2.57 A 3-m-wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Fig. P2.57. The gate is hinged at its bottom and held closed by a horizontal force, \( F_H \), located at the center of the gate. The maximum value for \( F_H \) is 3500 kN. (a) Determine the maximum water depth, \( h \), above the center of the gate that can exist without the gate opening. (b) Is the answer the same if the gate is hinged at the top? Explain your answer.

**FIGURE P2.57**

For gate hinged at bottom:

\[
\sum M_H = 0
\]

so that

\[
\begin{align*}
(4 \text{m}) F_H &= \ell F_R \quad \text{(see figure)} \quad (1) \\
F_R &= \Delta h_c A = \left(9.80 \frac{\text{kN}}{\text{m}^3}\right)(h)(3 \text{m} \times 8 \text{m}) \\
&= (9.80 \times 24 h_c) \Delta h_c \text{kN}
\end{align*}
\]

\[
\begin{align*}
y_R &= \frac{\ell \times c}{y_c A} + y_c = \frac{1}{h_c} \left(3 \text{m} \times 8 \text{m}\right)^2 + h_c +\frac{h}{h_c} \\
&= \frac{5.33}{h_c} + h
\end{align*}
\]

Thus,

\[
\ell (\text{m}) = h + 4 - \left(\frac{5.33}{h_c} + h \right) = 4 - \frac{5.33}{h_c}
\]

and from Eq. (1)

\[
(4 \text{m})(3500 \text{ kN}) = (4 - \frac{5.33}{h_c})(9.80 \times 24)(h_c) \text{ kN}
\]

so that

\[
\frac{1}{h_c} = 16.2 \text{ m}
\]

(cont')
For gate hinged at top

\[ \sum M_H = 0 \]

so that

\[ (4 \text{ m}) F_H = l_1 F_R \quad \text{(see figure)} \] (1)

where

\[ l_1 = y_F - (h - 4) = \left( \frac{5.33}{h} + 4 \right) - (h - 4) \]

\[ = \frac{5.33}{h} + 4 \]

Thus, from Eq. (1)

\[ (4 \text{ m})(3500 \text{ kN}) = \left( \frac{5.33}{h} + 4 \right)(9.80 \times 24)(h) \text{ kN} \]

and

\[ h = 13.5 \text{ m} \]

Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.