Period #12: Bending Behavior

A. Next Topic

Chapter 4 dealt with axial loading of axial members (rods)

Chapter 5 dealt with torsional loading of axial members (shafts)

Chapters 6 and 7 deals with transverse loading of axial members (beams).

Our goal in studying beams is to quantify the internal strains and stresses in members subjected to transverse loads.
B. Beam Sign Conventions for Loads, Transverse Shears, and Bending Moments

Beams subjected to transverse loads \( w(x) \) will have internal shear forces \( V \) and bending moments \( M \).

The sign conventions for \( w, V, \) and \( M \) are as shown below:

Loads acting upward are taken to be positive.

Internal shears on cut faces that tend to cause counter-clockwise rotation are positive.

Internal bending moments that tend to cause compression of the top fibers of a beam are positive.
C. Calculation of the Shear (V) and Moment (M) distributions

A shear diagram is a graphical representation of the transverse shear force V distribution in a beam.

A moment diagram is a graphical representation of the bending moment M distribution in a beam.

Given a loading that acts on specific beam with well-defined support conditions, it is necessary to be able to compute both V(x) and M(x).

1. Relation between w and V

Consider an infinitesimal beam segment of length dx:

\[ \sum F_y = 0 \quad [\text{Neglect terms of order 2 or higher}] \]

\[ V(x) + w(x)dx = V(x + dx) \]

\[ \frac{V(x + dx) - V(x)}{dx} = w(x) \]

\[ \frac{dV}{dx} = w(x) \leftrightarrow V_B - V_A = \int_A^B w(x)dx \]
In words, the shear in a beam is obtained by integrating the load function.

Alternatively, the slope of the shear diagram represents the intensity of the load function.
When a discrete load acts on a beam, the shear diagram increases or decreases with that load.
2. Relation Between V and M

Consider an infinitesimal segment of a beam dx:

\[ \sum M = 0 \quad \text{[Neglect all terms of order 2 or higher]} \]

\[ -M(x) + M(x + dx) - V(x)dx = 0 \]

\[ \frac{M(x + dx) - M(x)}{dx} = V(x) \]

\[ \frac{dM}{dx} = V(x) \quad \leftrightarrow \quad M_B - M_A = \int_{A}^{B} V(x)dx \]
In words, the moment in a beam is obtained by integrating the shear.

Alternatively, the slope of moment diagram at each point represents the shear at each point.
When a concentrated clockwise moment is applied to a beam, the moment distribution increases with that moment.
D. Examples

Example 12.1  Draw the shear and moment diagrams for the beam

![Beam with loads](image-url)
Example 12.2. Express the internal shear and moment in terms of $x$ and then draw the shear and moment diagrams.
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\[
R_A + R_B = \frac{1}{2} W_o \frac{L}{2} = \frac{W_o L}{4}
\]

\[
\Sigma M = 0 = R_B L - \left(\frac{W_o}{4}\right) \left(\frac{5L}{6}\right)
\]

\[
R_B = \frac{5}{24} W_o L
\]

\[
R_A = \frac{1}{24} W_o L
\]

\[
V(x) = \frac{W_o L}{24} - \frac{2W_o}{L}\left(\frac{x^2}{2} - \frac{xL}{2}\right) \quad x > \frac{L}{2}
\]

\[
V(x') = \frac{W_o L}{24} - \frac{W_o x'^2}{L}
\]

\[
V = 0 \quad \text{at} \quad x' = \frac{L}{24} = 0.204L = x'_{\text{max}} \quad \text{(location of max moment)}
\]

\[
M(x') = M_{\text{max}} = \frac{W_o L^2}{48} + \frac{W_o L}{24} \left(0.204L\right) - \frac{W_o}{3L} \left(0.204L\right)^3
\]

\[
M_{\text{max}} = 0.0265 W_o L^2 = M_{\text{max}}
\]