Problem 1: Linear momentum equation (Chapter 5)

Information and assumptions
- \( \rho = 999 \text{ kg/m}^3 \)
- \( V_j = 30 \text{ m/s} \)
- \( A_j = 0.01 \text{ m}^2 \)
- \( \theta = 30^\circ \)
- Frictionless cart
- Jet velocity magnitude remains constant along the vane surface

Find
- The restrain force \( F_x \)

Solution
For a control volume that surrounds the vane and cart, the continuity equation gives,

\[
\rho V_{in} A_{in} = \rho V_{out} A_{out}
\]

Since \( V_{in} = V_{out} = V_j \), thus

\[
A_{in} = A_{out} = A_j
\]

The linear momentum equation in the horizontal direction is

\[
\int_{CS} u \rho V \cdot \hat{n} dA = \sum F_{CV}
\]

or

\[
-V_j \rho V_j A_j + (V_j \cos \theta) \rho V_j A_j = -F_x
\]

Thus,

\[
F_x = \rho A_j V_j^2 (1 - \cos \theta)
\]

For \( \rho = 999 \text{ kg/m}^3 \), \( V_j = 30 \text{ m/s} \), \( A_j = 0.01 \text{ m}^2 \), and \( \theta = 30^\circ \),

\[
F_x = \left( 999 \text{ kg/m}^3 \right) (0.01 \text{ m}^2) (30 \text{ m/s})^2 (1 - \cos 30^\circ) = 1205 \text{ N}
\]
Problem 2: Energy equation (Chapter 5)

Information and assumptions
- $Q = 220 \text{ m}^3/\text{h}$
- $\rho = 999 \text{ kg/m}^3$
- Friction head loss $h_L = 5 \text{ m}$
- $\Delta z = 2 \text{ m}$
- $D_e = 5 \text{ cm}$
- $\alpha = 1$

Find
- Pump power in kW delivered to the water

Solution
Energy equation:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$  

Let “1” be at the reservoir surface and “2” be at the nozzle exit. Then,

- $p_1 = p_2 = 0 \text{ (gage)}$
- $z_2 - z_1 = 2 \text{ m}$
- $V_1 \approx 0$

$$V_2 = \frac{Q}{A} = \frac{Q}{\pi(D_e/2)^2} = \frac{(220 \text{ m}^3/\text{h})(\text{h}/3600 \text{s})}{\pi(0.025 \text{ m})^2} = 31.12 \text{ m/s}$$

$$h_L = 5 \text{ m}$$

The energy equation with $\alpha_1 = \alpha_2 = 1$ becomes

$$h_p = \frac{V_2^2}{2g} + (z_2 - z_1) + h_L$$

or

$$h_p = \frac{(31.12 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 2 \text{ m} + 5 \text{ m} = 56.4 \text{ m}$$

The pump power is

$$P = \dot{m}h_p = \rho g Q h_p$$

$$= \left(999 \text{ kg/m}^3\right) \left(9.81 \text{ m/s}^2\right) \left(\frac{220 \text{ m}^3}{\text{h}}\right) \left(\frac{3600 \text{ s}}{\text{h}}\right) \left(56.4 \text{ m}\right) = 33,778 \text{ W} = 33.8 \text{ kW}$$
Problem 3: Exact solution (Chapter 6)

Information and assumptions
- Steady, parallel, viscous flow
- No pressure gradient in the flow direction
- \( U = 0.2 \, \text{m/s} \)
- \( b = 5 \, \text{mm} \)

Find
- Velocity distribution across the plates and flow rate between the plates per unit depth

Solution
(a) For a steady flow (\( \partial / \partial t = 0 \)) with \( v = w = 0 \) and for a zero pressure gradient in the flow direction (\( \partial p / \partial x = 0 \)), the Navier-Stokes equations (also by using the continuity equation, \( \partial u / \partial x = 0 \)) reduce to
\[
\frac{\partial^2 u}{\partial y^2} = 0
\]
So that
\[
u = c_1 y + c_2
\]
By using the boundary conditions, \( u = 0 \) at \( y = 0 \) and \( u = U \) at \( y = b \),
\[
c_1 = \frac{U}{b}
\]
\[
c_2 = 0
\]
Therefore
\[
u = \frac{U}{b} y = \frac{0.2 \, \text{m/s}}{0.005 \, \text{m}} = 40 \cdot y \, (\text{m/s})
\]

(b) The flow rate per unit depth is
\[
q = \int_0^b u \, dy
\]
Thus,
\[
q = \int_0^b \frac{U}{b} y \, dy = \frac{Ub}{2}
\]
\[
= \frac{(0.2 \, \text{m/s})(0.005 \, \text{m})}{2} = 5 \times 10^{-4} \, \text{m}^2/\text{s}
\]
Problem 4: Dimensional analysis (Chapter 7)

Information and assumptions

- \( D = f(\rho, V, d) \)
- For model: \( D = 17 \text{ lb}, \ d = 1 \text{ ft}, \ V = 4 \text{ ft/s} \)
- For prototype: \( d = 30 \text{ ft} \)
- For water: \( \rho = 1.94 \text{ slugs/ft}^3 \)
- For air: \( \rho = 2.38 \times 10^{-3} \text{ slugs/ft}^3 \)

Find

- Pi parameter and the drag \( D \) of the prototype

Solution

(a) From the pi theorem, \( 4 - 3 = 1 \) pi term required.

\[
\Pi = D \cdot \rho^a \cdot V^b \cdot d^c
\]

\[
(F)(FL^{-4}T^2)^a(FL^{-1})^b(L)^c = F^0L^0T^0
\]

or

\[
(MLT^{-1})(ML^{-3})^a(ML^{-1})^b(L)^c = M^0L^0T^0
\]

\[
\therefore \Pi = \frac{D}{\rho V^2 d^2}
\]

(b) For similarity between model and prototype,

\[
\frac{D}{\rho V^2 d^2} = \frac{D_m}{\rho_m V_m^2 d_m^2}
\]

where, the subscript \( m \) stands for ‘model’. Then,

\[
D = \left( \frac{\rho}{\rho_m} \right) \left( \frac{V}{V_m} \right)^2 \left( \frac{d}{d_m} \right)^2 D_m
\]

\[
= \left( \frac{2.38 \times 10^{-3} \text{ slugs/ft}^3}{1.94 \text{ slugs/ft}^3} \right) \left( \frac{10 \text{ ft/s}}{4 \text{ ft/s}} \right)^2 \left( \frac{30 \text{ ft}}{1 \text{ ft}} \right)^2 (17 \text{ lb}) = 117 \text{ lb}
\]