Problem 1:
Consider a plate containing a circular hole of radius \( a \), as shown in the figure below. The plate, which has geometric dimensions large enough to be idealized as an infinite plate, is subjected to a uniform far-field tensile stress \( \sigma^\infty \). In the class, we showed that the linear-elastic stress field in this body is

\[
\begin{align*}
\sigma_r &= \frac{\sigma^\infty}{2} \left[ \left(1 - \frac{a^2}{r^2}\right) - \left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right) \cos 2\theta \right] \\
\sigma_\theta &= \frac{\sigma^\infty}{2} \left[ \left(1 + \frac{a^2}{r^2}\right) + \left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta \right] \\
\tau_{r\theta} &= \frac{\sigma^\infty}{2} \left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right) \sin 2\theta
\end{align*}
\]

where \( r \) and \( \theta \) are polar coordinates with the center of the hole as the origin.

1. Plot \( \sigma_r \) and \( \sigma_\theta \) as a function of \( r \) at the horizontal (line AB) and vertical (line CD) planes through the center of the hole.

2. Plot \( \sigma_\theta \) as a function of \( \theta \) (\( 0 \leq \theta \leq 2\pi \)) at the free surface of the hole (\( r = a \)).
**Problem 2:**
Consider an infinite plate, shown in the figure below, containing an elliptical hole with $2a$ and $2b$ as the lengths of its major and minor axes. If the plate is subjected to a uniform far-field tensile stress $\sigma^\infty$, we showed in the class that the normal stress at the tip of the ellipse (i.e., at point A) under linear-elastic condition is

$$\sigma_x \bigg|_{x=a,y=0} = \sigma^\infty \left[ 1 + 2 \frac{a}{b} \right].$$

Now, let $\rho$ denote the radius of curvature at the tip of the ellipse. Show that $\sigma_y \big|_{x=a,y=0}$ can also be expressed by

$$\sigma_y \big|_{x=a,y=0} = \sigma^\infty \left[ 1 + 2 \frac{a}{\sqrt{\rho}} \right].$$

Hint: using equation of ellipse in cartesian coordinates ($x$-$y$), calculate $\rho$ from

$$\rho = \left[ \frac{1 + (dy/dx)^2}{d^2y/dx^2} \right]_{x=a,y=0}.$$