Course Syllabus

1. **Color**
2. **Camera models, camera calibration**
3. **Advanced image pre-processing**
   - **Line detection**
   - **Corner detection**
   - **Maximally stable extremal regions**
4. **Mathematical Morphology**
   - **binary**
   - **gray-scale**
   - **skeletonization**
   - **granulometry**
   - **morphological segmentation**
   - **Scale in image processing**
5. **Wavelet theory in image processing**
6. **Image Compression**
7. **Texture**
8. **Image Registration**
   - **rigid**
   - **non-rigid**
   - **RANSAC**

13.4 Gray-scale dilation and erosion

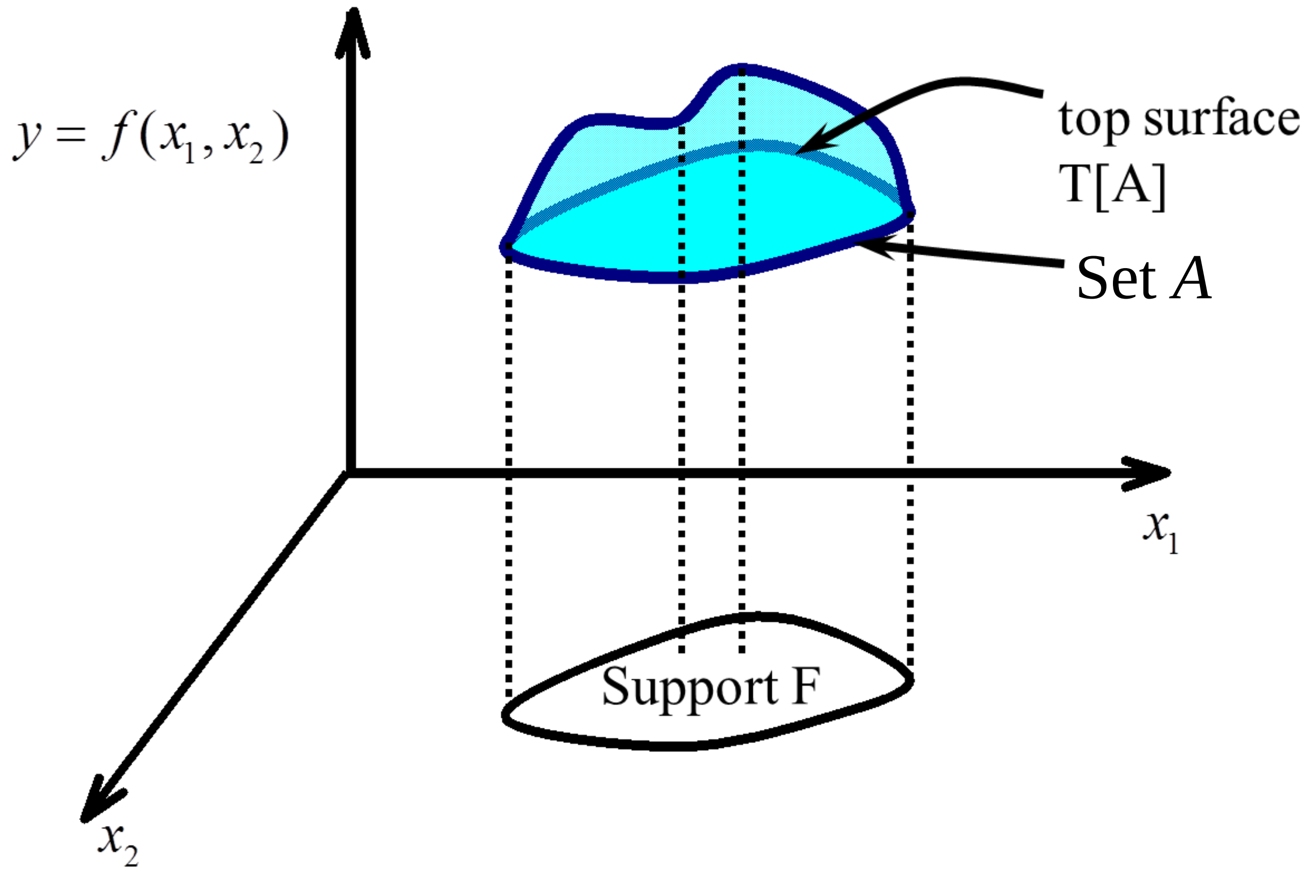
- Binary morphological operations are extendible to gray-scale images using the ‘min’ and ‘max’ operations.
- Erosion – assigns to each pixel minimum value in a neighborhood of corresponding pixel in input image
    - structuring element is richer than in binary case
    - structuring element is a function of two variables, specifies desired local gray-level property
    - value of structuring element is subtracted when minimum is calculated in the neighborhood
- Dilation – assigns maximum value in neighborhood of corresponding pixel in input image
    - value of structuring element is added when maximum is calculated in the neighborhood

13.4 Gray-scale dilation and erosion *continued*

- Such extension permits **topographic view** of gray-scale images
  - gray-level is interpreted as height of a particular location of a hypothetical landscape
  - light and dark spots in the image correspond to hills and valleys
  - such morphological approach permits the location of global properties of the image
    - ✓ valleys
    - ✓ mountain ridges (crests)
    - ✓ watersheds

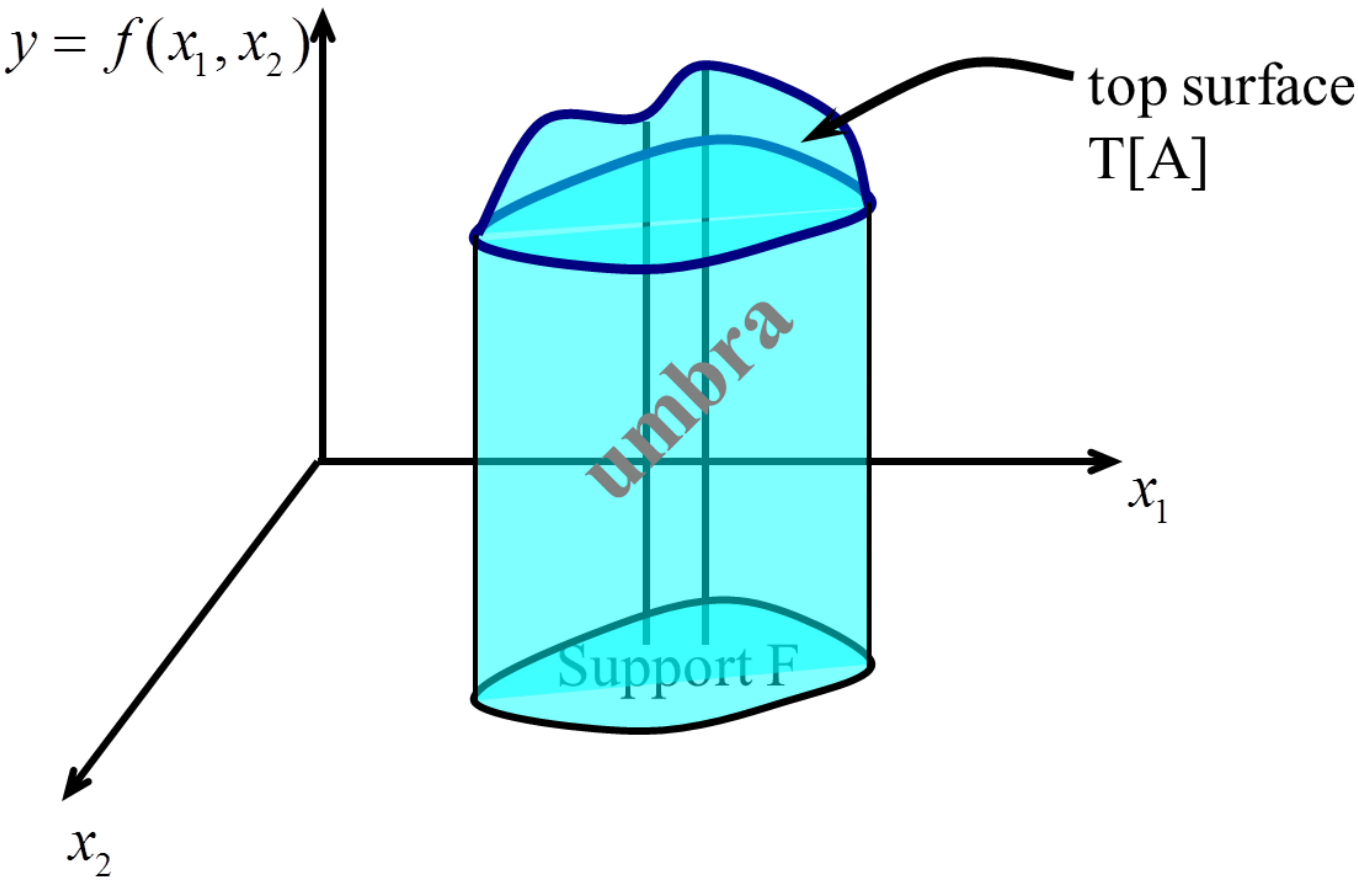
13.4 Gray-scale dilation and erosion *continued*

Concepts of **umbra** and **top** of the point set



13.4 Gray-scale dilation and erosion *continued*

Concepts of **umbra** and **top** of the point set



Gray-scale dilation is expressed as the dilation of umbras.

13.4.1 Top surface, umbra, and gray-scale dilation and erosion

- point set $A \subset \mathcal{E}^n$
- first $(n-1)$ co-ordinates ... spatial domain
- n^{th} co-ordinate ... function value (brightness)
- The **top surface** of set A = function defined on the $(n-1)$-dimensional support
- for each $(n-1)$-tuple, top surface is the highest value of the last co-ordinate of A for each $(n-1)$-tuple
- if A is Euclidean, highest value means supremum

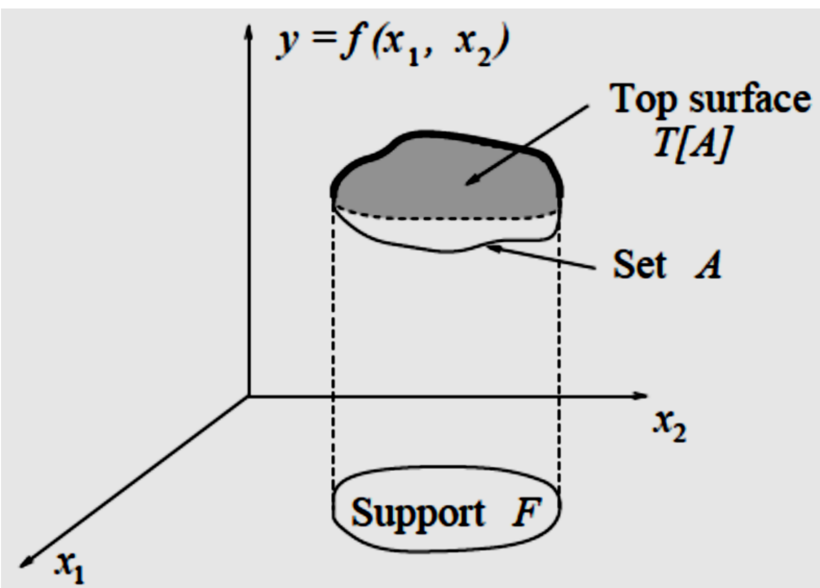


Figure 13.12: Top surface of the set A corresponds to maximal values of the function $f(x_1, x_2)$.

13.4.1 Top surface, umbra, and gray-scale dilation and erosion

- point set $A \subset \mathcal{E}^n$
- **support** $F = \{x \in \mathcal{E}^{n-1}$ for some $y \in \mathcal{E}, (x, y) \in A\}$
- **top surface** $T[A]$ is mapping $F \longrightarrow \mathcal{E}$

$$T[A](x) = \max\{y, (x, y) \in A\}$$

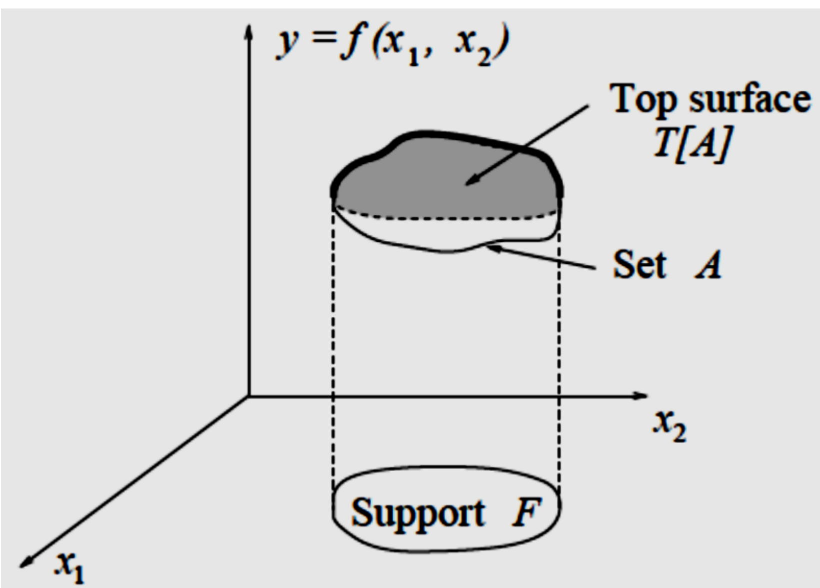


Figure 13.12: Top surface of the set A corresponds to maximal values of the function $f(x_1, x_2)$.

- **umbra** of function *f* is defined on some subset *F* (support) of (n−1)-dimensional space
- umbra – region of complete shadow when obstructing light by non-transparent object
- umbra of *f* … set consisting of top surface of *f* and everything below it

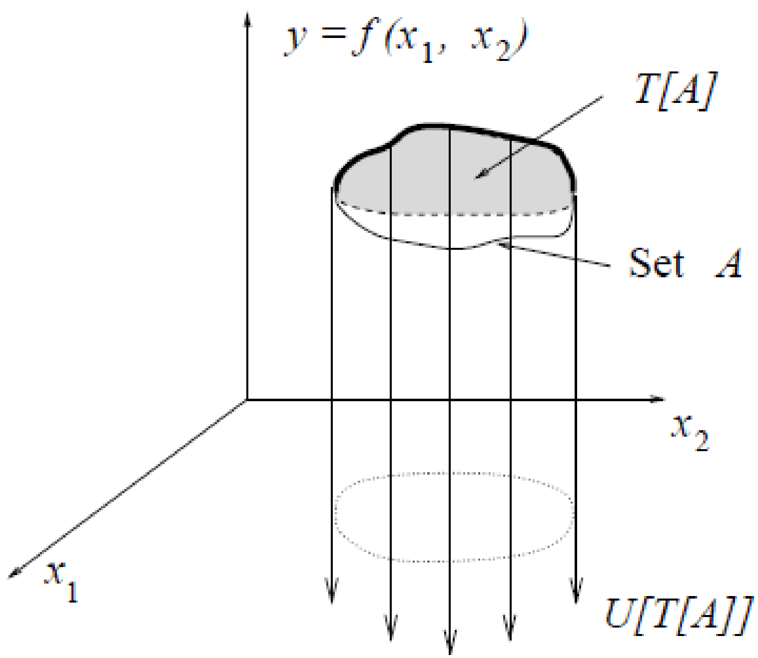


Figure 13.13: Umbra of the top surface of a set is the whole subspace below it.

- let $F \subseteq \mathcal{E}^{n-1}$ and $f\,:\ F \rightarrow \mathcal{E}^{n-1}$
- umbra $U[f] \subseteq F \times \mathcal{E}$

$$U[f] = \{(x, y)|\, x \in F, y \leq f(x)\}$$

- umbra of an umbra of *f* is an umbra.

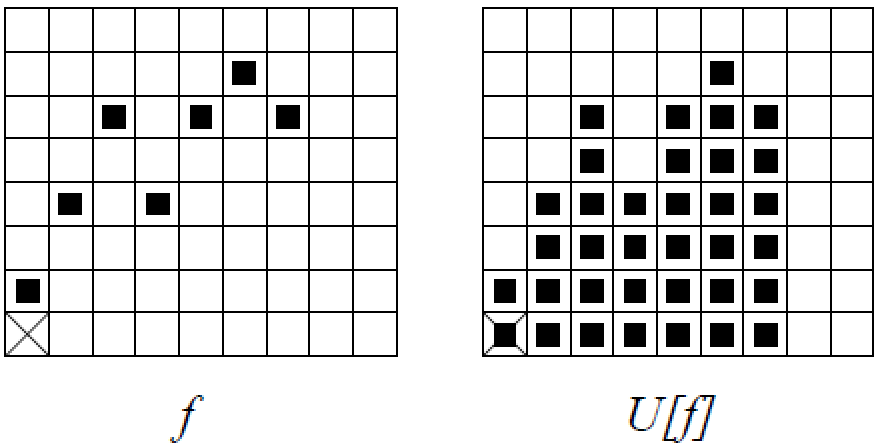


Figure 13.14: Example of a 1D function (left) and its umbra (right).

- umbra of an umbra of *f* is an umbra.

Gray-scale Dilation

- gray-scale dilation of two functions
  ... top surface of the dilation of their umbras

- let $F, K \subseteq \mathcal{E}^{n-1}$ and $f : F \to \mathcal{E}$ and $k : K \to \mathcal{E}$

- **dilation** $\oplus$ of f by k, $f \oplus k : F \oplus K \longrightarrow \mathcal{E}$ is defined by
$$f \oplus k = T(U[f] \oplus U[k])$$
  $\oplus$ on the left-hand side is dilation in the gray-scale image domain
  $\oplus$ on the right-hand side is dilation in the binary image

- no new symbol introduced
- the same applies to erosion $\ominus$ later
- similar to binary dilation
  - first function f represents image
  - second function k represents structuring element

Gray-scale Dilation: Illustration

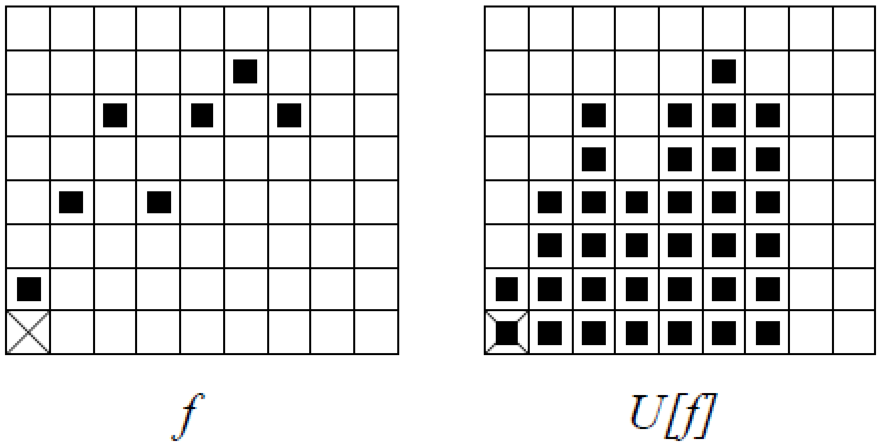


Figure 13.14: Example of a 1D function (left) and its umbra (right).

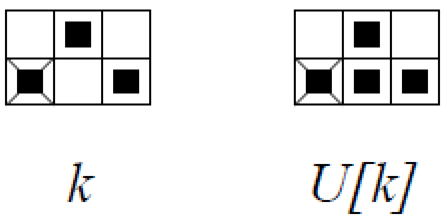


Figure 13.15: A structuring element: 1D function (left) and its umbra (right).

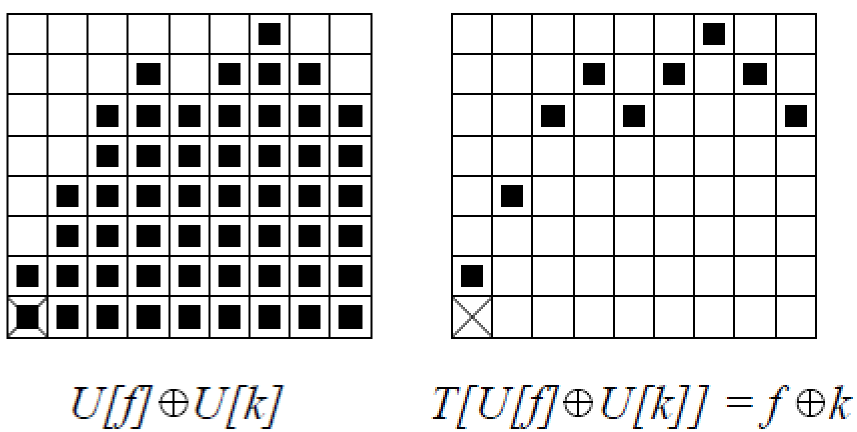


Figure 13.16: 1D example of gray-scale dilation. The umbras of the 1D function f and structuring element k are dilated first, $U[f] \oplus U[k]$. The top surface of this dilated set gives the result,

$$f \oplus k = T(U[f] \oplus U[k])$$

.

- This explains what gray-scale dilation means
- does not give a reasonable algorithm for actual computations in hardware

- computationally plausible way to calculate dilation ... taking the maximum of a set of sums:

$$(f \oplus k)(x) = \max\{f(x-z) + k(z), x-z \in F, z \in K\}$$

- computational complexity is the same as for convolution in linear filtering, where a summation of products is performed

- Case: when the structuring element is binary

$$(f \oplus K)(x) = \max\{f(x-z), x-z \in F, z \in K\}$$

Gray-scale Erosion

- definition of **gray-scale erosion** is analogous to gray-scale dilation.
- gray-scale erosion of two functions (point sets)
- Takes their umbras.
- Erodes them using binary erosion.
- Gives the result as the top surface.

- let $F, K \subseteq \mathcal{E}^{n-1}$ and $f : F \rightarrow \mathcal{E}$ and $k : K \rightarrow \mathcal{E}$
- **erosion** $\ominus$ of f by k, $f \ominus k : F \ominus K \rightarrow \mathcal{E}$ is defined by

$$f \ominus k = T(U[f] \ominus U[k])$$

- to decrease computational complexity, the actual computations performed as the minimum of a set of differences (notice similarity to correlation)

$$(f \ominus k)(x) = \min\{f(x-z) - k(z), x - z \in F, z \in K\}$$

- Case: when the structuring element is binary

$$(f \ominus K)(x) = \min\{f(x-z), x - z \in F, z \in K\}$$

Gray-scale Erosion: Illustration

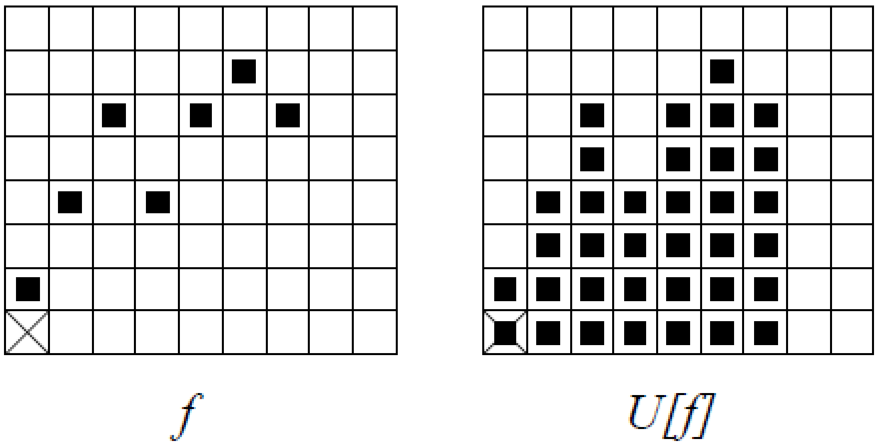


Figure 13.14: Example of a 1D function (left) and its umbra (right).

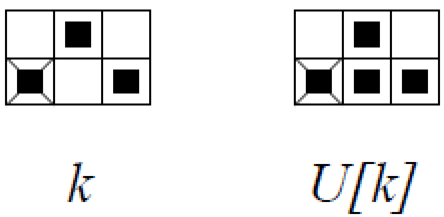


Figure 13.15: A structuring element: 1D function (left) and its umbra (right).

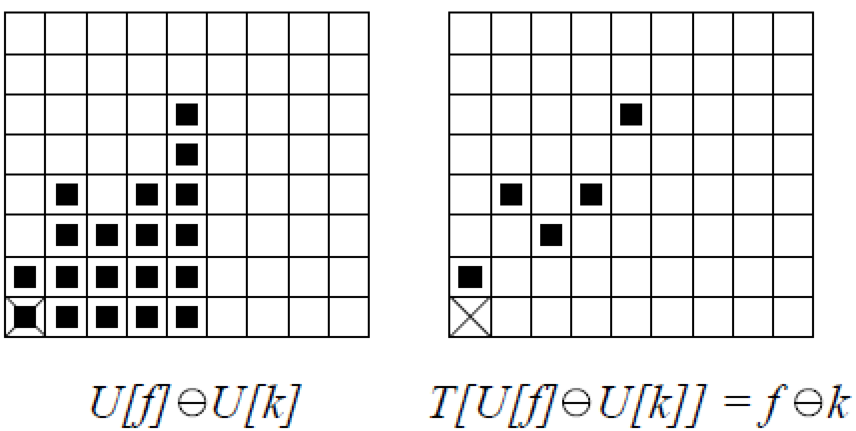


Figure 13.17: 1D example of gray-scale erosion. The umbras of 1D function f and structuring element k are eroded first, $U[f] \ominus U[k]$. The top surface of this eroded set gives the result,

$$f \ominus k = T(\, U[f] \ominus U[k]\,)$$

Example

- microscopic image of cells corrupted by noise
- aim is to reduce noise and locate individual cells
- 3×3 structuring element used for erosion/dilation
- individual cells can be located by the reconstruction operation (Section 13.5.4)
- original image is used as a mask and the dilated image in Figure 13.18c is an input for reconstruction
- black spots in (d) panel depict cells

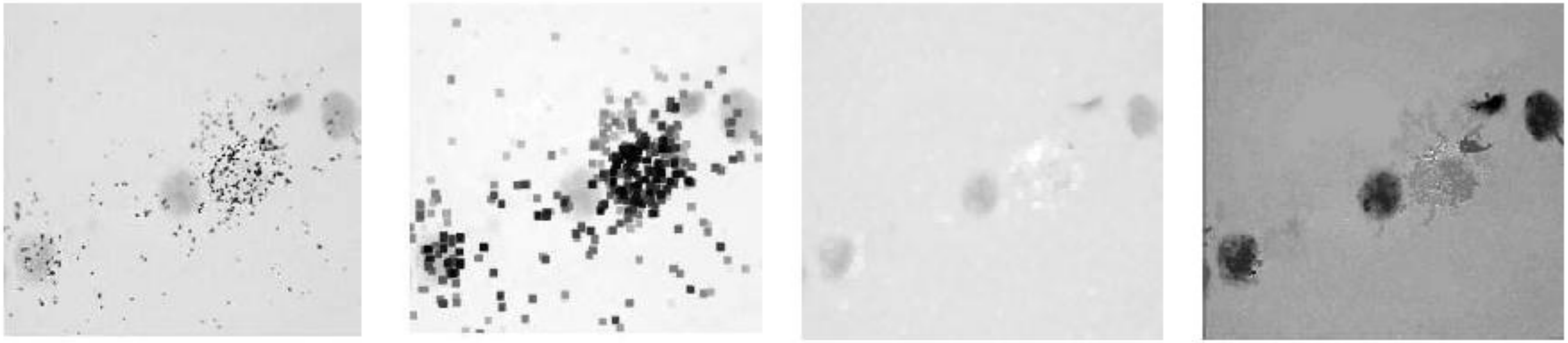


Figure 13.18: Morphological pre-processing: (a) cells in a microscopic image corrupted by noise; (b) eroded image; (c) dilation of (b), the noise has disappeared; (d) reconstructed cells. *Courtesy of P. Kodl, Rockwell Automation Research Center, Prague, Czech Republic.*

13.4.2 Opening and Closing

Gray-scale opening and closing

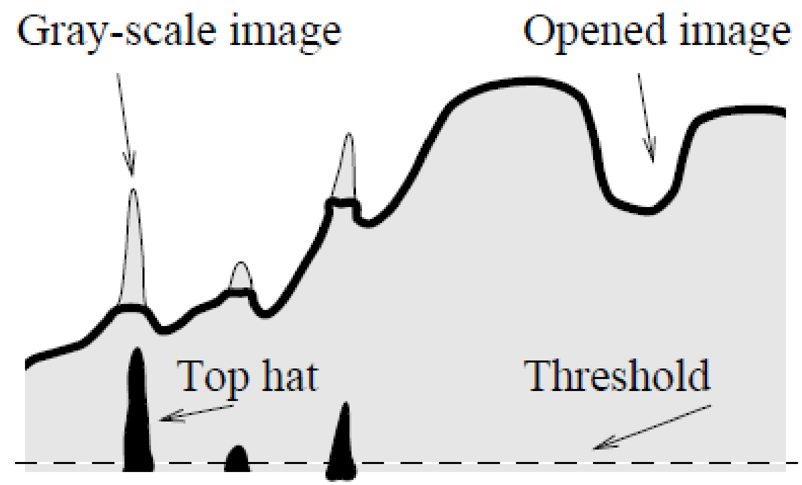
- defined as in binary morphology
- **Gray-scale opening** $f \circ k = (f \ominus k) \oplus k$
- **gray-scale closing** $f \bullet k = (f \oplus k) \ominus k$
- **duality** between opening and closing is expressed as ($\breve{k}$ means transpose)

$$-(f \circ k)(x) = \left((-f) \bullet \breve{k}\right)(x)$$

- opening of f by structuring element k can be interpreted as sliding k on the landscape f
- position of all highest points reached by some part of k during the slide gives the opening,
- similar interpretation exists for erosion
- Gray-scale opening and closing often used to extract parts of a gray-scale image with given shape and gray-scale structure

13.4.3 Top hat transformation

- simple tool for segmenting objects in gray-scale images that differ in brightness from background even when background is uneven
- top-hat transform superseded by watershed segmentation for more complicated backgrounds
- gray-level image *X*, structuring element *K*



- residue of opening as compared to original image $X \,/\, (X \circ K)$ is **top hat transformation**
- good tool for extracting light (or dark) objects on dark (light) possibly slowly changing background
- parts of image that cannot fit into structuring element *K* are removed by opening
- Subtracting opened image from original – removed objects stand out clearly
- actual segmentation performed by simple thresholding

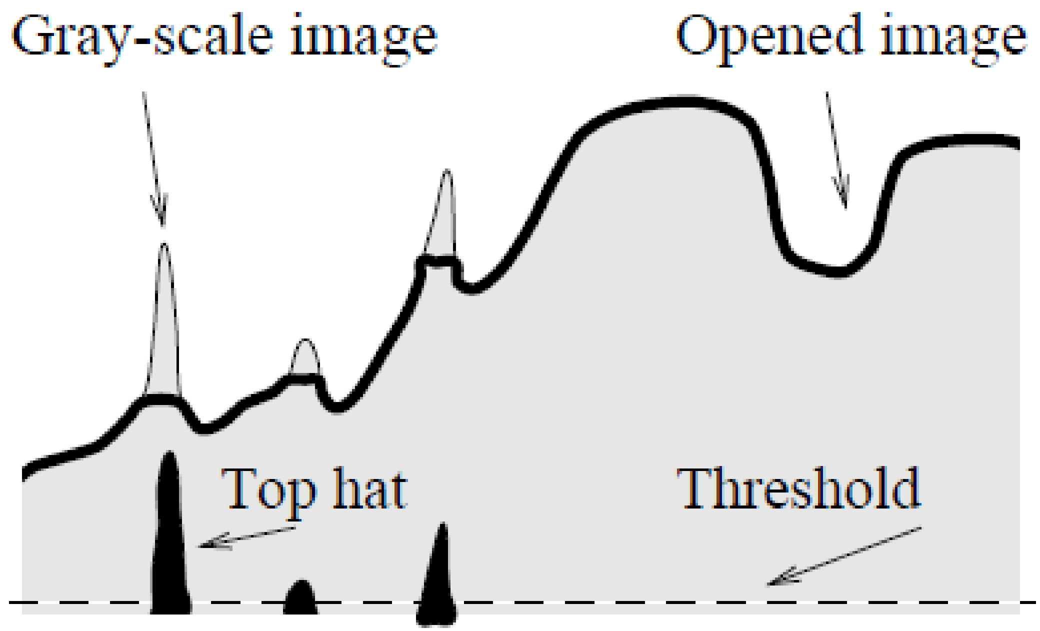


Figure 13.19: The top hat transform permits the extraction of light objects from an uneven background.

Example from visual industrial inspection

- glass capillaries for mercury maximal thermometers had the following problem: thin glass tube should be narrowed in one particular place to prevent mercury falling back when the temperature decreases from the maximal value done by using a narrow gas flame and low pressure in the capillary

- capillary is illuminated by a collimated light beam—when the capillary wall collapses due to heat and low pressure, an instant specular reflection is observed and serves as a trigger to cover the gas flame

- Originally, machine was controlled by a human operator who looked at the tube image projected optically on the screen; the gas flame was covered when the specular reflection was observed

- task had to be automated and the trigger signal obtained from a digitized image

  ⇒ specular reflection is detected by a morphological procedure

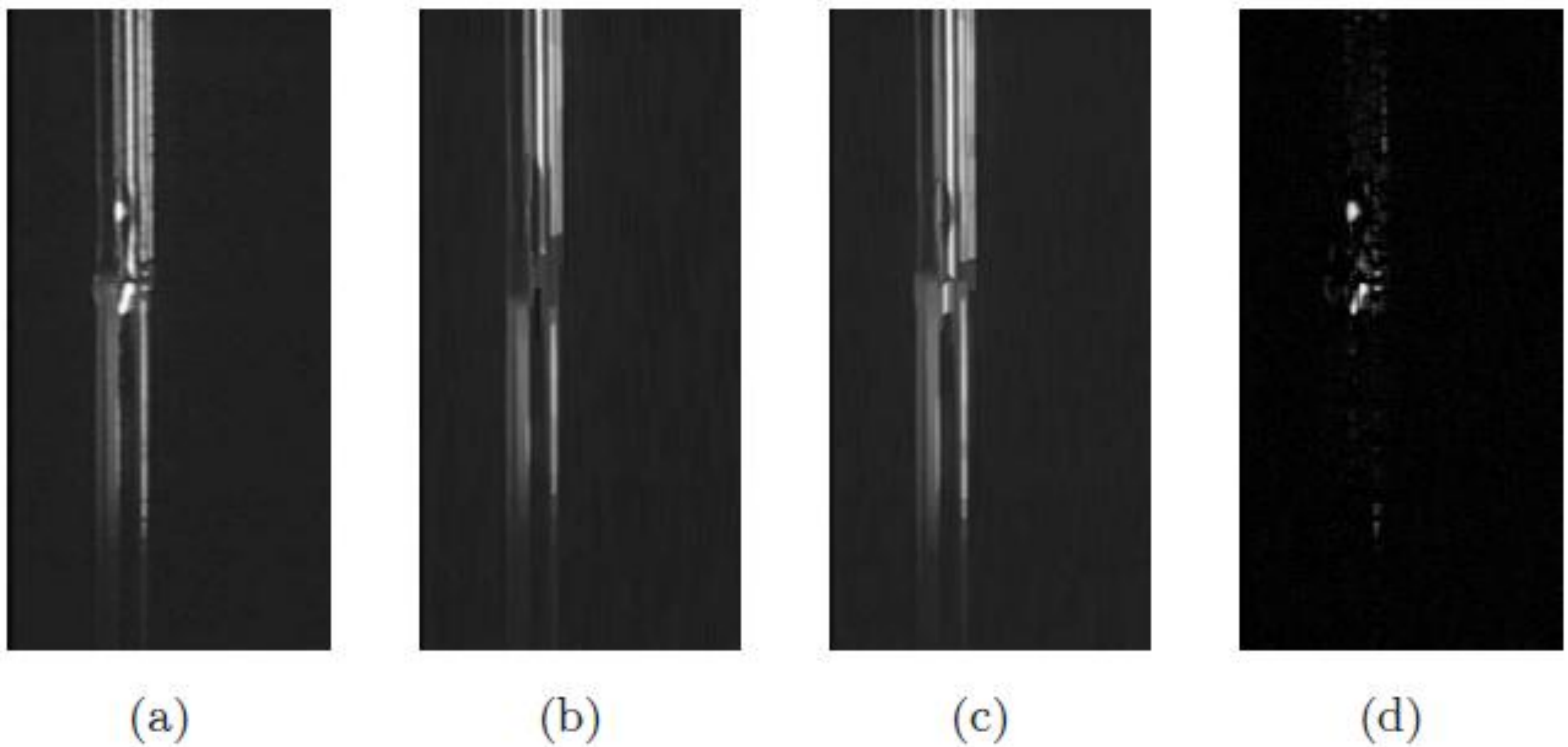


Figure 13.20: An industrial example of gray-scale opening and top hat segmentation, i.e., image-based control of glass tube narrowing by gas flame. (a) Original image of the glass tube, 512×256 pixels. (b) Erosion by a one-pixel-wide vertical structuring element 20 pixels long. (c) Opening with the same element. (d) Final specular reflection segmentation by the top hat transformation. *Courtesy of V. Smutný, R. Šára, CTU Prague, P. Kodl, Rockwell Automation Research Center, Prague, Czech Republic.*

13.5 Skeletons and object marking

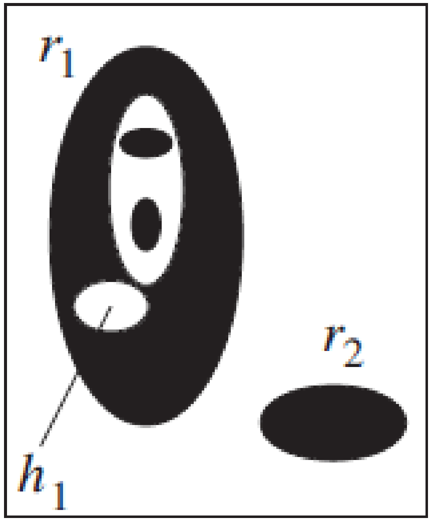
13.5.1 Homotopic transformations

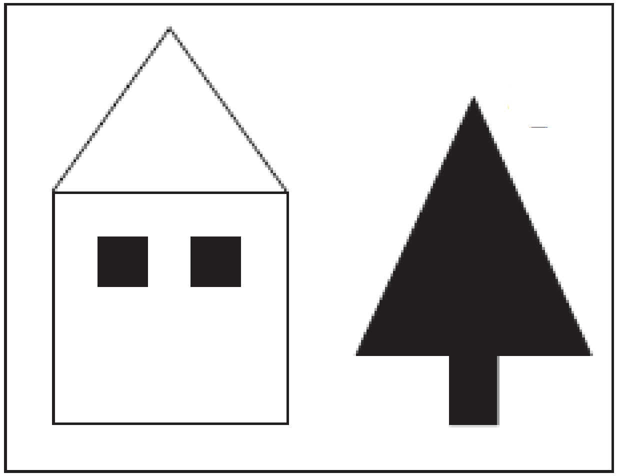
- transformation is homotopic if it does not change the continuity relation between regions and holes in the image.

- this relation expressed by homotopic tree
  - its root ... image background
  - first-level branches ... objects (regions)
  - second-level branches ... holes
  - etc.

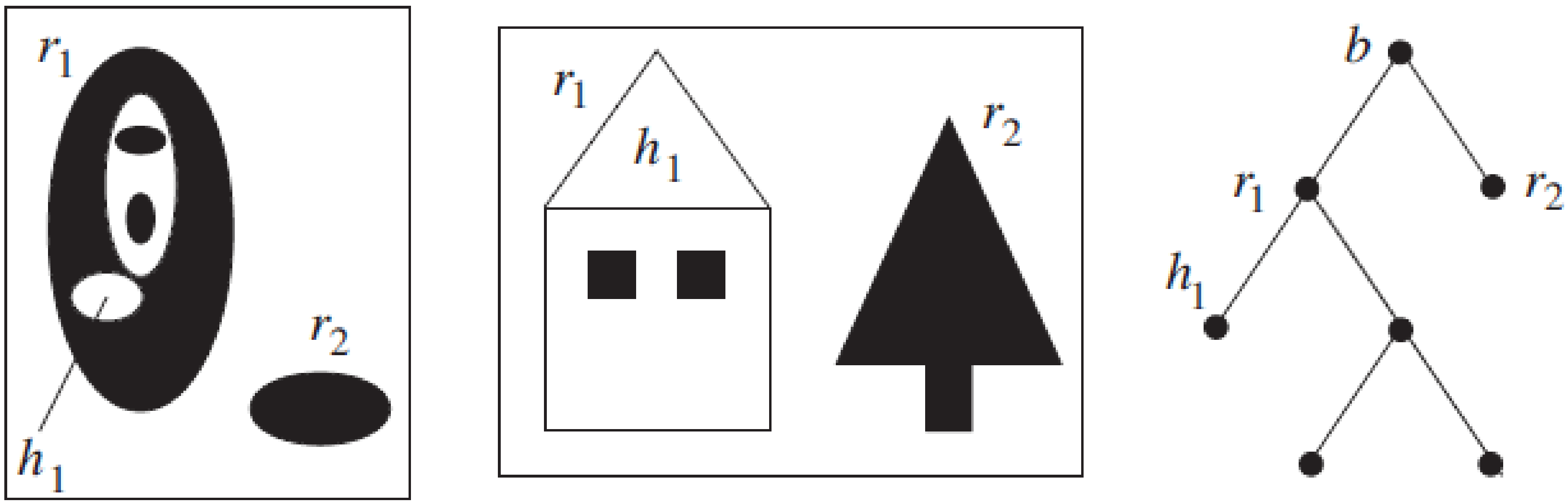
- transformation is homotopic if it does not change homotopic tree

Homotopic Tree

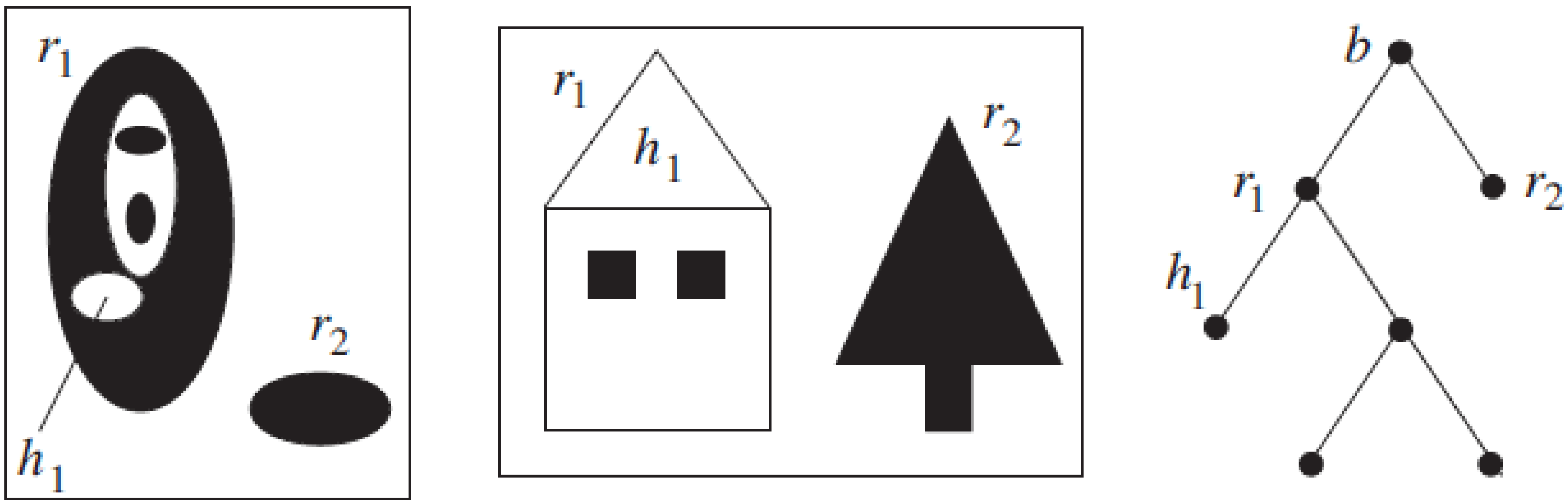




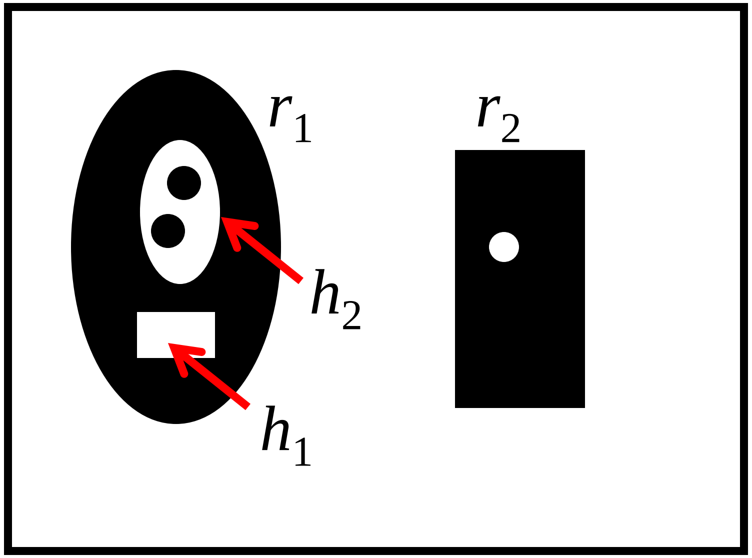
Homotopic Tree

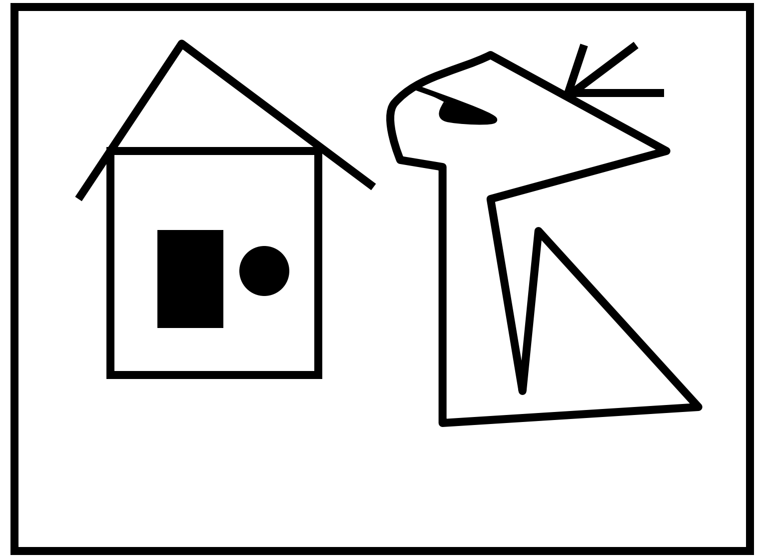


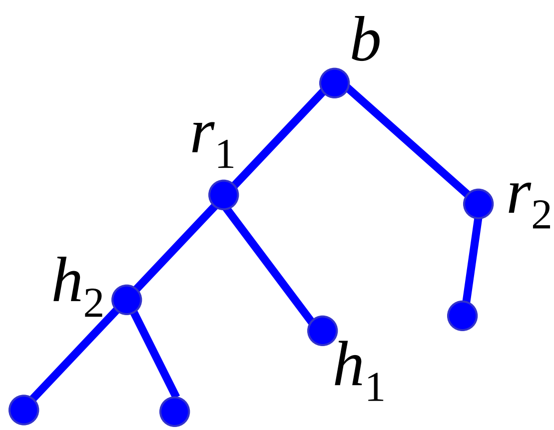
Homotopic Tree



Homotopic Tree

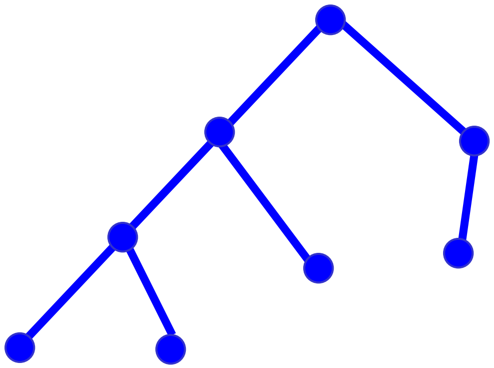






Quitz: Homotopic Transformation

- What is the relation between an element in the ith and i+1th levels?



13.5.2 Skeleton, maximal ball

- skeletonization = **medial axis transform**
- ‘grassfire’ scenario
- A grassfire starts on the entire region boundary at the same instant – propagates towards the region interior with constant speed
- **skeleton** $S(X)$ ... set of points where two or more fire-fronts meet

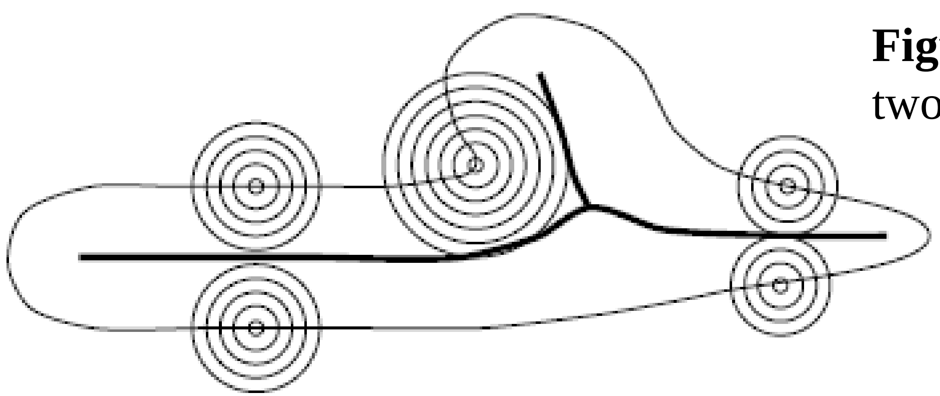


Figure 13.22: Skeleton as points where two or more fire-fronts of grassfire meet.

- Formal definition of skeleton based on maximal ball concept
- **ball** $B(p, r)$, $r \geq 0$ ... set of points with distances d from center $\leq r$
- ball B included in a set X is **maximal** if and only if there is no larger ball included in X that contains B

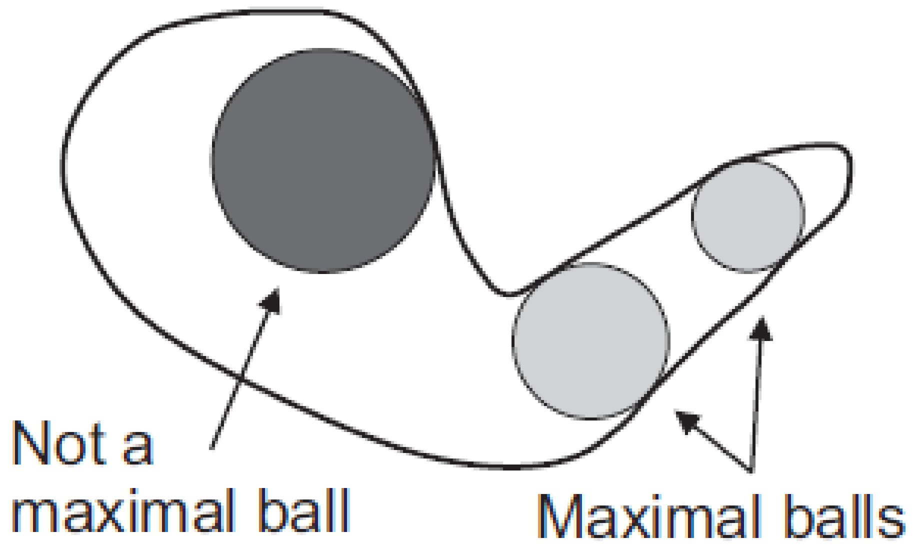


Figure 13.23: Ball and two maximal balls in a Euclidean plane.

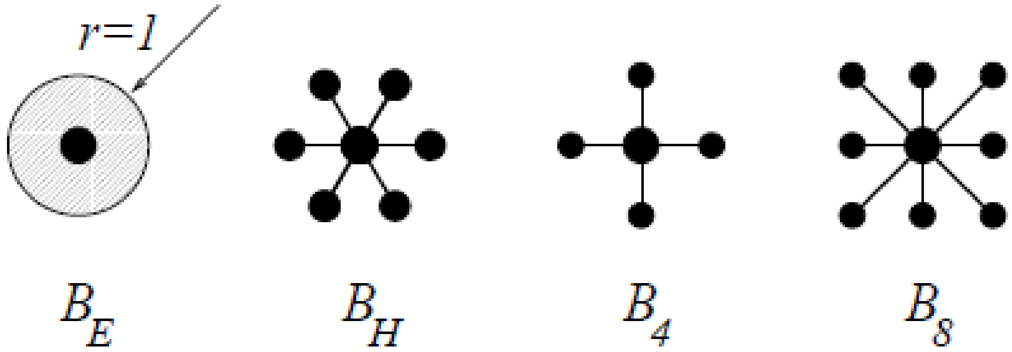


Figure 13.24: Unit-size disk for different distances, from left side: Euclidean distance, 6-, 4-, and 8-connectivity, respectively.

- plane $\mathbb{R}^2$ with usual Euclidean distance gives unit ball B_E

- three distances and balls are often defined in the discrete plane $\mathbb{Z}^2$
- if support is a square grid, two unit balls are possible:
- B_4 for 4-connectivity
- B_8 for 8-connectivity

- **skeleton by maximal balls** $S(X)$ of a set $X \subset \mathbb{Z}^2$ is the set of centers p of maximal balls

$$S(X) = \{p \in X : \exists r \geq 0, B(p,r) \text{ is a maximal ball of } X\}$$

- this definition of skeleton has intuitive meaning in Euclidean plane
- skeleton of a disk reduces to its center
- skeleton of a stripe with rounded endings is a unit thickness line at its center
- etc.

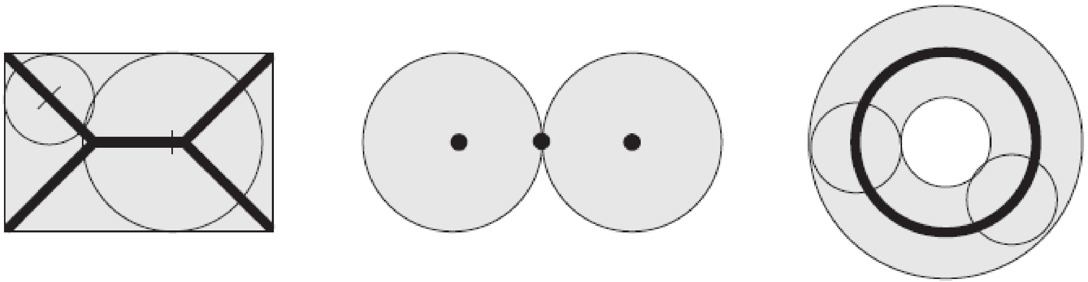


Figure 13.25: Skeletons of rectangle, two touching balls, and a ring.

- skeleton by maximal balls – two unfortunate properties
- does not necessarily preserve homotopy (connectivity)
- some of skeleton lines may be wider than one pixel
- skeleton is often substituted by sequential homotopic thinning that does not have these two properties
- dilation can be used in any of the discrete connectivities to create balls of varying radii
- nB = ball of radius n

$$nB = B \oplus B \oplus \ldots \oplus B$$

- skeleton by maximal balls ... union of the residues of opening of set X at all scales

$$S(X) = \bigcup_{n=0}^{\infty} \Big((X \ominus nB) \setminus \big((X \ominus nB) \circ B \big) \Big)$$

- trouble : skeletons are disconnected - a property is not useful in many applications
- **homotopic skeletons** that preserve connectivity are preferred