3.51 Water flows through the pipe contraction shown in Fig. P3.51. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, \( D \).

\[
\frac{P_1}{g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{g} + \frac{V_2^2}{2g} + Z_2
\]

With \( A_1 V_1 = A_2 V_2 \)

Thus, with \( Z_1 = Z_2 \)

\[
\frac{P_1 - P_2}{g} = \frac{V_1^2 - V_2^2}{2g} = \frac{\left(\frac{V_1}{D_1}\right)^2 - \left(\frac{V_2}{D_2}\right)^2}{2g}
\]

but \( \rho_1 = \rho \) and \( \rho_2 = \rho \) so that \( \rho_1 - \rho_2 = \rho (h_1 - h_2) = 0.2 \rho \)

Thus

\[
\frac{0.2 \rho}{g} \left[ \left(\frac{V_1}{D_1}\right)^2 - \left(\frac{V_2}{D_2}\right)^2 \right] = \frac{V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{\frac{0.2 \left(\frac{2g}{\rho}\right)}{\left[ \left(\frac{V_1}{D_1}\right)^2 - \left(\frac{V_2}{D_2}\right)^2 \right]}}
\]

and

\[ Q = A_1 V_1 = \frac{\pi (0.1)^2}{2} \sqrt{\frac{0.2 \left(\frac{2g}{\rho}\right)}{\left[ \left(\frac{V_1}{D_1}\right)^2 - \left(\frac{V_2}{D_2}\right)^2 \right]}} \]

or

\[ Q = \frac{0.0156 D^2}{V(0.1)^2 - D^2} \quad \text{when} \quad D \sim m \]