Mathematically Based Ergonomics

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Executive Summary

An analytical approach to the study of human motion and ergonomic design based upon a novel method for determining the workspace of human motion has been established. The method was recently developed at the University of Iowa Center for Computer Aided Design (CCAD) and yields the exact boundary of the workspace in closed-form. This method is employed towards addressing problems of interest in human motion analysis in terms of ergonomic design, workspace visualization, posture prediction, layout design, and placement. Because of the closed-form formulation of the workspace, we are able to delineate the exact reach envelope (boundary of the workspace) of human limbs while taking into consideration the ranges of motion. As a result of this work, it is possible to (1) Visualize the exact workspace of human limbs, (2) Define and plan trajectories in the workspace, (3) Design ergonomic workplaces subject to specified cost functions, (4) Facilitate the design of layouts and packaging, (5) Verify measured data and validate human models, (6) Predict realistic postures, and (7) Optimize designs based on specified cost functions. Cost functions representing dexterity, reacheability, energy, force, and others have been developed and integrated with optimization code to address ergonomics design problems. Using this method, we will address the following problems.

(a) Visualization of the exact workspace
(b) Placement
(c) Layout Design
(d) Posture Prediction

Introduction

The study of human motion has been to a great extent based upon rules of thumb and empirical measurements. This work is aimed at utilizing a recently developed analytical approach for addressing standing problems in anthropometry and to provide a systematic approach to ergonomic design.

Because this method yields the exact workspace while taking into account all ranges of motion and the exact dimensions, it enables the design of complex systems that have never been imagined before. This is achieved by manipulating the workspace (because equations are available) whereby cost functions are optimized. For example, we are able to calculate a new posture of an arm given a cost function of minimizing the energy of motion, maximizing dexterity (all possible orientations at a point), or reducing induced stress at a joint (used to reduce the risk for cumulative trauma).

Imagine an assembly line in a manufacturing environment where certain tasks are to be accomplished by a person. Using this mathematical algorithm, it is possible to place the person (i.e., define the most convenient location) such that to achieve the most comfortable work posture and to minimize the required energy. Furthermore, an appropriate function has also been created to place the person in this environment whereby reducing the stress induced at his/her joints.

To date, there has not been a single analytical method for ergonomic design that is based on a fundamental mathematical approach. While this method has been developed over the past few years by the PI and graduate students for the field of robotics and computer aided design (CAD), computer software with a user-friendly interface is not yet available but is still under development.

The proposed work aims to utilize this approach to address issues important to the automotive, office furniture designers, and manufacturing industries. Specific examples have been illustrated and numerous results have been obtained and validated.

This group is currently working to further expand the formulation, to develop an interactive software system that enables designers to explore the workspace generated by human limbs, and to predict realistic postures using various cost functions (we have demonstrated a minimization of stress and energy approach to posture prediction).

As a result of this work, developers and users of human motion simulation software will have an additional mathematically-based tool for better understanding human motion and ergonomic design. This will be an added capability that never existed. Because this approach is mathematical in nature, it can be expanded to any articulated human appendage and to the modeling of human motion without the need for extensive empirical measurements. The ultimate goal is to use this technology as complimentary work to that under investigation at the University of Michigan Center for Ergonomics and as added capability to that provided by commercial software systems (e.g., Boeing

Human Modeling System, EAI JACK, Genicom Safework, TecMath Ramsis, Technomatix ROBCAD/man, Deneb Ergo, etc.).

Background

A rigorous mathematical formulation has been developed to promote understanding of the human motion. This formulation is the result of approximately 30 man-years of research and development at the University of Iowa Center for Computer Aided Design. The potential impact of this research in the ergonomic design field is significant as it presents the only method to date that is based on a fundamental rigorous formulation. By way of demonstration, consider a forearm modeled as a four degree of freedom system, where the spherical joint at the shoulder is modeled as three intersecting revolute joints and the elbow as a single revolute joint (Fig. 1). This representation of motion adapts a kinematics modeling method developed by the PI (Abdel-Malek, *et al.* 1997; 1999). We have limited the motion of the shoulder to that of the glenohumeral joint. Figure 1a depicts the motion to be modeled where each joint is given an independent coordinate q_i and Fig. 1b depicts the equivalent kinematic diagram of the system.

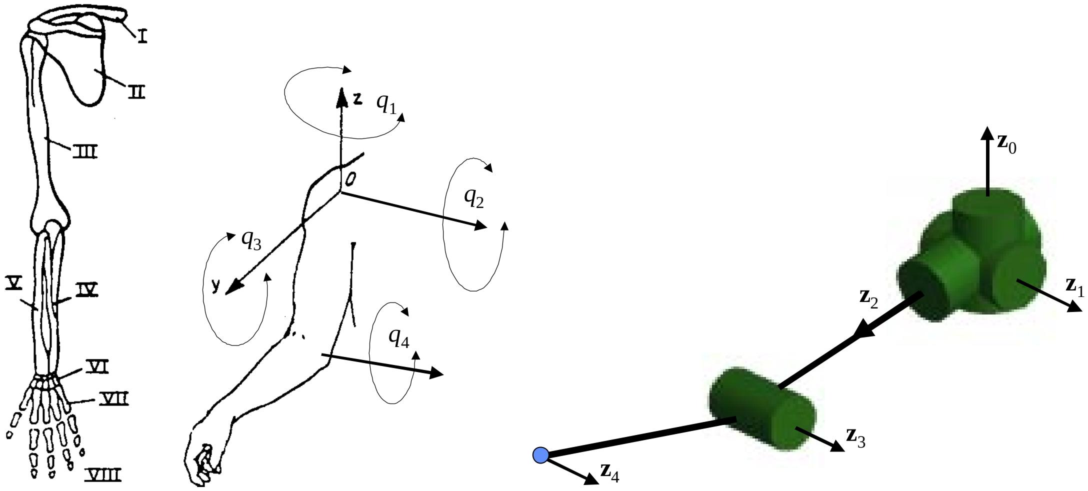


Fig. 1 (a) A schematic of the forearm (b) Kinematic modeling of the forearm as a spherical joint (shoulder) and a revolute joint (elbow)

Perhaps the most beneficial aspect of this approach is the ability to include joint limits in the formulation as inequality constraints. For example, for a person with joint ranges in the form of $q_i^L \le q_i \le q_i^U$, we convert (parameterize) the inequalities to equality constraints in order to incorporate ranges of motion in the formulation. We will use this information in addition to the physical dimensions to visualize the *exact* reach envelope, to predict postures, to better understand human motion, and to utilize this technology in the ergonomic design process.

The underlying mathematical formulation is based upon a rank deficiency condition developed to stratify the Manifold generated by the human appendage. By applying an appropriate theory to the constraint Jaconbian matrix, we generate singular surfaces that may or may not be on the boundary of the workspace envelope. Indeed, those surfaces play an important role in understanding the trajectories chosen by a human to perform a given task. The theory is based on mature algorithms adapted to ergonomics from the field of Differential Geometry and Robotics.

Whether an arm, leg, torso, or line of sight with respect to the torso, the method models any motion as a series of bodies connected as a kinematic chain. Manifold stratification is used to delineate singular behavior (also called barriers) which may or may not be on the boundary envelope. Because of the rigorous nature of this work, it is then possible to obtain cross-sections, optimize, and calculate postures.

We summarize our results and the potential impact this formulation will have in the field of ergonomics.

Visualization of the Exact Workspace

Combining all barriers yields the workspace of the forearm as shown in Fig. 2a. The workspace oriented with respect to the torso is shown in Fig. 2b. It must be noted that the result shown in Fig. 2b is exact and in closed form (i.e., we have equations that represent the exact boundary). Therefore, it is now possible to manipulate these equations and interrogate the model for valuable engineering design data. Because of the closed formulation, it is also possible to calculate properties such as volume, mass, and moments of inertia. It is also possible to determine collisions and interference with the boundary. As a result of this work, it will be possible to plan trajectories inside the workspace and perform ergonomic analysis in virtual space, test, evaluate, and design thus allowing users to evaluate products very early in the design process.

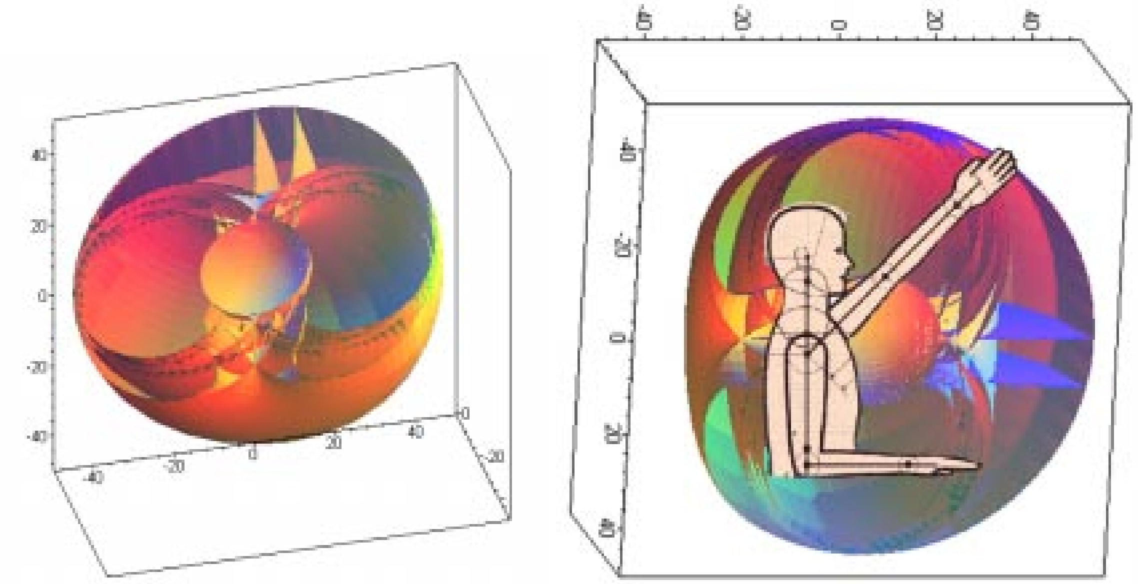


Fig. 2 (a) Two cross-sectional views of the workspace of the forearm (b) Depicting the exact workspace of the shoulder-arm example

Placement

The problem addressed in this section is that of placing the person in order to achieve a certain goal. This issue has become important in recent years in view of the rising costs of disability due to lack of occupational safety. Cumulative trauma (e.g., repetitive injuries) are one of the leading causes for occupational disorders). The aim of this work is to address such issues, which pertain to placement of a person in such a way to reduce such factors as repetitive injuries. One may think of the *Placement* problem as the inverse of the *Layout Design* problem.

Typically, the user defines points in space that are of interest (e.g., push buttons, levers, keyboard, etc.). This goal is characterized in terms of a cost function as maximizing reacheability, maximizing dexterity, minimizing energy, or reducing stress induced at a joint. In order to achieve this goal, we have developed an optimization algorithm that uses the results of Section 1 (exact workspace envelope in closed form), whereby the workspace is manipulated in space subject to the pre-defined cost functions. The simplified algorithm is shown in Fig. 3. This numerical algorithm is designed to move the workspace envelope in such a way to include the target points, to optimize the given cost function, and to satisfy all constraints.

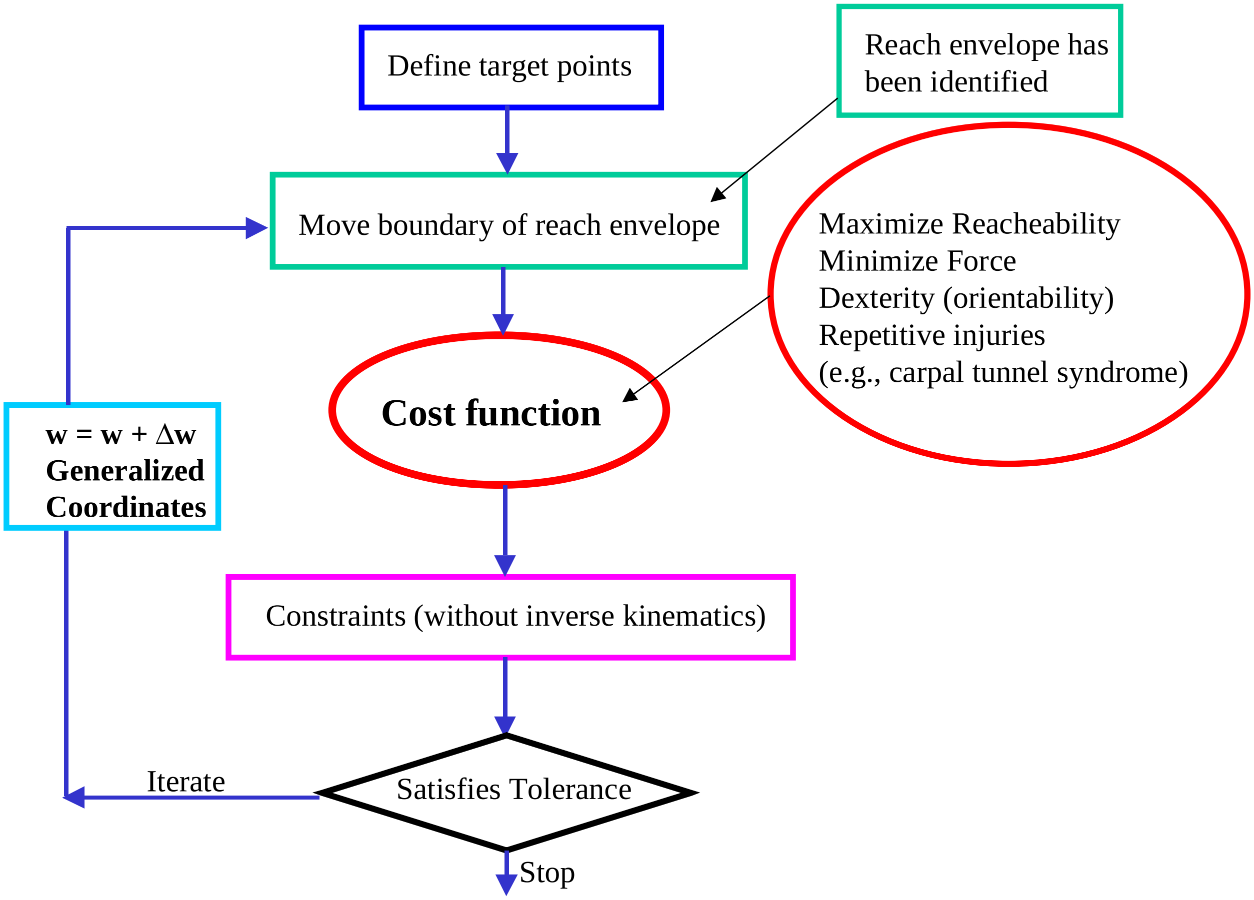


Fig. 3 Algorithm for placement

In order to demonstrate the formulation, we illustrate the planar case again. Consider the shoulder, forearm, elbow, and wrist model shown in Fig. 4a. Because it is possible to obtain the reach envelope in closed form, it is also possible to obtain cross sections through the workspace. For example, Fig. 4b the cross section at a given depth.

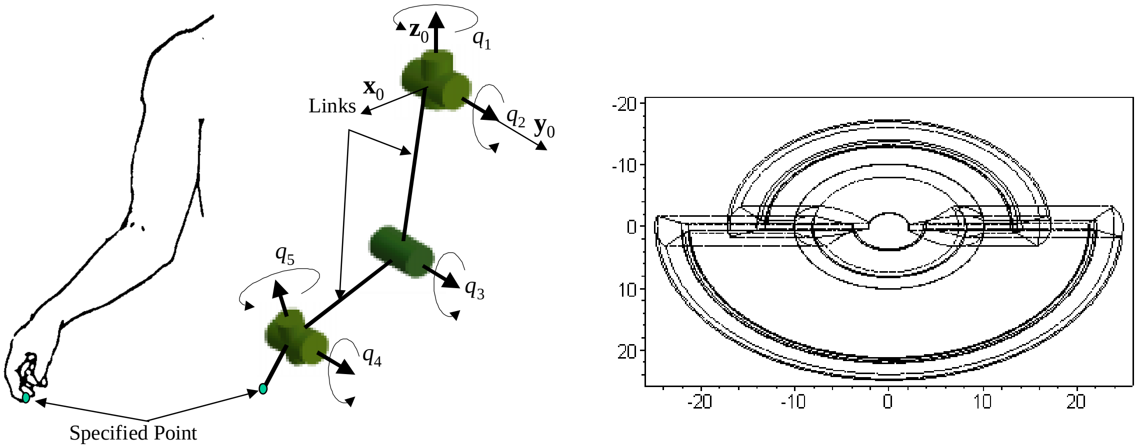


Fig. 4 (a) A model of the arm (b) A cross-section through the workspace

In order to define cost functions, we introduce measures that enable the quantification of the property to be optimized. For example, if the goal is to maximize dexterity of the person to be placed, then a dexterity measure is developed. Examples of such measures that we have developed are listed below (Note that those were derived based on the mathematical models developed earlier and utilize the concept of an exact reach envelope).

a. Maximizing dexterity (a measure of dexterity has been proposed)

$$Dex = \sqrt{\left|\mathbf{H_q}\mathbf{H_q}^T\right|} \qquad \text{where} \qquad \mathbf{H_q} = \left[\begin{array}{c:c} \xi_{\mathbf{q}^*} & \mathbf{0} \\ \hdashline \mathbf{I} & \mathbf{q}_\lambda^* \end{array}\right]$$

b. Minimizing stress at a joint (to reduce repetitive injury)

Displacement from Neutral

$$Neutralx = \sum_{i=1}^{n}(q_i - q_i^N) \qquad q_i^L \leq q_i \leq q_i^U$$

Kinetic Energy

$$K = \sum_{i=1}^{n} K_i = \frac{1}{2}\sum_{i=1}^{n} Tr\left(\sum_{p=1}^{i}\sum_{r=1}^{i}\mathbf{U}_{ip}\mathbf{H_q}\mathbf{U}_{ir}^T\dot{q}_p\dot{q}_r\right)$$

Potential Energy

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n}\left(-m_i\mathbf{g}({}^0\mathbf{A}_i\,{}^i\bar{\mathbf{r}}_i\right)$$

Based on one of the above cost functions, we then employ the algorithm to drive (move) the workspace towards optimizing the specified cost function and define a new placement for the human as illustrated for the cross section of the arm where it was driven to include the target points.

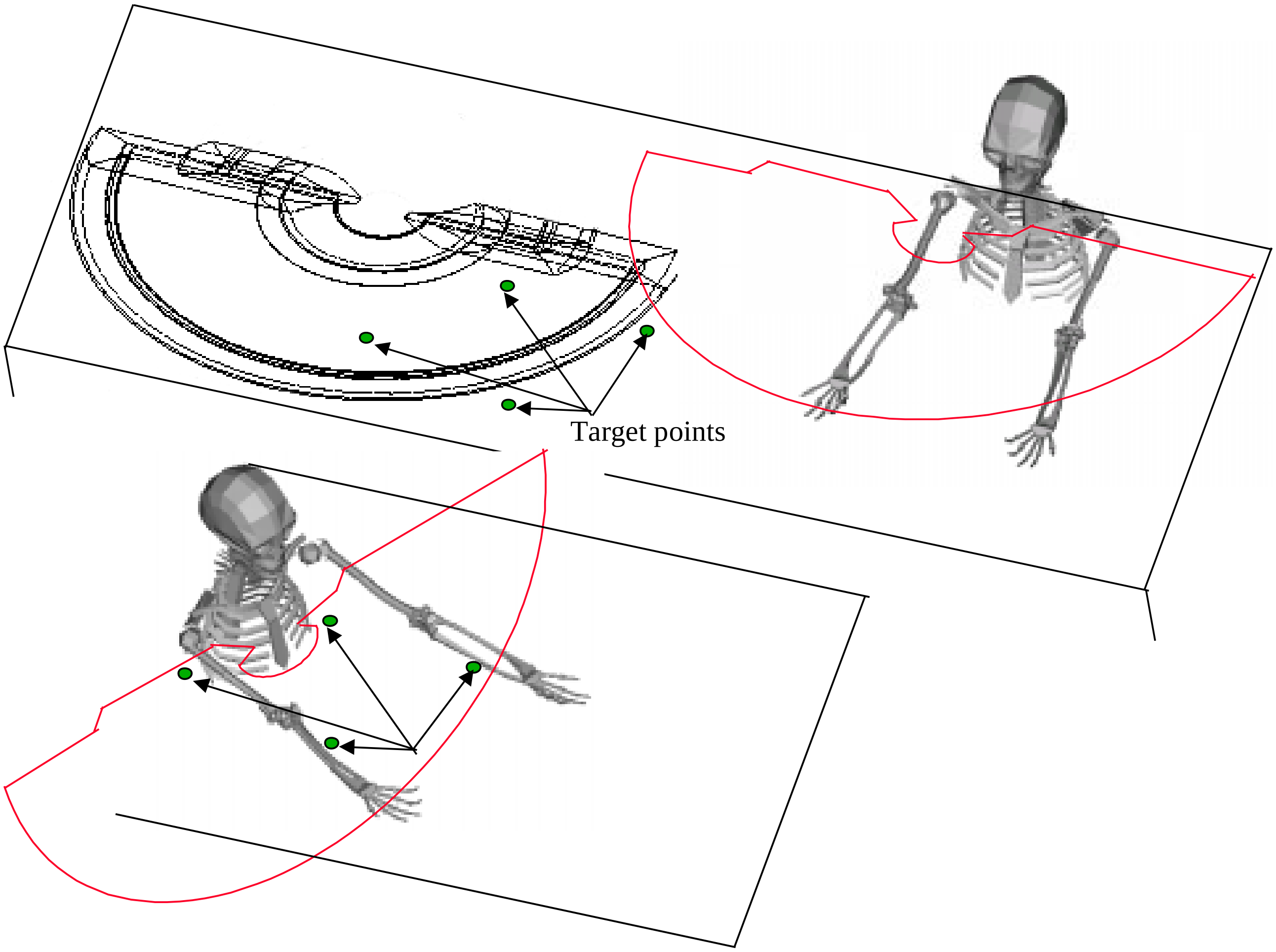


Fig. 5 Motion of the workspace (i.e., placement of the human) is computed as a result of the numerical algorithm

Layout Design

Layout design addresses an important problem in ergonomics. This problem arises in the design and packaging of vehicles (e.g., dashboards), avionics (design of cockpits), manufacturing (placement of tools, controls, etc.), and many other fields.

Given the position of a human, the problem requires selecting the most suitable positions for the target points. Moreover, in human motion simulation packages, it is important to design the surrounding environment such that an animated mannequin can perform the required tasks, verify the design, and explore potential difficulties.

Because of our closed-form results, we are able to optimize the location of the target points with respect to the workspace envelope while optimizing a given cost function. The problem is illustrated in Fig. 6, where two target points must be located with respect to a human operator. Since equations of the boundary envelope exist, it is now possible to determine where the points can be located while satisfying all constraints. These constraints include closure (point inside the envelope) and embedding (points embedded a distance from the boundary). In order to place the target points, a cost function is used to drive the iterative algorithm towards maximizing/minimizing a cost function. A cost function that has been successfully attempted is the total displacement from neutral position for each of the joints.

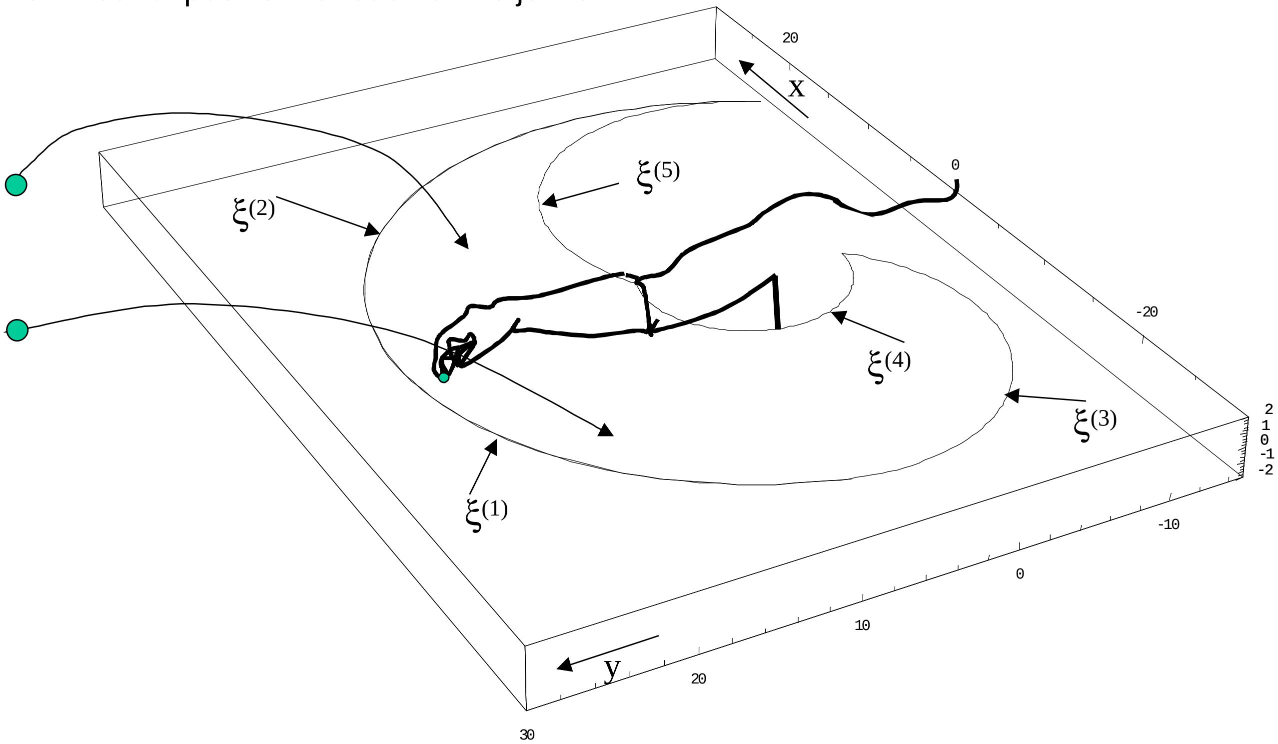


Fig. 6 Illustration of the placement problem (locating the target points with respect to a human)

Posture Prediction

In the fields of kinematics and robotics, posture prediction is called inverse kinematics, where a position is specified (and possible orientation) of the end-effector (e.g., hand), the algorithm is required to determine the joint variables (e.g., angles of the shoulder, elbow, and wrist) to predict what posture must be used to reach the given point in the given orientation. While inverse kinematics algorithms have been able to address problems especially those with minimum degrees of freedom, the inverse solutions are difficult to obtain. The difficulties become more apparent for higher degrees of freedom where the inverse kinematics yields a large number of solutions, some of which are imaginary, and others are difficult to choose from. Moreover, it is even more difficult using traditional inverse kinematic techniques to calculate the best posture based on the initial configuration of the arm (initial conditions).

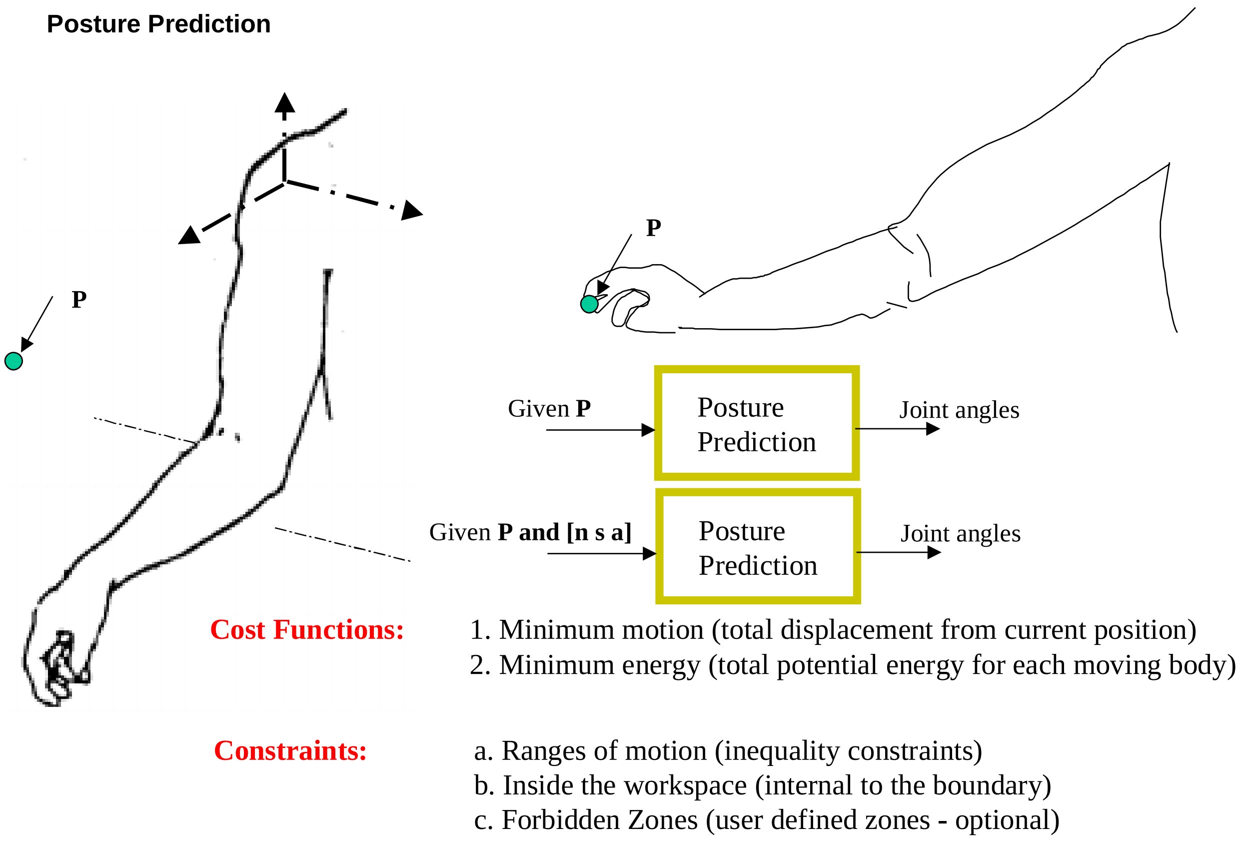
In our approach and because of the underlying mathematical formulation, we are able to perform the following:

(1) Readily determine if a point is reacheable or not by a simple geometric check for the point (whether or not it is in the workspace).

(2) Calculate a realistic posture given the coordinates of a target point. If the point is in the workspace, a cost function is used to drive (move) the limb (e.g., arm) towards

the point while satisfying all necessary constraints. This cost function is designed to emulate human motion, to reduce the effort needed by the person (realistic calculations), and depends on the initial posture (initial configuration of the arm). It can handle any number of degrees of freedom and incorporates joint angles (ranges of motion).

(3) Calculate the posture if both the position and orientation are given (e.g., the index finger in a given orientation). The orientation could be a complete triad or simply one vector (e.g., direction of index finger).



Cost Functions:
1. Minimum motion (total displacement from current position)
2. Minimum energy (total potential energy for each moving body)

Constraints:
a. Ranges of motion (inequality constraints)
b. Inside the workspace (internal to the boundary)
c. Forbidden Zones (user defined zones - optional)

Fig. 7 Illustration of posture prediction

References

The formulation and code were developed during the past six years for the field of robotics. More recently, because of the ability to determine the workspace in closed-form, the method was successfully applied to geometric modeling (swept volumes), manufacturing automation (numerically controlled verification), and computer-aided design (solid modeling).

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