55:148 Digital Image Processing
Chapter 11
3D Vision, Geometry

Topics:

- Basics of projective geometry
  - Points and hyperplanes in projective space
  - Homography
  - Estimating homography from point correspondence
- The single perspective camera
  - An overview of single camera calibration
  - Calibration of one camera from the known scene
- Scene reconstruction from multiple views
  - Triangulation
  - Projective reconstruction
  - Matching constraints
  - Bundle adjustment
- Two cameras, stereopsis
  - The geometry of two cameras. The fundamental matrix
  - Relative motion of the camera; the essential matrix
  - Estimation of a fundamental matrix from image point correspondences
  - Camera Image rectification
  - Applications of the epipolar geometry in vision
- Three and more cameras
  - Stereo correspondence algorithms

Image epipolar rectification

Illustrative description

- Both image planes being coplanar
- Baseline along the horizontal axis (x-axis) of the image plane
- Two epipoles at infinity along the horizontal direction
- Horizontal epipolar lines

- **Method:** Apply suitable homographic transformation on each image

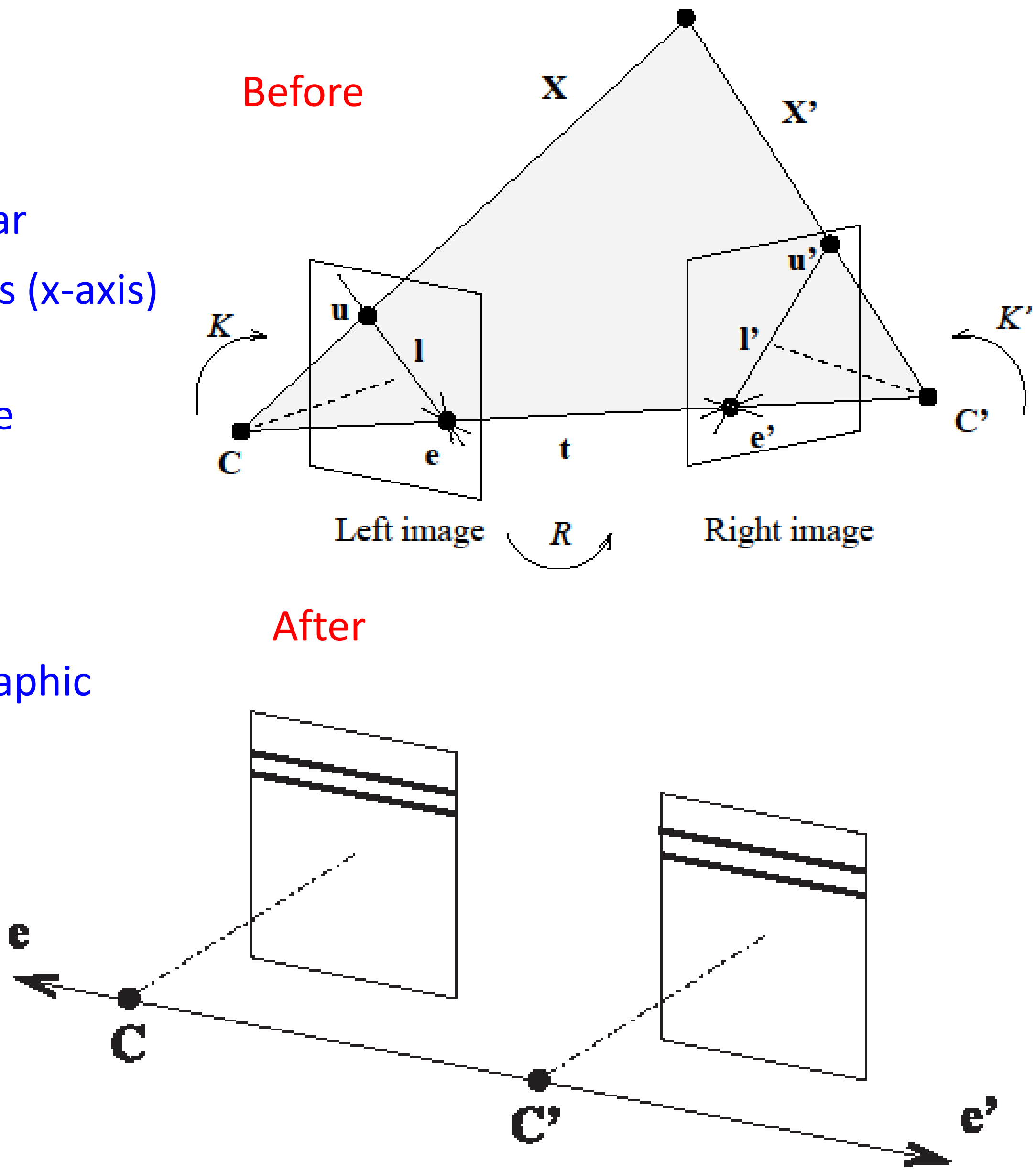

Image rectification (before)

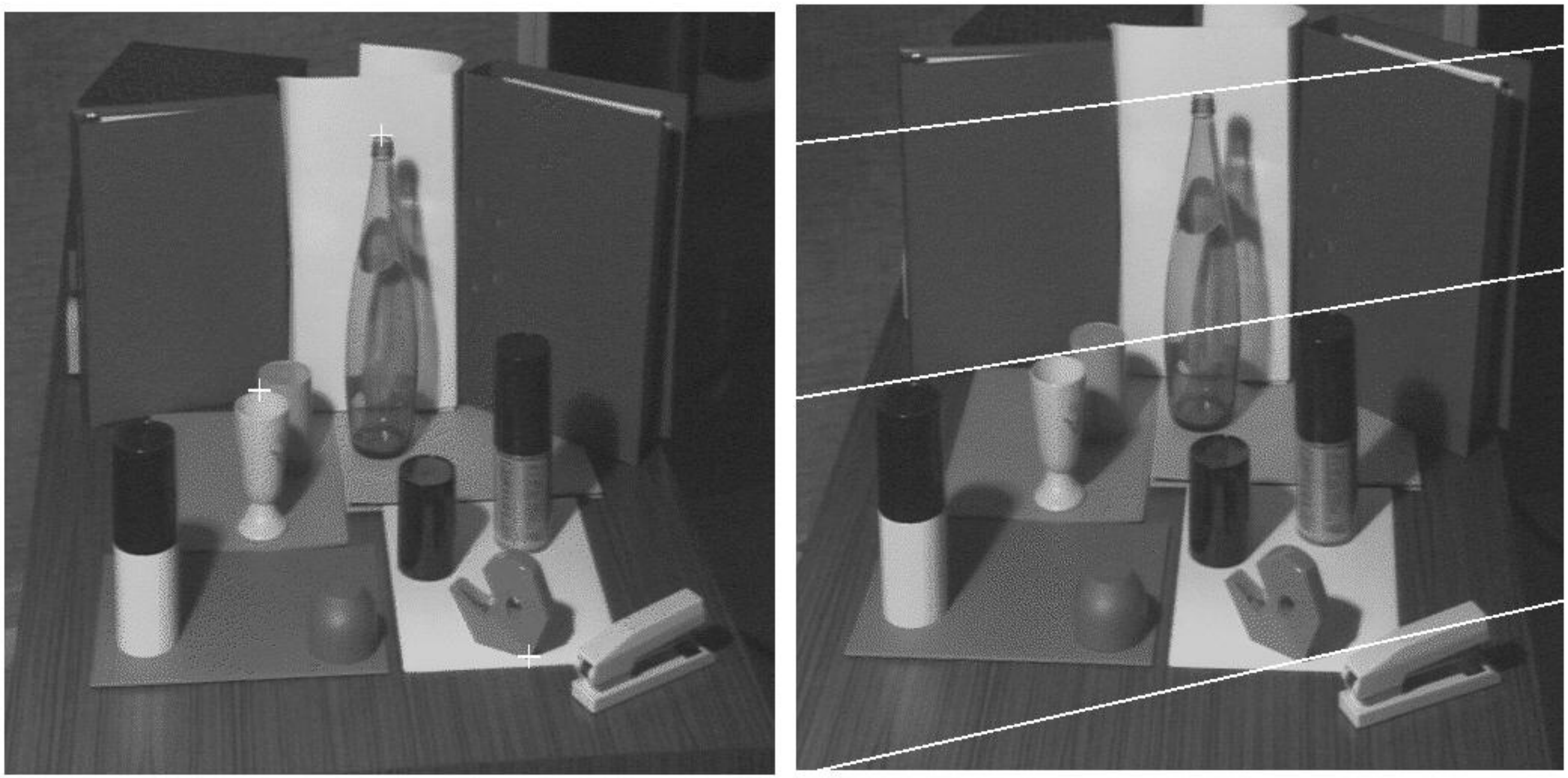

Image rectification (after)

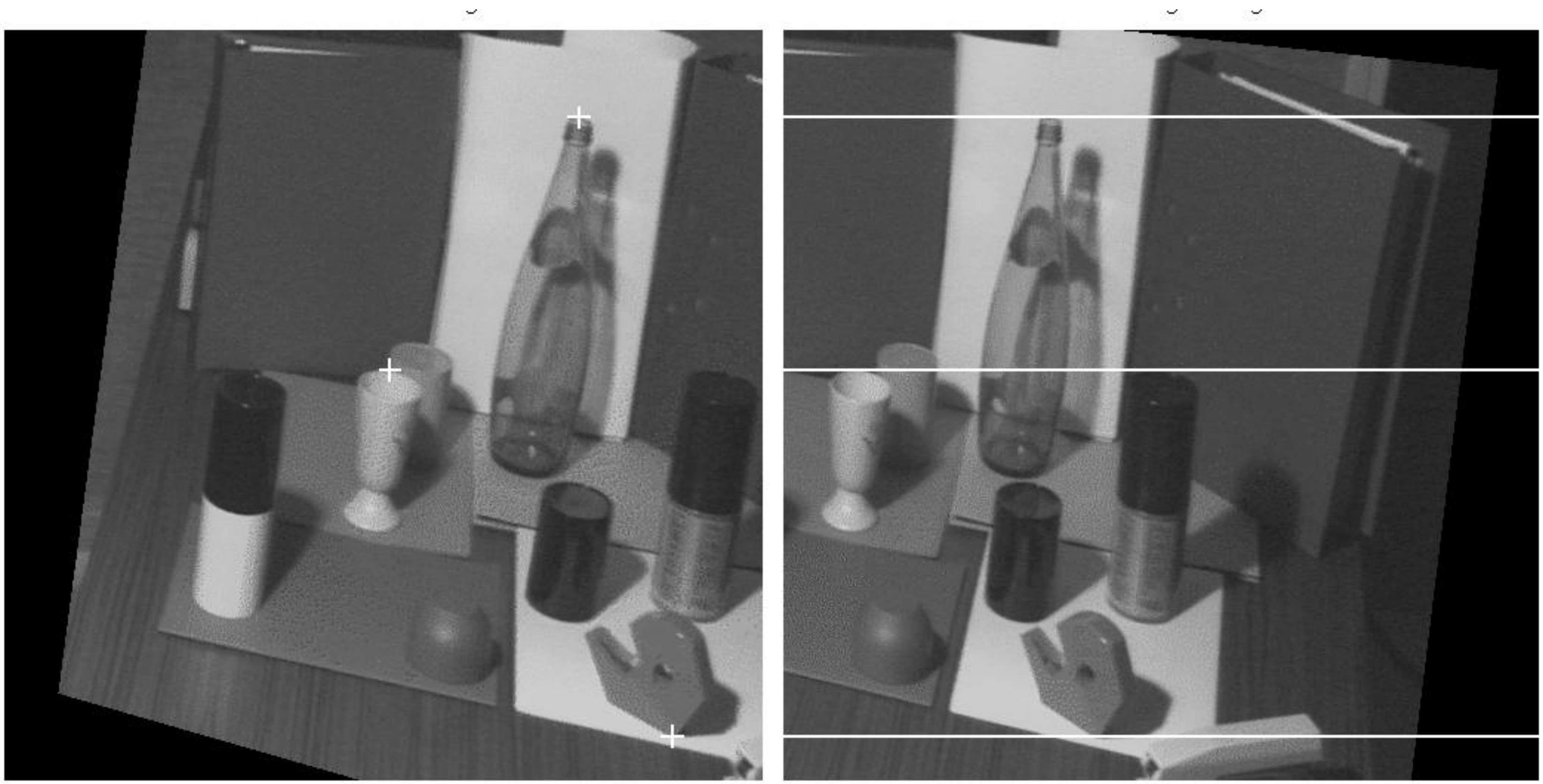

Image rectification

Analytic description

- Suppose that we know camera matrices M and M' for the two cameras
- The objective is to rotate each image planes around their optical centers until their focal planes becomes coplanar
  - Thereby, containing the baseline
  - More specifically, the new x-axis is parallel to the baseline
- Both images have y-axis $\perp$ x-axis

Thus, new camera matrices

$$M^* = K[R \mid -R\mathbf{c}_1] \qquad \text{and} \qquad M^{*\prime} = K[R \mid -R\mathbf{c}_2]$$

where, $R = \begin{bmatrix} \mathbf{r}_1^{\mathrm{T}} \\ \mathbf{r}_2^{\mathrm{T}} \\ \mathbf{r}_3^{\mathrm{T}} \end{bmatrix}$

1) The new x-axis is parallel to the baseline $\Rightarrow \mathbf{r}_1 = \frac{\mathbf{c}_1 - \mathbf{c}_2}{|\mathbf{c}_1 - \mathbf{c}_2|}$
2) The new y-axis is orthogonal to the new x-axis $\Rightarrow \mathbf{r}_2 = \mathbf{q} \times \mathbf{r}_1$, for some $\mathbf{q} \neq \mathbf{r}_1$ and $|\mathbf{q}| = 1$
3) The new z-axis is orthogonal to the new xy-plane $\Rightarrow \mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$

A proof that Image rectification ≅ a suitable homographic transformation

Let

$$M = [\widetilde{M} \mid -\widetilde{M}\mathbf{c}_1] \qquad \text{and} \qquad M^* = [\widetilde{M}^* \mid -\widetilde{M}^*\mathbf{c}_1]$$

For any 3D scene point $\mathbf{X}$, projected image points using camera matrices are:

$$\widetilde{\mathbf{u}} = M\mathbf{X},$$
$$\widetilde{\mathbf{u}}^* = M^*\mathbf{X}$$

Using the relation in homogeneous coordinate system,

$$\mathbf{X} = \mathbf{c}_1 + \lambda_0 \widetilde{M}^{-1}\widetilde{\mathbf{u}}$$
$$\mathbf{X} = \mathbf{c}_1 + \lambda_1 \widetilde{M}^{*-1}\widetilde{\mathbf{u}}^*, \quad \lambda_0, \lambda_0 \in \mathbb{R}$$

i.e.,

$$\lambda_1 \widetilde{M}^{*-1}\widetilde{\mathbf{u}}^* = \lambda_0 \widetilde{M}^{-1}\widetilde{\mathbf{u}} \quad \Rightarrow \quad \widetilde{\mathbf{u}}^* = \lambda \widetilde{M}^* \widetilde{M}^{-1}\widetilde{\mathbf{u}}$$

λ: scale factor

$\widetilde{M}^*\widetilde{M}^{-1}$: 3-by-3 matrix ≈ perspective transformation ≈ homography equivalent

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Image rectification

Algorithmic description

- As we have seen from analytic discussion
  Given the knowledge of M and M', image rectification process is precisely defined
  So, the first task is to determine camera matrices M and M'

Algorithm Compute camera matrices M and M'

Task 1: Solve F using $m \geq 8$ corresponding point pairs in two images and algebraic error minimization

Input: A linear system: $\mathbf{u}_i'^{\mathrm{T}} F \mathbf{u}_i = 0, \quad i = 1,2,\ldots,m$

Using Kronecker product identity: $AB\mathbf{c} = \left(\mathbf{c}^{\mathrm{T}} \otimes A\right)\mathbf{b}$, we get $\mathbf{u}_i'^{\mathrm{T}} F \mathbf{u}_i = \left[\mathbf{u}_i^{\mathrm{T}} \otimes \mathbf{u}_i'^{\mathrm{T}}\right]\mathbf{f} = 0$

Put together all correspondences

$$\begin{bmatrix} \mathbf{u}_{i,1}^{\mathrm{T}} \otimes \mathbf{u}_{i,1}'^{\mathrm{T}} \\ \vdots \\ \mathbf{u}_{i,m}^{\mathrm{T}} \otimes \mathbf{u}_{i,m}'^{\mathrm{T}} \end{bmatrix} \mathbf{f} = W\mathbf{f} = 0$$

Compute $W^T W$ and apply singular value decomposition; choose $\mathbf{f}$ along the eigenvector corresponding to the smallest eigenvalue

For F to be a valid fundamental matrix, its rank must be 2; but it may not satisfy in reality (**why?**); thus needs correction

SVD decomposition: $F = UDV^{\mathrm{T}}$; set the smallest eigenvalue of D to zero giving a new diagonal matrix $\widetilde{D}$; use the new matrix $\tilde{F} = U\widetilde{D}V^{\mathrm{T}}$

Note that ML estimation method incorporate the validity condition of F into the optimization process

Image rectification

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Algorithm Compute camera matrices M and M'

Task 1: Solve F using $m \geq 8$ corresponding point pairs in two images and algebraic error minimization

Input: A linear system: $\mathbf{u}_i'^{\mathrm{T}} F \mathbf{u}_i = 0, \quad i = 1,2, \dots, m$

Output: Fundamental matrix $\tilde{F}$

Task 2: Decompose the fundamental $\tilde{F}$ to matrix camera matrices

$$M = [I \mid \mathbf{0}]$$
$$M' = [S(\mathbf{e}')\tilde{F} \mid \mathbf{e}']$$

Input of the algorithm : A linear system: $\mathbf{u}_i'^{\mathrm{T}} F \mathbf{u}_i = 0, \quad i = 1,2, \dots, m$

Output of the algorithm: camera matrices M and M'

Now what?

Image rectification

Algorithmic description

Algorithm Compute image rectification

Input: M and M'

Output: M^* and M'^* and two homography transformations: H and H'

Decompose each camera matrices: $M = A * [R; \mathbf{t}]$ and $M' = A' * [R'; \mathbf{t}']$

Determine two optical centers $\mathbf{c}$ and $\mathbf{c}'$ using M and M' and their decompositions

Determine new rotation matrix R^* common to M^* and M'^*

Determine new intrinsic parameters A^* by averaging A and A' and then annulling skew component

Compute new projection matrices:

$$M^* = A^* * [R^* \mid -R^*\mathbf{c}] \text{ and } M'^* = A^* * [R^* \mid -R^*\mathbf{c}']$$

Compute homography transformations

$$H = \widetilde{M}^* \widetilde{M}^{-1} \quad \text{and} \quad H' = \widetilde{M}'^* \widetilde{M}'^{-1}$$

Image rectification (before)

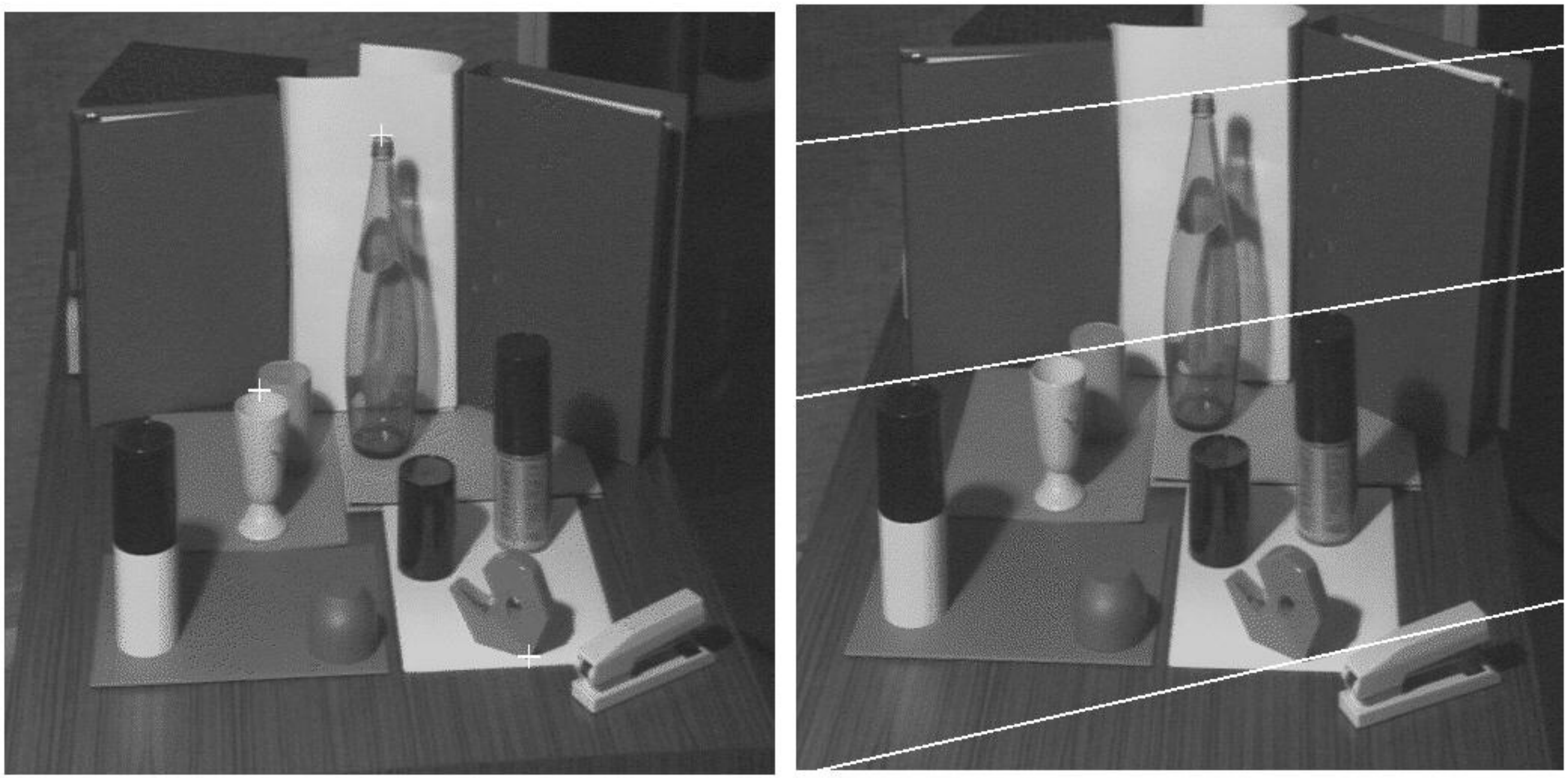


Image rectification (after)

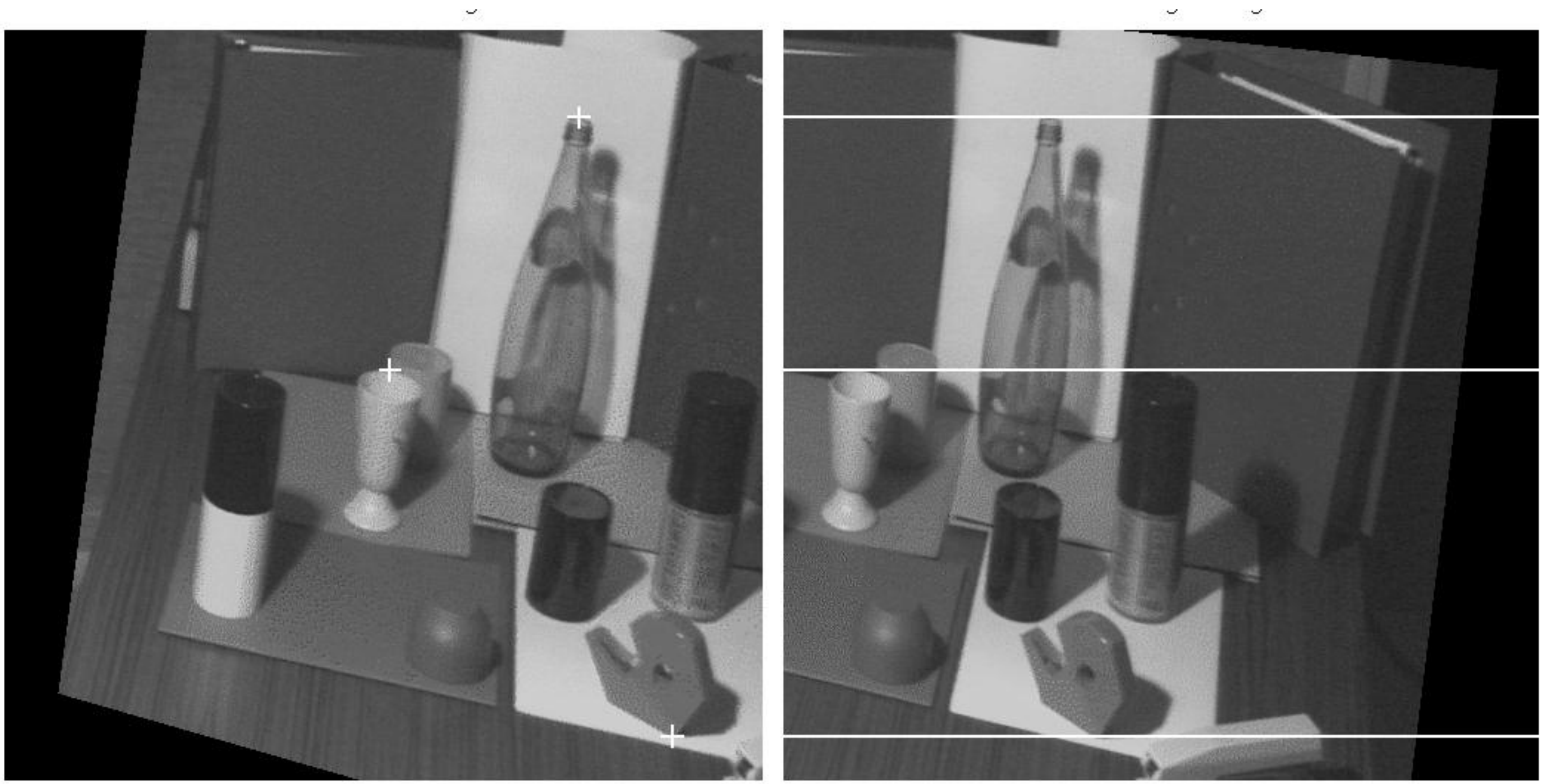
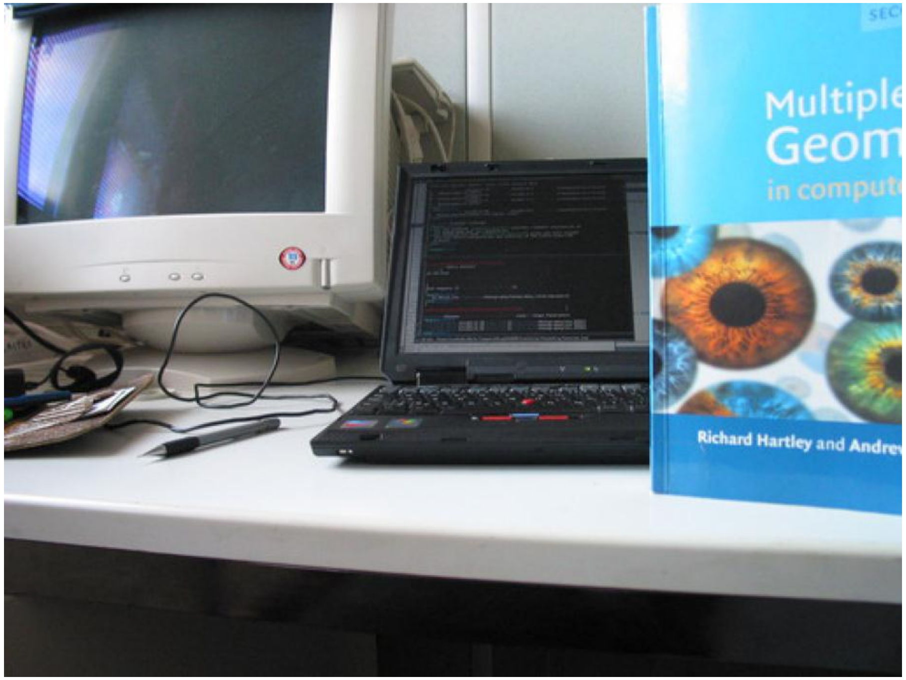


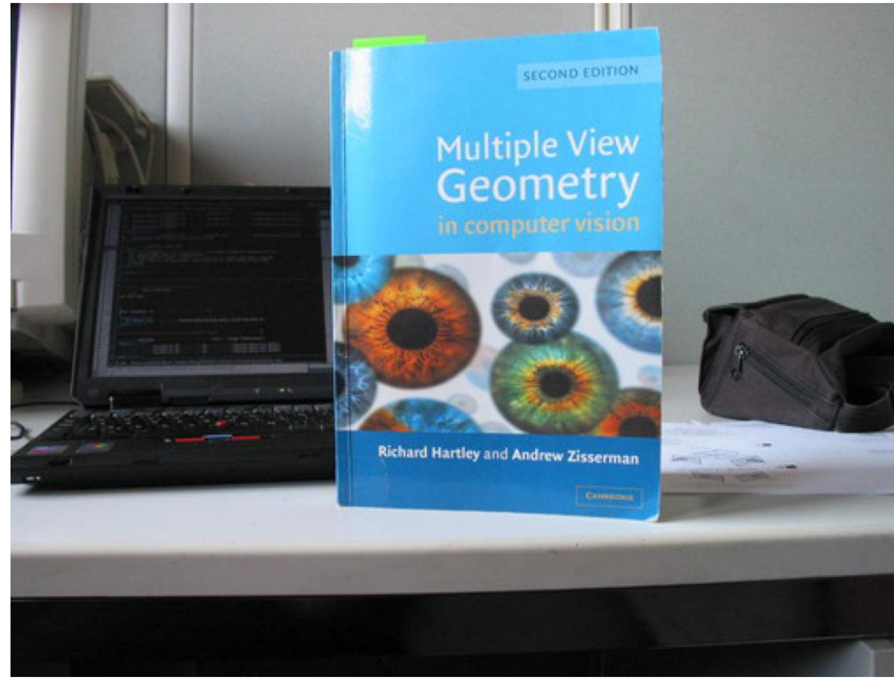
Image rectification: application

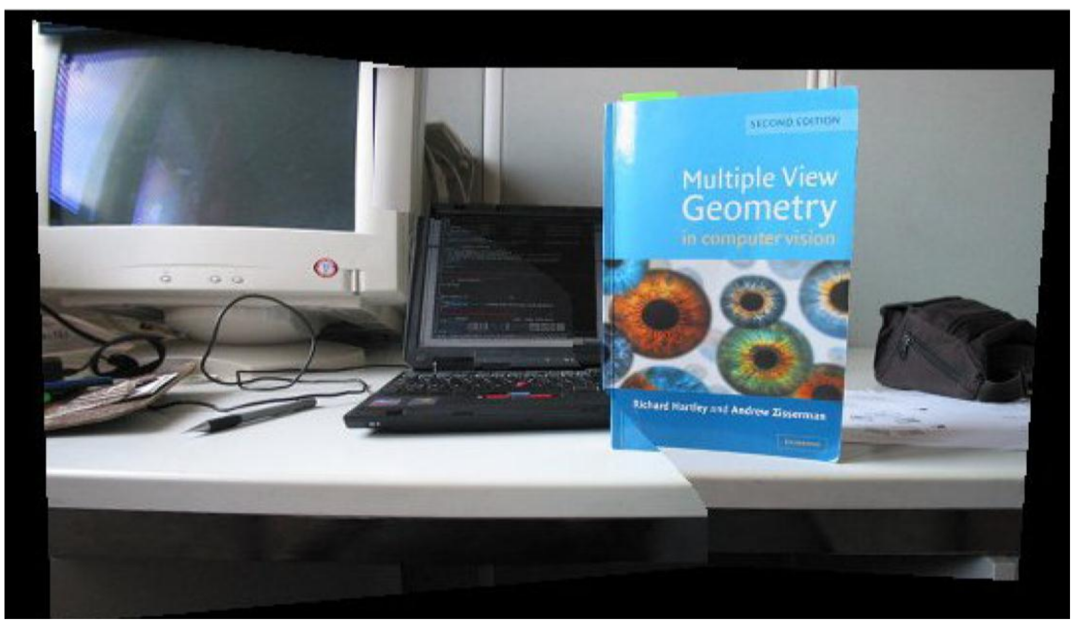
3D reconstruction becomes easier

Panoramic view

Panoramic view: challenges

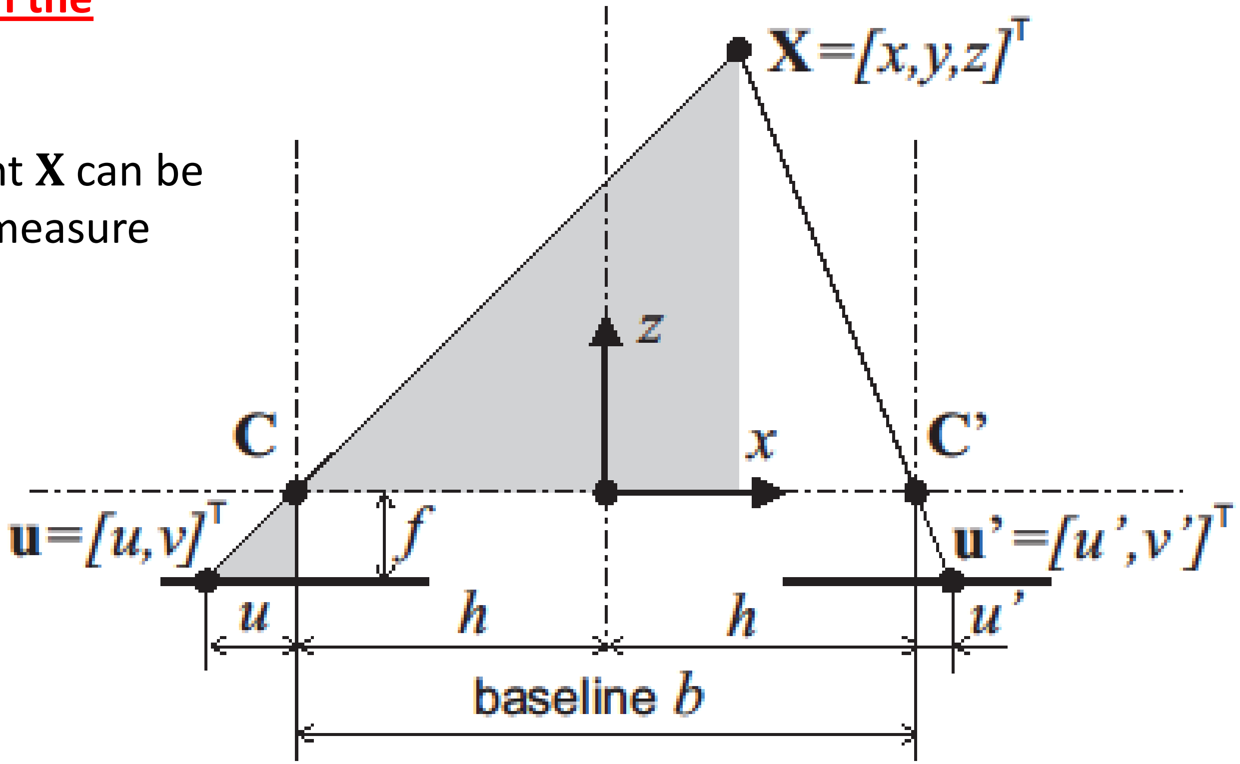






Elementary stereo geometry in the rectified image configuration

The depth z of a 3D scene point $\mathbf{X}$ can be calculated using the disparity measure $d = u' - u$



$$\frac{u}{f} = -\frac{h+x}{z}, \qquad \frac{u'}{f} = \frac{h-x}{z}$$

$$\Rightarrow \quad z = \frac{2hf}{u'-u} = \frac{bf}{u'-u} = \frac{bf}{d}$$

$$x = \frac{-b(u+u')}{2d}, \qquad y = \frac{bv}{d}$$