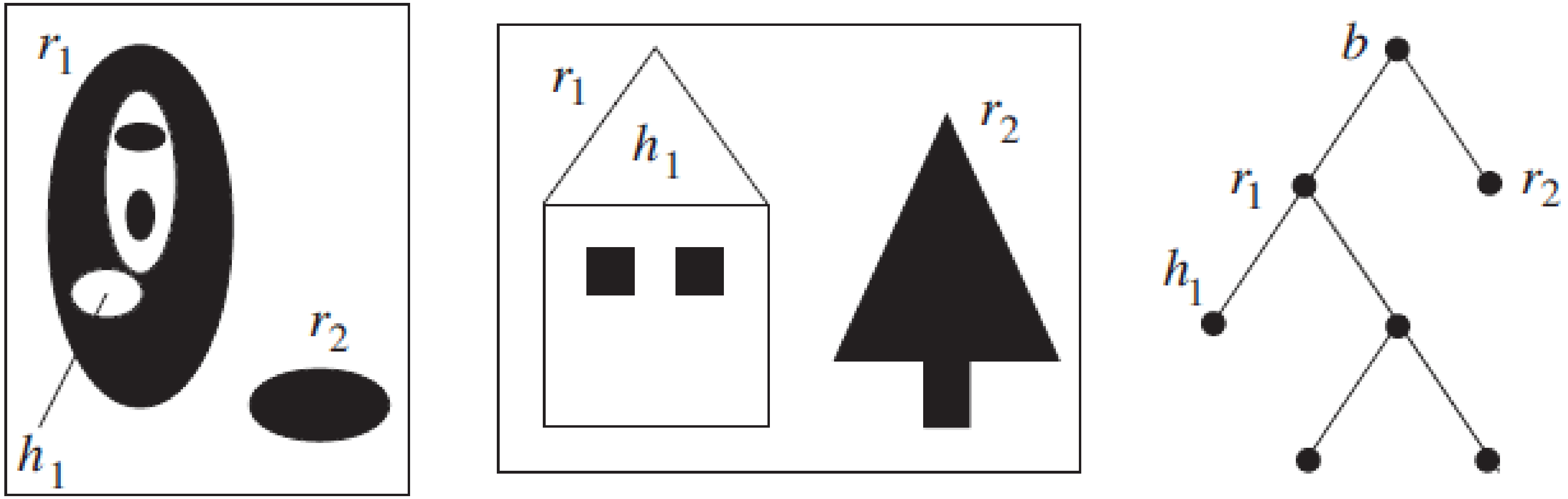
- transformation is homotopic if it does not change homotopic tree

Homotopic Tree

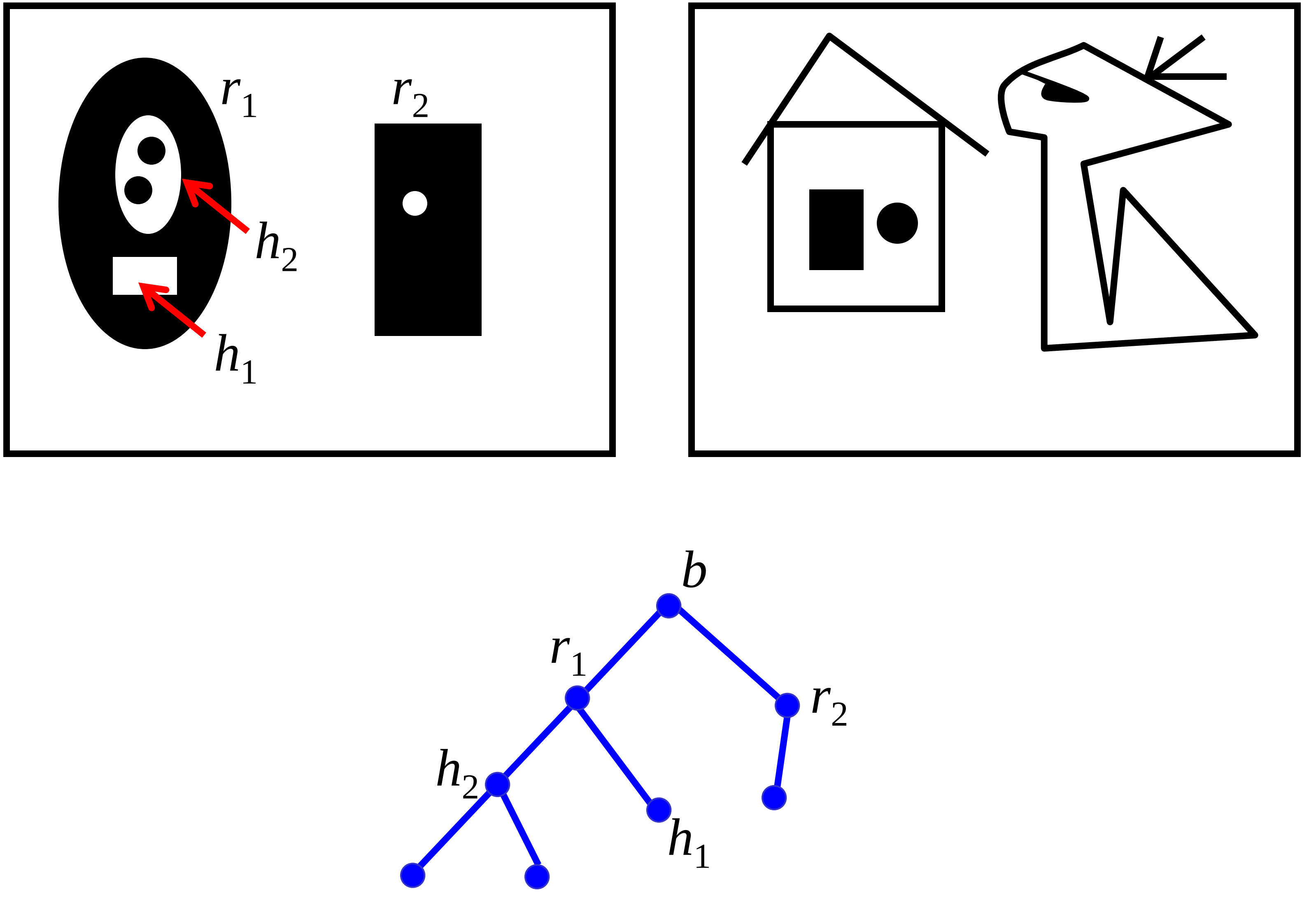




Homotopic Tree

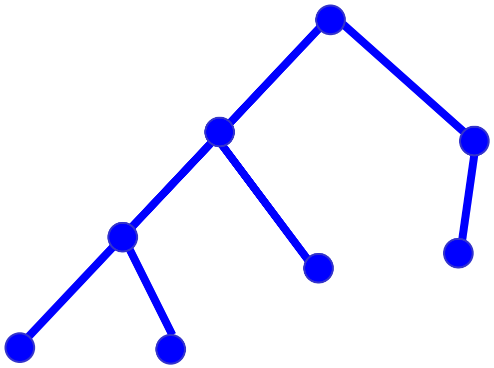


Homotopic Tree



Quitz: Homotopic Transformation

- What is the relation between an element in the ith and i+1th levels?



13.5.2 Skeleton, maximal ball

- skeletonization = **medial axis transform**
- ‘grassfire’ scenario
- A grassfire starts on the entire region boundary at the same instant – propagates towards the region interior with constant speed
- **skeleton** $S(X)$ ... set of points where two or more fire-fronts meet

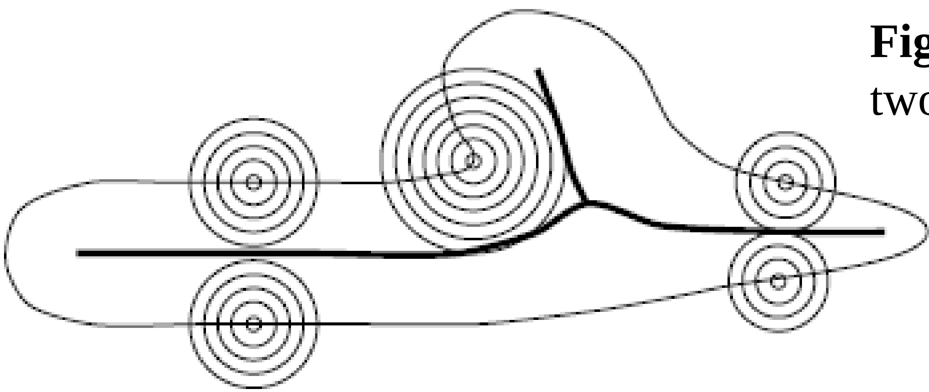


Figure 13.22: Skeleton as points where two or more fire-fronts of grassfire meet.

- Formal definition of skeleton based on maximal ball concept
- **ball** $B(p, r)$, $r \geq 0$ ... set of points with distances d from center $\leq r$
- ball B included in a set X is **maximal** if and only if there is no larger ball included in X that contains B

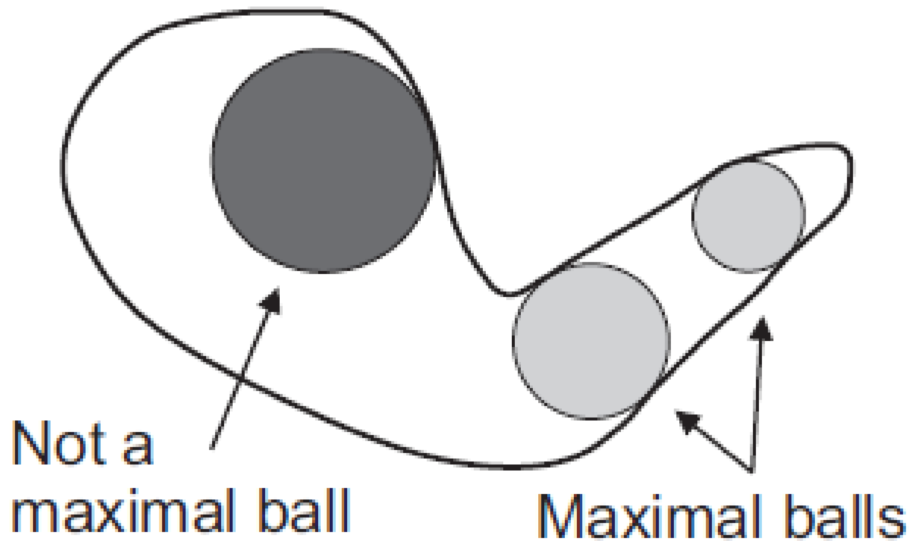


Figure 13.23: Ball and two maximal balls in a Euclidean plane.

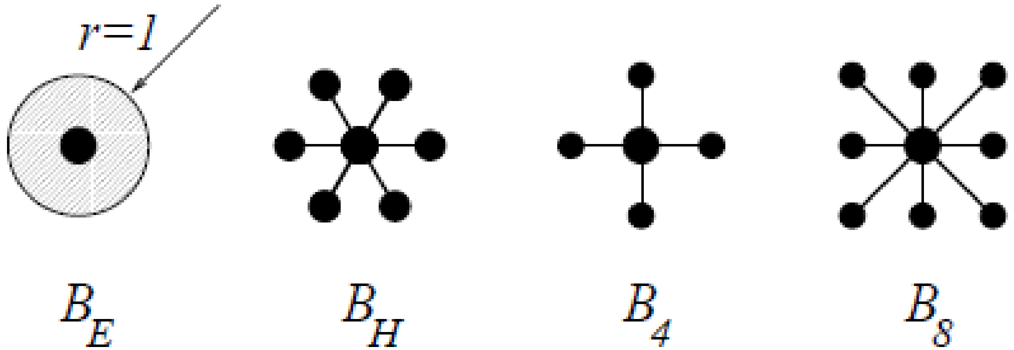


Figure 13.24: Unit-size disk for different distances, from left side: Euclidean distance, 6-, 4-, and 8-connectivity, respectively.

- plane $\mathbb{R}^2$ with usual Euclidean distance gives unit ball B_E

- three distances and balls are often defined in the discrete plane $\mathbb{Z}^2$
- if support is a square grid, two unit balls are possible:
- B_4 for 4-connectivity
- B_8 for 8-connectivity

- **skeleton by maximal balls** $S(X)$ of a set $X \subset \mathbb{Z}^2$ is the set of centers p of maximal balls

$$S(X) = \{p \in X : \exists r \geq 0, B(p,r) \text{ is a maximal ball of } X\}$$

- this definition of skeleton has intuitive meaning in Euclidean plane
- skeleton of a disk reduces to its center
- skeleton of a stripe with rounded endings is a unit thickness line at its center
- etc.

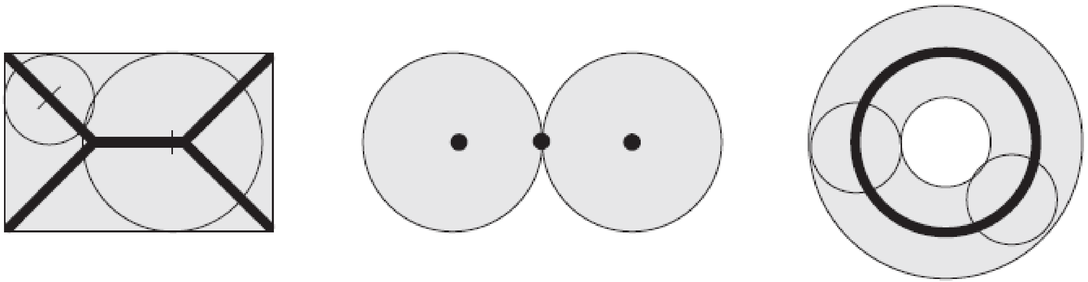


Figure 13.25: Skeletons of rectangle, two touching balls, and a ring.

- skeleton by maximal balls – two unfortunate properties
- does not necessarily preserve homotopy (connectivity)
- some of skeleton lines may be wider than one pixel
- skeleton is often substituted by sequential homotopic thinning that does not have these two properties
- dilation can be used in any of the discrete connectivities to create balls of varying radii
- nB = ball of radius n

$$nB = B \oplus B \oplus \ldots \oplus B$$

- skeleton by maximal balls ... union of the residues of opening of set X at all scales

$$S(X) = \bigcup_{n=0}^{\infty} \Big((X \ominus nB) \setminus \big((X \ominus nB) \circ B \big) \Big)$$

- trouble : skeletons are disconnected - a property is not useful in many applications
- **homotopic skeletons** that preserve connectivity are preferred

13.5.3 Thinning, thickening, and homotopic skeleton

- hit-or-miss transformation can be used for **thinning** and **thickening** of point
- sets
- image X and a composite structuring element $B = (B_1, B_2)$
- notice that B here is not a ball
- *Thinning*

$$X \oslash B = X \setminus (X \otimes B)$$

- *Thickening*

$$X \odot B = X \setminus (X \otimes B)$$

- thinning – part of object boundary is subtracted by set difference operation
- thickening – part of background boundary is added

- Thinning and thickening are dual transformations

$$(X \odot B)^c = X^c \oslash (B_2, B_1)$$

- Thinning and thickening often used sequentially
- Let $B = \{B_{(1)}, B_{(2)}, B_{(3)}, \dots, B_{(n)}\}$ denote a sequence of composite structuring
- elements $B_{(i)} = (B_{i_1}, B_{i_2})$
- **Sequential thinning** – sequence of *n* structuring elements

$$X \oslash B = \left(\left(\left(X \oslash B_{(1)}\right) \oslash B_{(1)}\right) \dots \oslash B_{(n)}\right)$$

- **sequential thickening**

$$X \odot B = \left(\left(\left(X \odot B_{(1)}\right) \odot B_{(1)}\right) \dots \odot B_{(n)}\right)$$

- several sequences of structuring elements $\{B_{(i)}\}$ are useful in practice
- e.g., permissible rotation of structuring element in digital raster (e.g., hexagonal, square, or octagonal)
- These sequences are called the **Golay alphabet**
- composite structuring element – expressed by a single matrix
- “one” means that this element belongs to *B*1 (it is a subset of objects in the
- hit-or-miss transformation)
- “zero” belongs to *B*2 and is a subset of the background
- $*$ ... element not used in matching process = its value is not significant

- Thinning and thickening sequential transformations converge to some image — the number of iterations needed depends on the objects in the image and the structuring element used

- if two successive images in the sequence are identical, the thinning (or thickening) is stopped

Sequential thinning by structuring element *L*

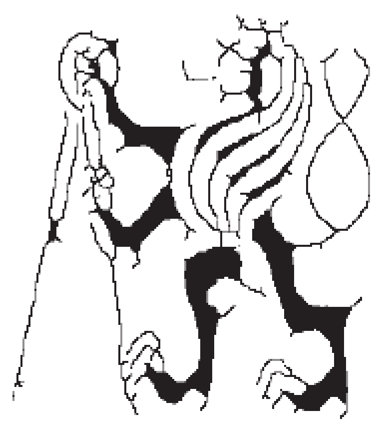
- thinning by *L* serves as homotopic substitute of the skeleton;
- final thinned image consists only of lines of width one and isolated points

- structuring element *L* from the Golay alphabet is given by

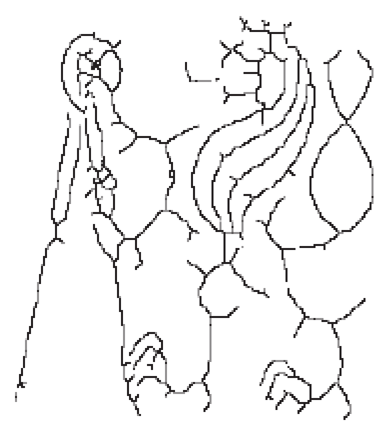
$$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ d & 1 & d \\ 1 & 1 & 1 \end{bmatrix}, \qquad L_1 = \begin{bmatrix} d & 0 & 0 \\ 1 & 1 & 0 \\ d & 1 & d \end{bmatrix}$$

- (The other six elements are given by rotation).

Original



after 5 iteration



final result

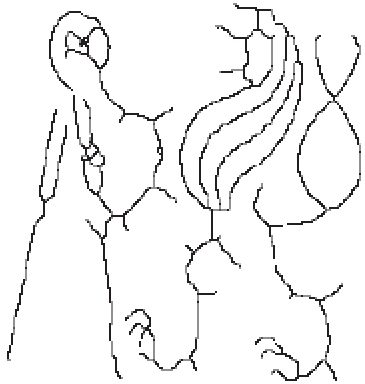
Sequential thinning by structuring element *E*

- structuring element *E* from the Golay alphabet is given by

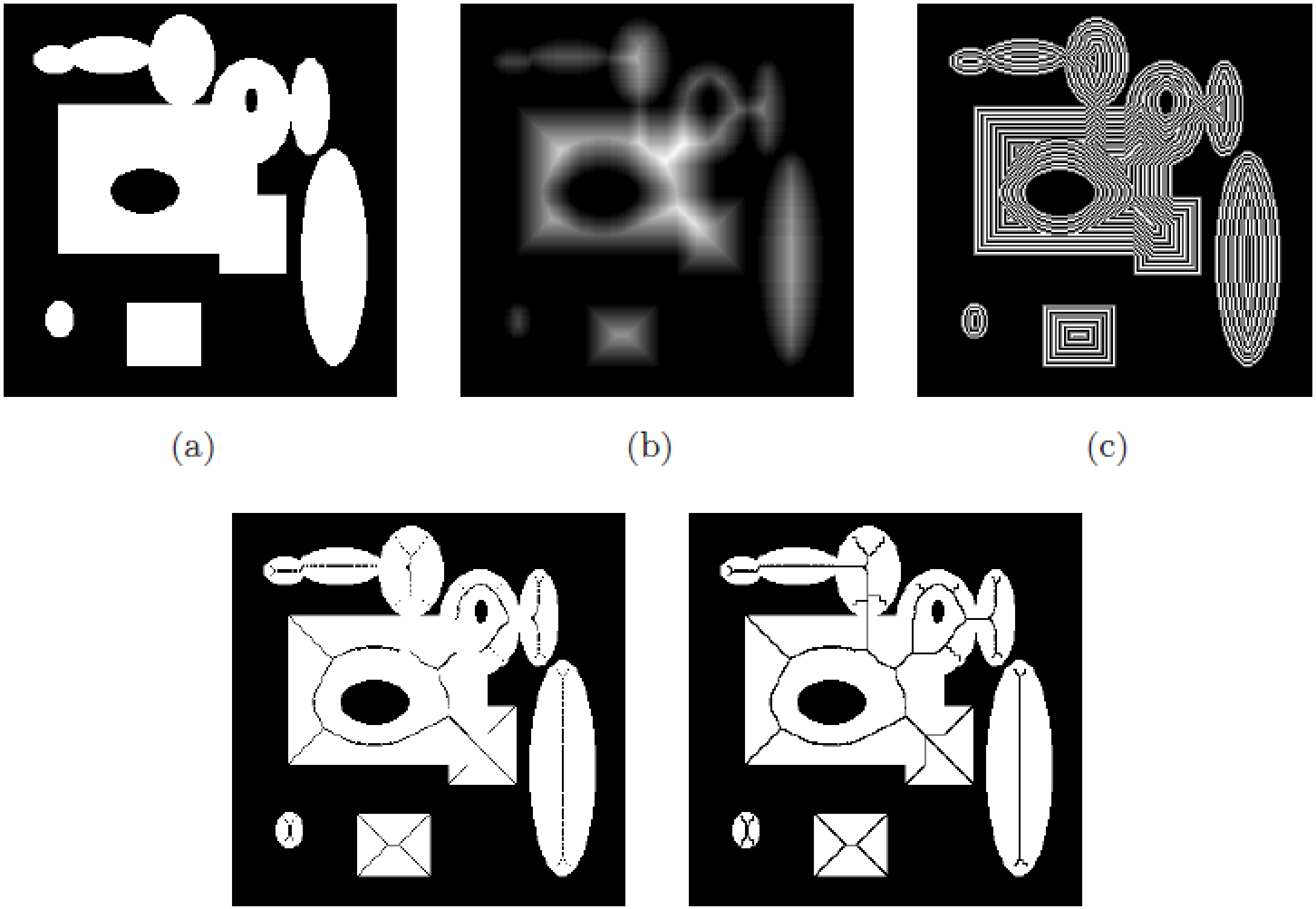
$$E_1 = \begin{bmatrix} d & 1 & d \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad L_1 = \begin{bmatrix} 0 & d & d \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Less jagged skeletons

Original



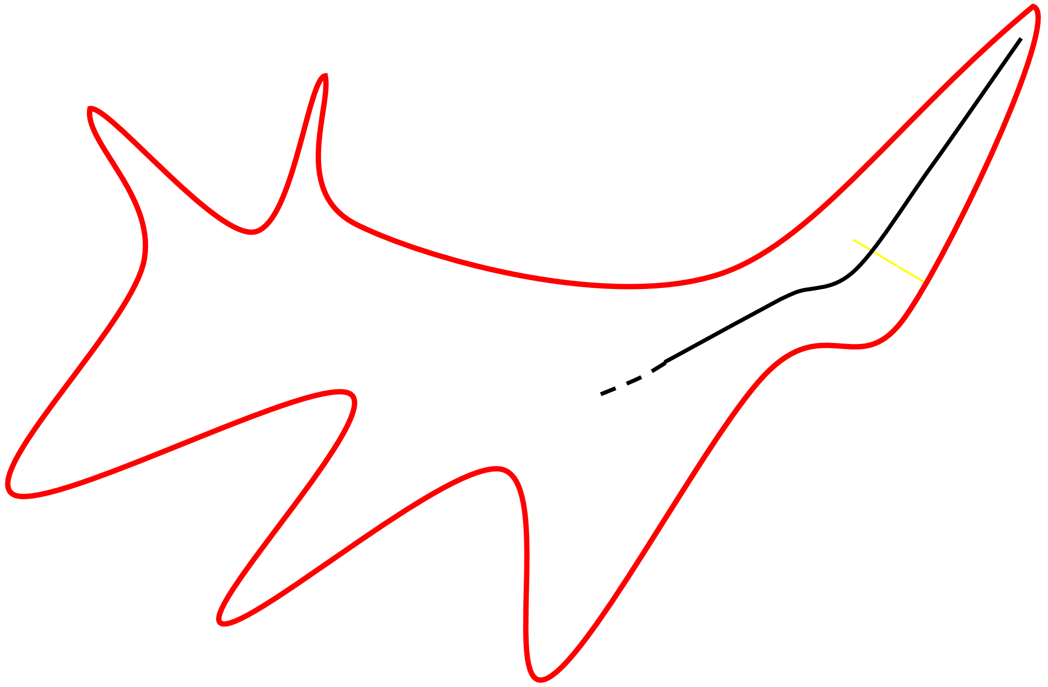
skeleton



(a) (b) (c)

(d) (e)

Axial line detection using Distance transform

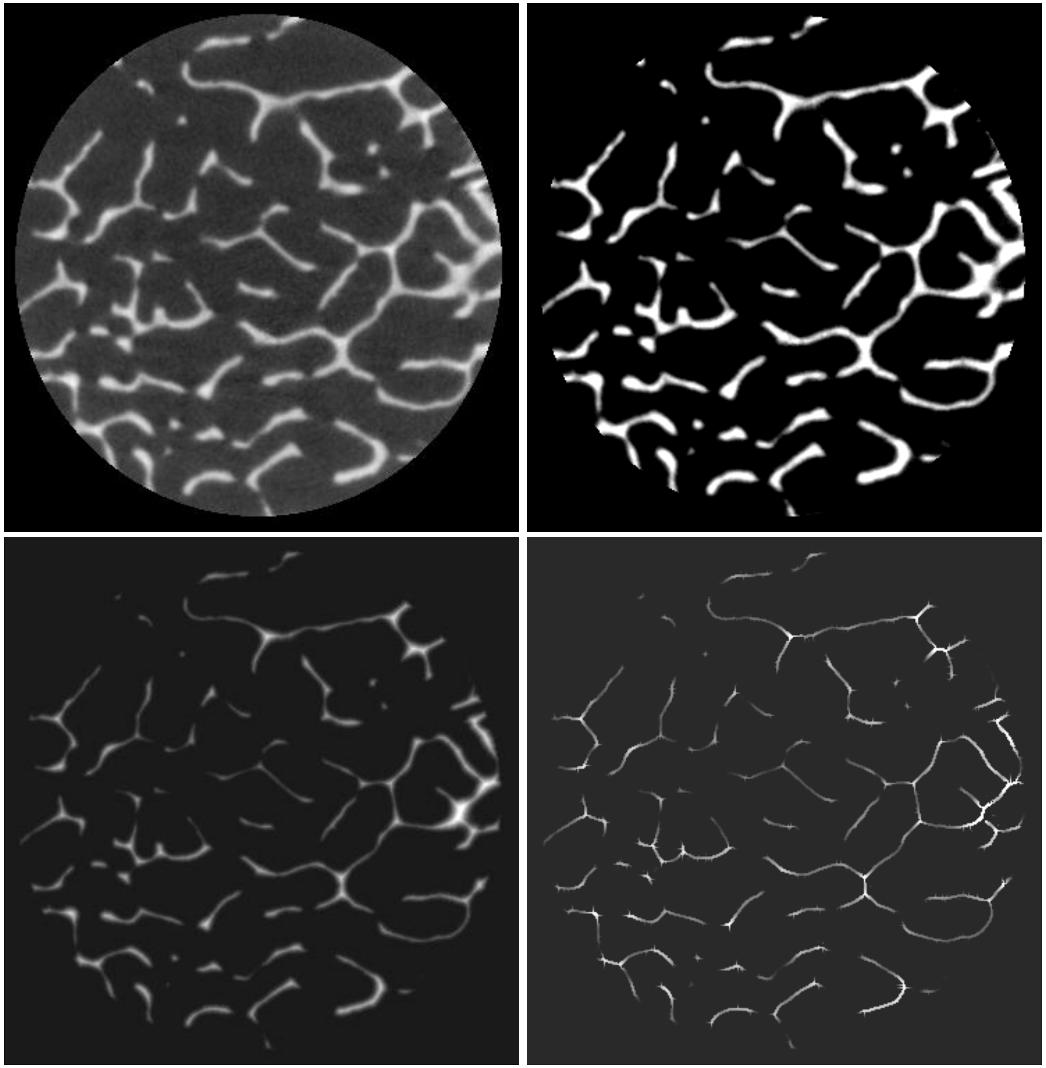


a point p is an axial point if there is no point p' such that a shortest path from p' to the boundary passes through p.

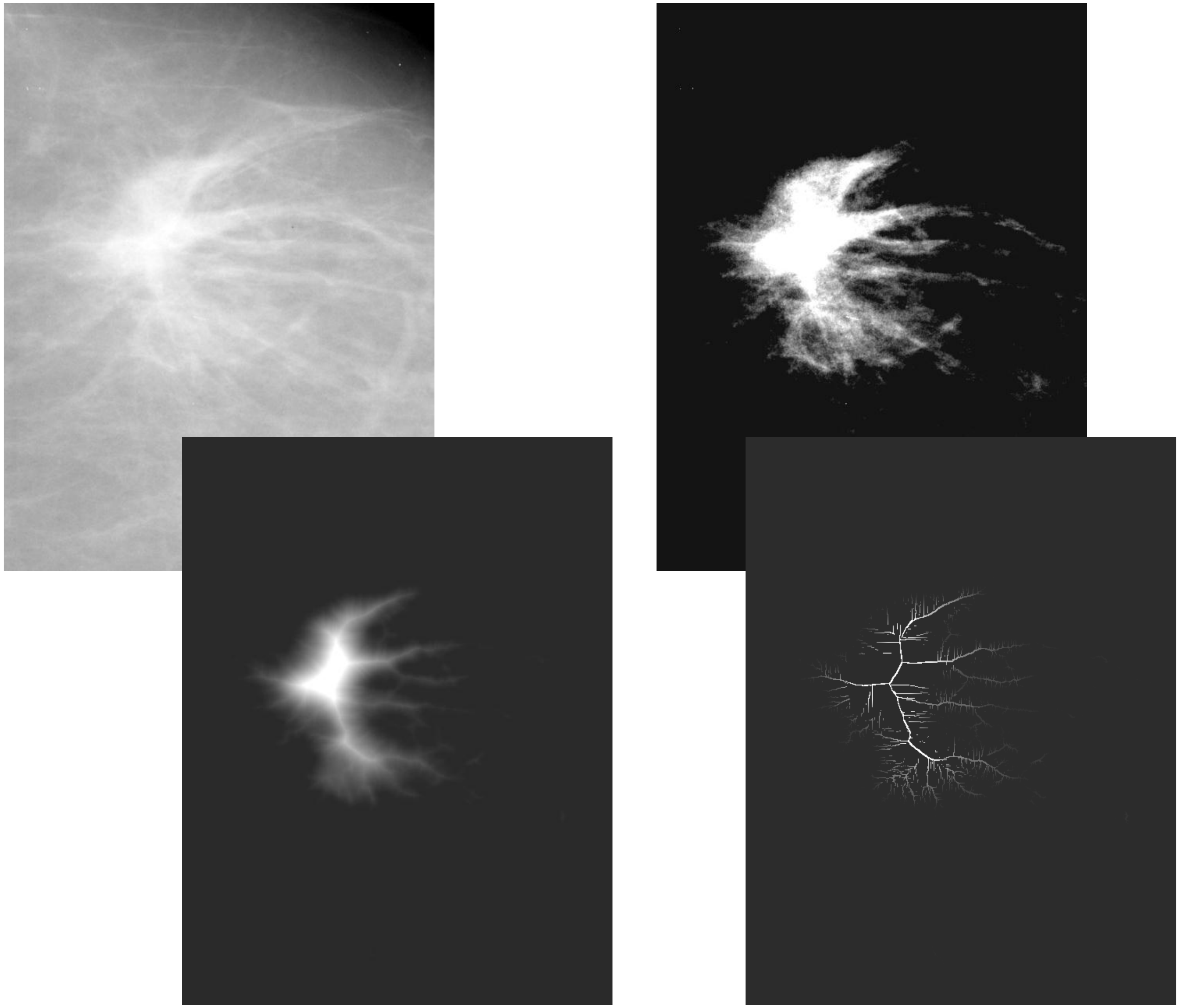
a point p is an axial point if there is no point q in the neighborhood of p such that

$$DT(q) = DT(p) + |p - q|$$

Axial line detection using Distance transform



Axial line detection using Distance transform



Axial line detection using Distance transform