55:148 Digital Image Processing

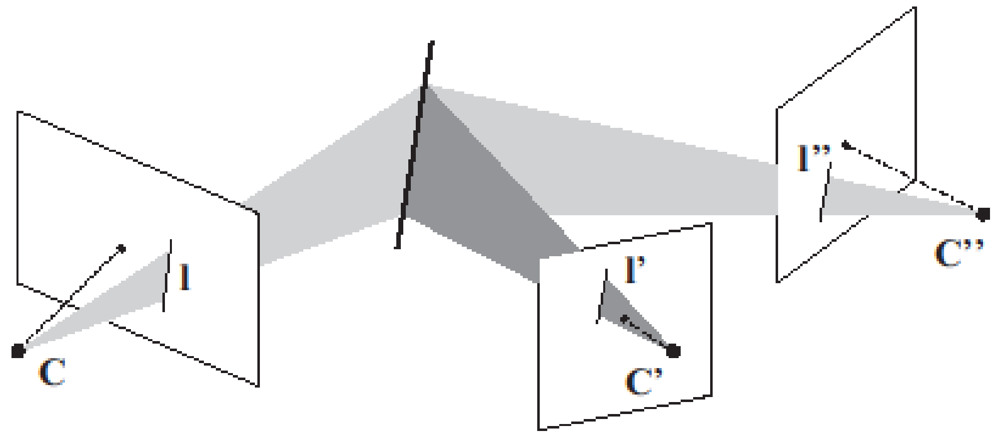
Chapter 11

3D Vision, Geometry

Topics:

- Basics of projective geometry
    - Points and hyperplanes in projective space
    - Homography
    - Estimating homography from point correspondence
- The single perspective camera
    - An overview of single camera calibration
    - Calibration of one camera from the known scene
- Scene reconstruction from multiple views
    - Triangulation
    - Projective reconstruction
    - Matching constraints
    - Bundle adjustment
- Two cameras, stereopsis
    - The geometry of two cameras. The fundamental matrix
    - Relative motion of the camera; the essential matrix
    - Estimation of a fundamental matrix from image point correspondences
    - Camera Image rectification
    - Applications of the epipolar geometry in vision
- Three and more cameras
    - Stereo correspondence algorithms
- Review

Three cameras and trifocal tensor



Three camera matrices:

$$M = [I \mid \mathbf{0}]; \quad M' = \left[\widetilde{M}' \mid e'\right]; \quad M'' = \left[\widetilde{M}'' \mid e''\right]$$

e', e'': projection of the optical center C on respective camera

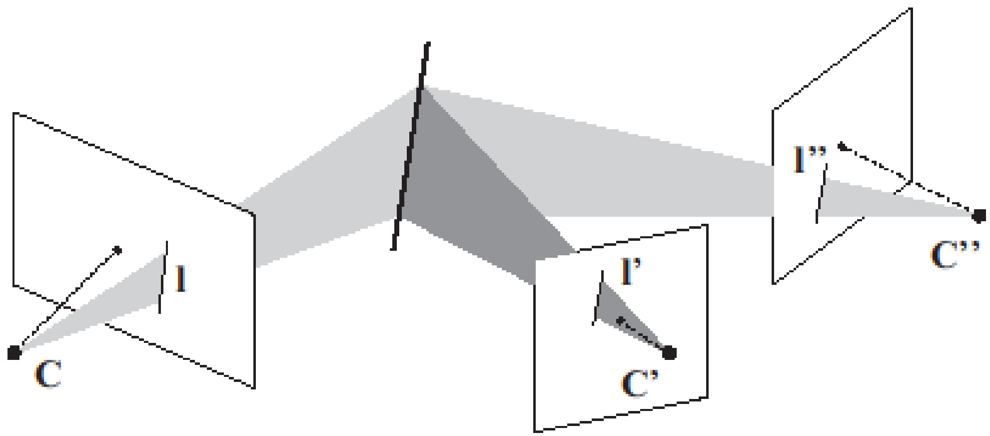
Scene planes derived by three matching projection lines $\mathbf{l}, \mathbf{l}', \mathbf{l}''$

$$\mathbf{a} = M^{\mathrm{T}}\mathbf{l} = \begin{bmatrix} \mathbf{l} \\ 0 \end{bmatrix}, \qquad \mathbf{a}' = M'^{\mathrm{T}}\mathbf{l}' = \begin{bmatrix} \widetilde{M}'^{\mathrm{T}}\mathbf{l}' \\ e'^{\mathrm{T}}\mathbf{l}' \end{bmatrix}, \qquad \mathbf{a}'' = M''^{\mathrm{T}}\mathbf{l}'' = \begin{bmatrix} \widetilde{M}''^{\mathrm{T}}\mathbf{l}'' \\ e''^{\mathrm{T}}\mathbf{l}'' \end{bmatrix}$$

Following that the three planes intersect on a common line

$$\mathbf{a} = \lambda'\mathbf{a}' + \lambda''\mathbf{a}'' \qquad \text{for some scalar values } \lambda' \text{ and } \lambda''$$

Three cameras and trifocal tensor *Continued…..*



Using

$$\mathbf{a} = \begin{bmatrix} \mathbf{l} \\ 0 \end{bmatrix}, \quad \mathbf{a}' = \begin{bmatrix} \widetilde{M}'^{\mathrm{T}}\mathbf{l}' \\ e'^{\mathrm{T}}\mathbf{l}' \end{bmatrix}, \quad \mathbf{a}'' = \begin{bmatrix} \widetilde{M}''^{\mathrm{T}}\mathbf{l}'' \\ e''^{\mathrm{T}}\mathbf{l}'' \end{bmatrix} \quad \text{and} \quad \mathbf{a} = \lambda'\mathbf{a}' + \lambda''\mathbf{a}''$$

We get

$$0 = \lambda' e'^{\mathrm{T}}\mathbf{l}' + \lambda'' e''^{\mathrm{T}}\mathbf{l}'' \Rightarrow \lambda' e'^{\mathrm{T}}\mathbf{l}' = -\lambda'' e''^{\mathrm{T}}\mathbf{l}''$$

$$\mathbf{l} = \lambda'\widetilde{M}'^{\mathrm{T}}\mathbf{l}' + \lambda''\widetilde{M}''^{\mathrm{T}}\mathbf{l}'' = \lambda\left[\left(e'^{\mathrm{T}}\mathbf{l}'\right)\widetilde{M}'^{\mathrm{T}}\mathbf{l}' - \left(e''^{\mathrm{T}}\mathbf{l}''\right)\widetilde{M}''^{\mathrm{T}}\mathbf{l}''\right] \; |\lambda\text{: scalar}$$

Leads to

$$\mathbf{l} = \lambda[\mathbf{l}'^{\mathrm{T}}T_1\mathbf{l}'' \quad \mathbf{l}'^{\mathrm{T}}T_2\mathbf{l}'' \quad \mathbf{l}'^{\mathrm{T}}T_3\mathbf{l}'']$$

Where, $T_i = \mathbf{m}_i' e''^{\mathrm{T}} - \mathbf{m}_i'' e'^{\mathrm{T}}$ for $i = 1,2,3;$ $\quad M = [\mathbf{m}_1 \quad \mathbf{m}_2 \quad \mathbf{m}_3]$

Putting it into $\mathbf{l}^{\mathrm{T}}\mathbf{u} = 0$

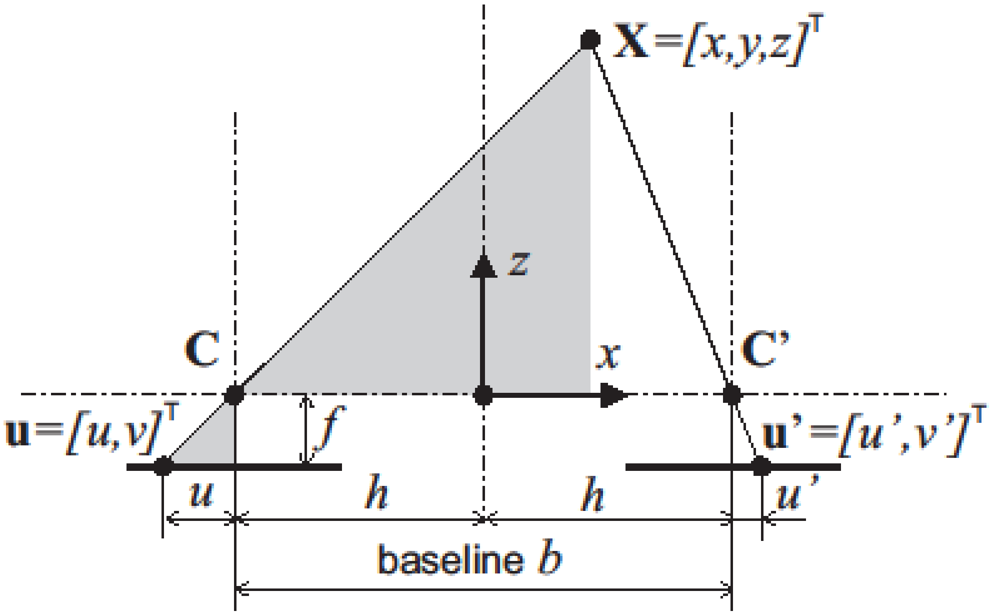
Trifocal constraint: $[\mathbf{l}'^{\mathrm{T}}T_1\mathbf{l}'' \quad \mathbf{l}'^{\mathrm{T}}T_2\mathbf{l}'' \quad \mathbf{l}'^{\mathrm{T}}T_3\mathbf{l}'']\mathbf{u} = 0$

Stereo correspondence algorithms

We know how to determine 3D depth using point correspondence after image rectification

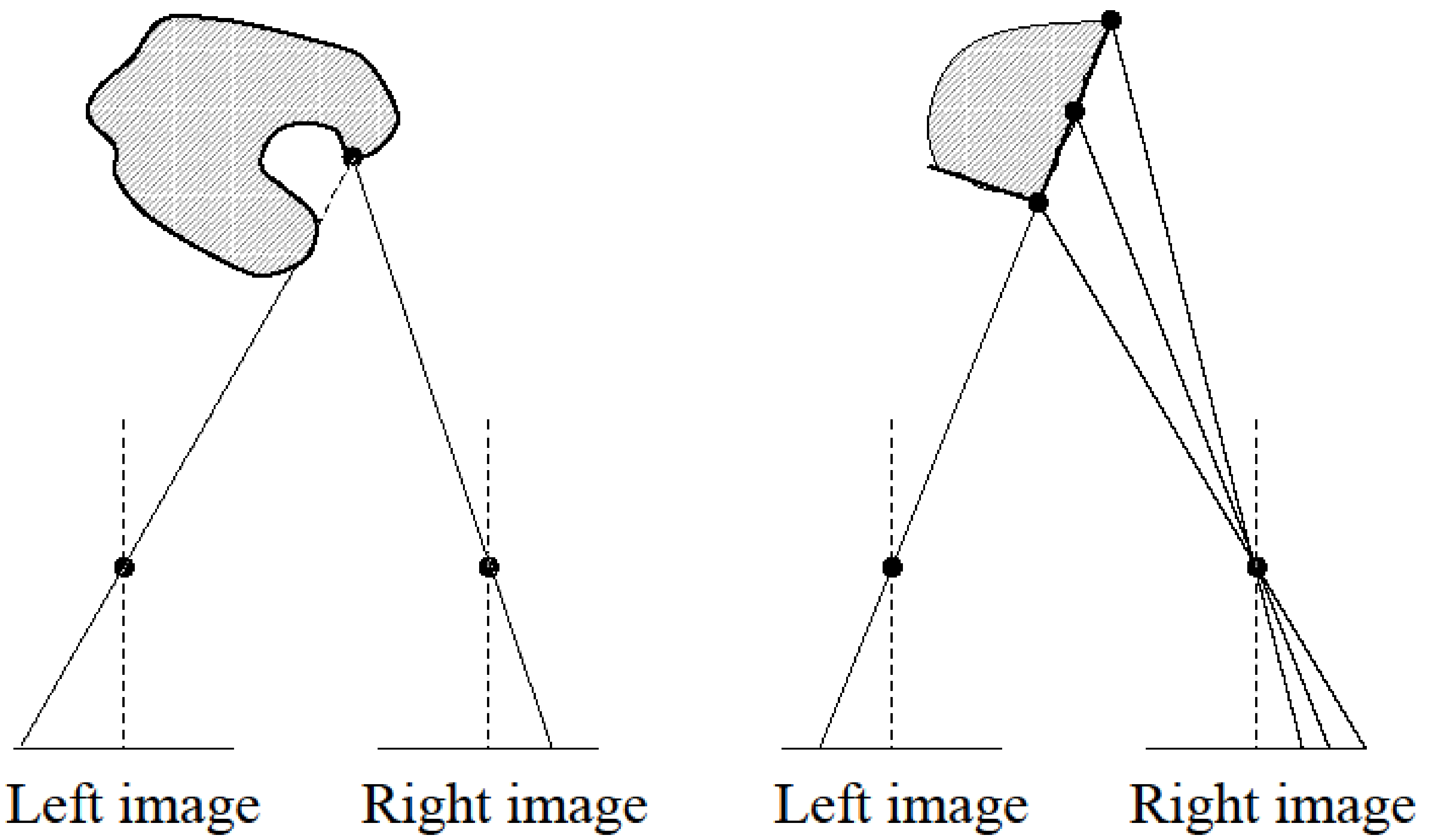
$$z = \frac{bf}{d}$$

$$x = \frac{-b(u+u')}{2d}, \qquad y = \frac{bv}{d}$$



So, the problem boils down to solving point correspondence among multiple stereo camera images

Challenges: Self occlusion and inherent ambiguity

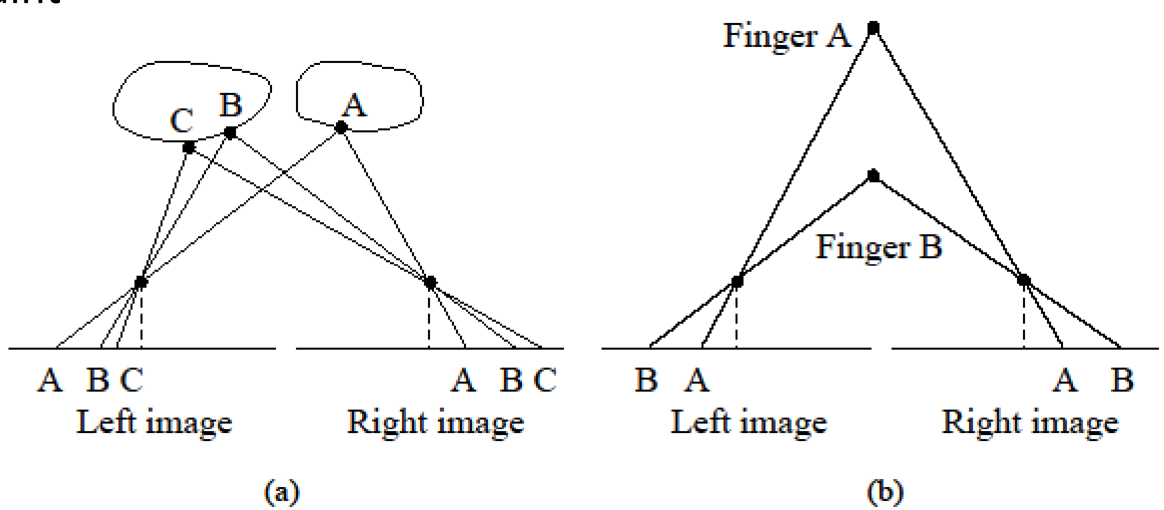


Approach

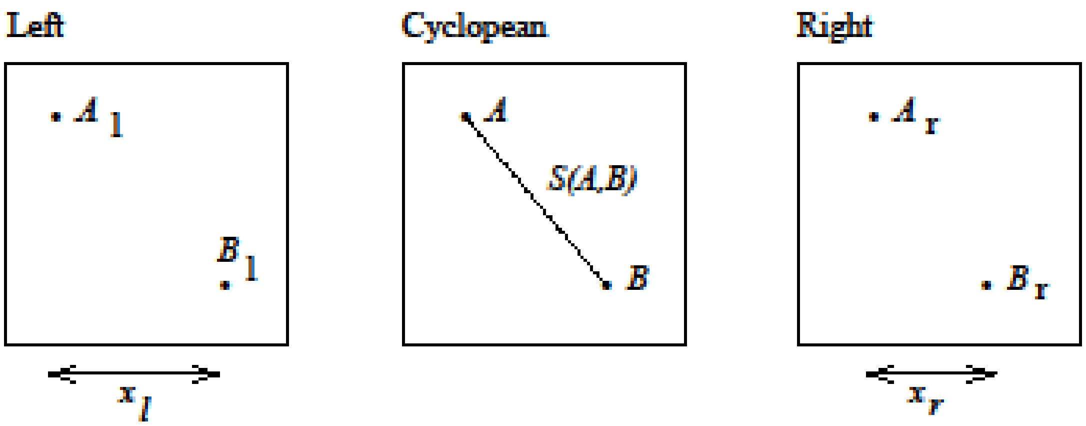
1. Reduce possible candidate pairs of corresponding points using various geometric and photometric constraints
2. Built point correspondence using “correspondence scoring” and optimization techniques

Different constraints:

1) Epipolar constraint
2) Uniqueness constraint
3) Symmetry constraint
4) Photometric compatibility constraint
5) Geometry similarity constraint
6) Disparity smoothness constraint
7) Disparity search range
8) Disparity gradient limit
9) Ordering Limit



Feature based stereo correspondence



Objective: determine the disparity gradient of a pair of matches and use that to score “goodness” of a matching pair

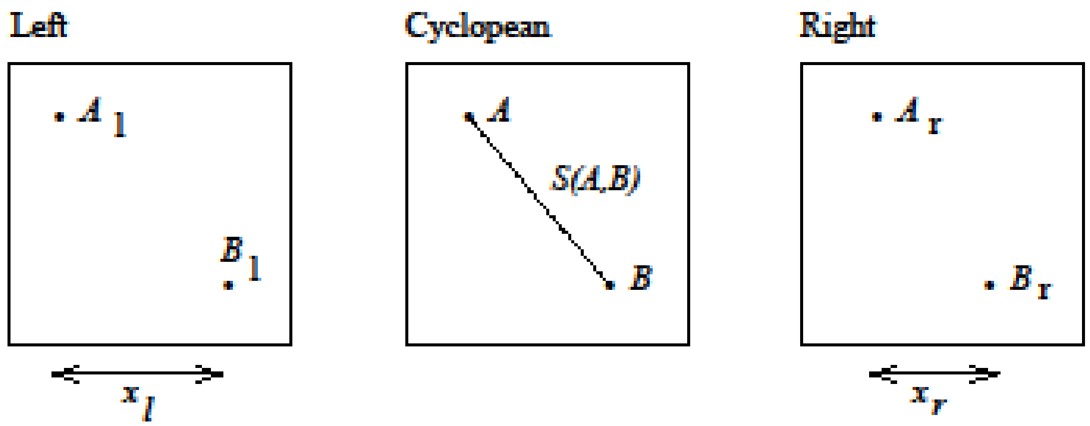
Compute average coordinates as the coordinates of A and B in the Cyclopean image
Determine the cyclopean difference

$$S(A,B) = \sqrt{\frac{1}{4}(x_l + x_r)^2 + \left(a_y - b_y\right)^2} \;\; | \; x_l = a_{xl} - b_{xl}, x_r = a_{xr} - b_{xr}$$

Compute the difference in disparity between the matches of A and B

$$D(A,B) = (a_{xl} - a_{xr}) - (b_{xl} - b_{xr}) = x_l - x_r$$

Disparity gradient constraint



Disparity gradient:

$$\Gamma(A,B) = \frac{D(A,B)}{S(A,B)} = \frac{x_l - x_r}{\sqrt{\frac{1}{4}(x_l + x_r)^2 + \left(a_y - b_y\right)^2}}$$

Disparity constraint: Impose a constraint on the disparity gradient measure that Γ should not exceed a preset threshold (say, '1')

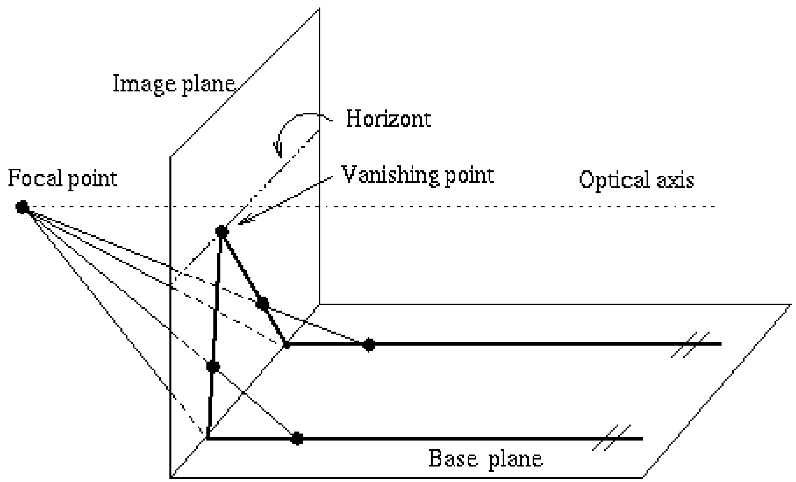
Algorithm

1. Extract feature-based points in the left and right images to build the correspondence
2. For each point in the left image, build initial list of correspondence points in the right image
3. For each possible candidate match, find the number of other possible matches fulfilling the disparity gradient constraint and use the number as the “likelihood” measure for the candidate match
4. Find the highest scoring match; remove all matches involving any of the two points impose uniqueness constraint
5. Repeat Step 2 until all possible matches are established

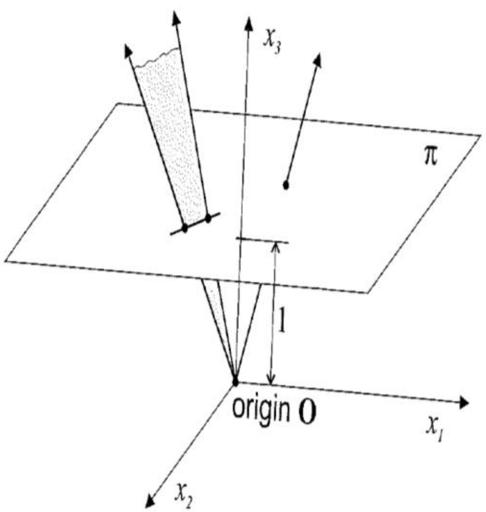
Review

1. Basics of projective geometry
    - Points and hyperplanes in projective space

Perspective projection of parallel lines



Properties of projection



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Homography ≈ Collineation ≈ Projective transformation
is a mapping from one projection plane to another projection plane for the same $(d+1)$-dimensional scene and the common origin

$$\mathcal{P}^d \xrightarrow{H} \acute{\mathcal{P}}^d.$$

Also, expressed as

$$\mathrm{u}' \cong H\mathrm{u},$$

Property:
Any three collinear points in $\mathcal{P}^d$ remain collinear in $\acute{\mathcal{P}}^d$
Prove!

Method:
Minimize algebraic distance errors (a canonical soln.)
Apply ML estimation

Review

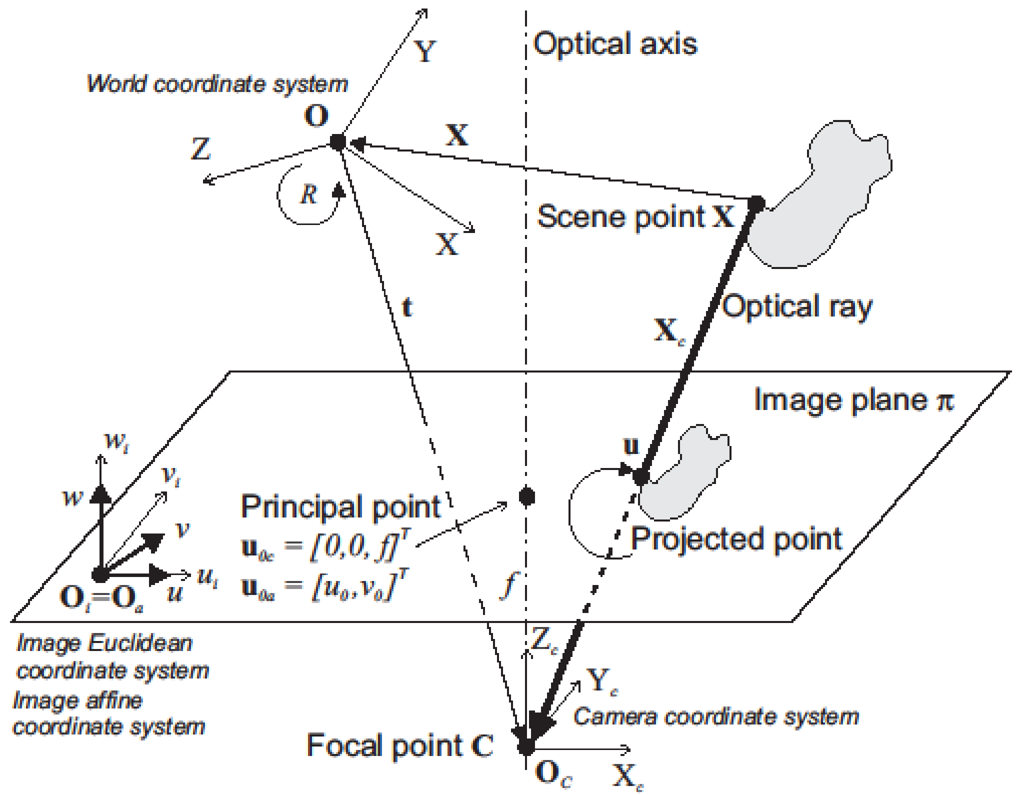
1. The single perspective camera

- An overview of single camera calibration
- Calibration of one camera from the known scene

Camera Model

Different coordinate systems
Different transformation matrices

$$\mathbf{u} \cong M\mathbf{X}$$



Single camera calibration using known 3D scene points

Given: a set of image-scene point correspondences $\{(\mathbf{u}_i, \mathbf{X}_i)\}_{i=1}^{m}$

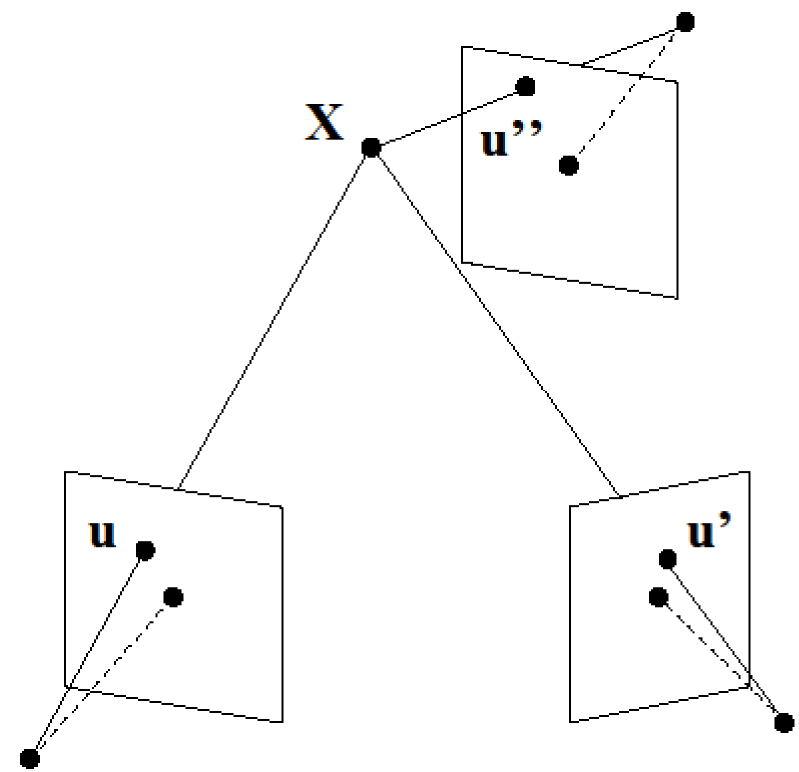
Output: the projection matrix M

Step 1 Initialization using linear estimation

Step 2 Optimization using maximum likelihood estimation

Review

1. Scene reconstruction from multiple views
    - Triangulation
    - Projective reconstruction
    - Matching constraints
    - Bundle adjustment



Scene reconstruction from multiple views

Task: Given matching points in n images. Determine the 3D scene point.

Basic Principle: Back-trace the ray in 3D scene for each image. The scene point is the common intersection of all rays.

Information needed: To back-trace a ray in the scene space, we need to know the corresponding camera matrix M_j.

Review

1. Two cameras, stereopsis
   - The geometry of two cameras.
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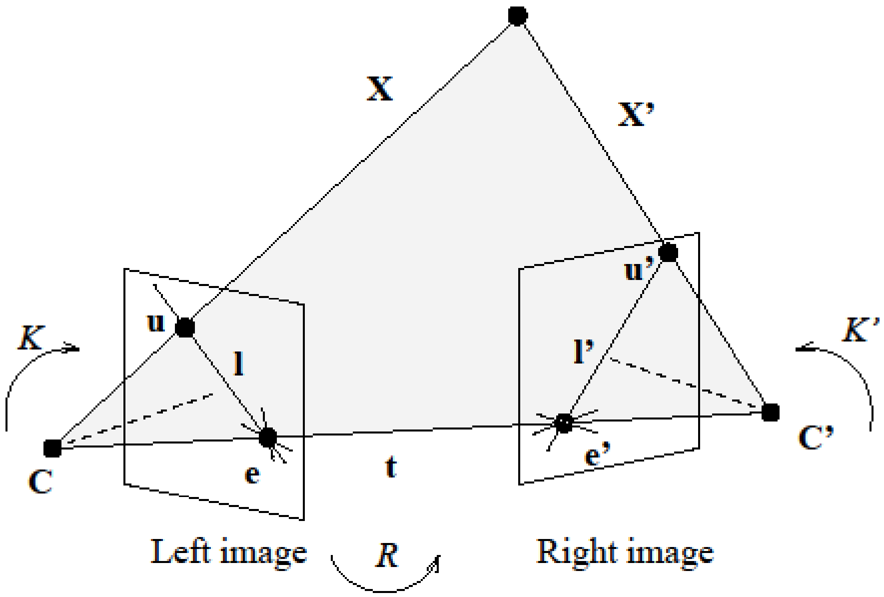
   - Estimation of a fundamental matrix
   - Applications



Epipolar constraint

$$\mathbf{u}'^{\mathrm{T}} F \mathbf{u} = 0$$

Fundamental matrix

First camera

$$M = K[I|\mathbf{0}] \;\; | \; K\text{: intrinsic callib. matrix}$$

Second camera

$$M' = K'[R| - R\mathbf{t}]$$

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Epipolar lines

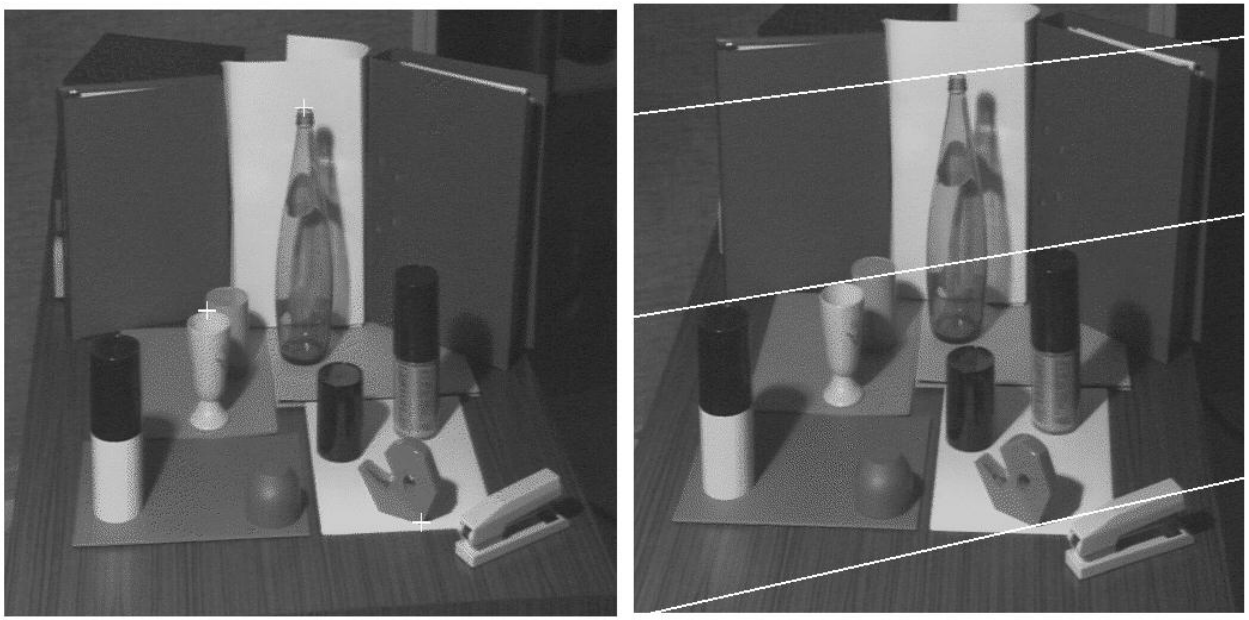

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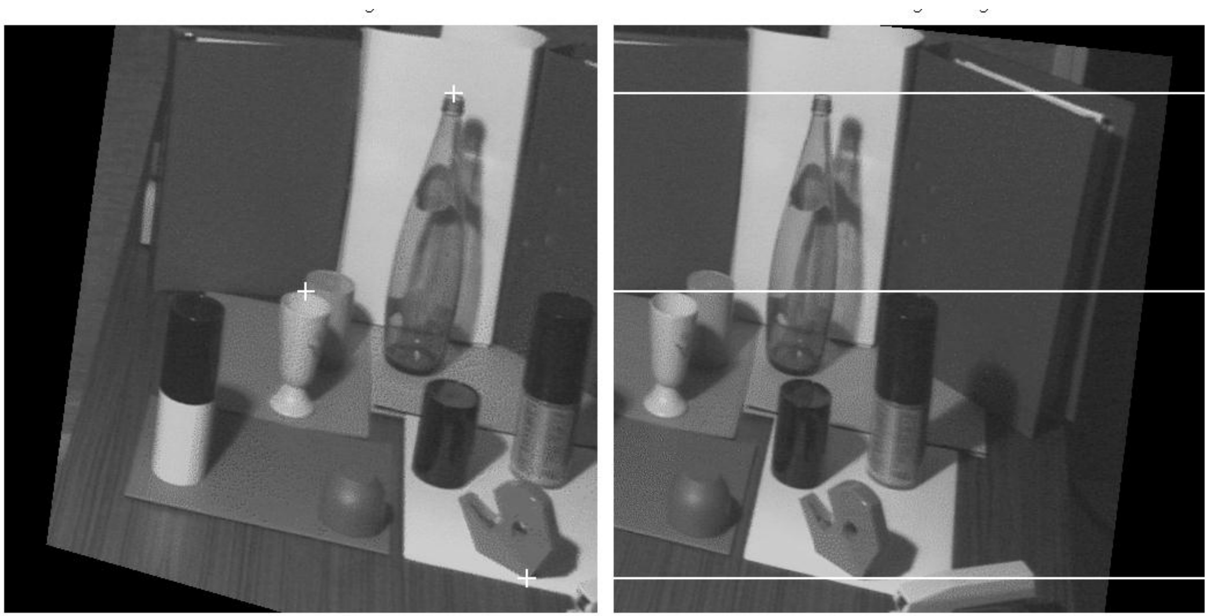
Image Rectification



Review

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Panoramic view

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Panoramic view