Chapter 1 INTRODUCTION AND BASIC CONCEPTS

1. Fluids and no-slip condition
   - Fluid: a substance that deforms continuously when subjected to shear stresses
   - No-slip condition: no relative motion between fluid and boundary

2. Basic units

<table>
<thead>
<tr>
<th>Dimension</th>
<th>SI unit</th>
<th>BG unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity $V$</td>
<td>$L/t$</td>
<td>m/s</td>
</tr>
<tr>
<td>Acceleration $a$</td>
<td>$L/t^2$</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>Force $F$</td>
<td>$ML/t^2$</td>
<td>N (Kg $\cdot$ m/s$^2$)</td>
</tr>
<tr>
<td>Pressure $p$</td>
<td>$F/L^2$</td>
<td>Pa (N/m$^2$)</td>
</tr>
<tr>
<td>Density $\rho$</td>
<td>$M/L^3$</td>
<td>Kg/m$^3$</td>
</tr>
<tr>
<td>Internal energy $u$</td>
<td>$FL/M$</td>
<td>1/Kg (N $\cdot$ m/kg)</td>
</tr>
</tbody>
</table>

3. Weight and mass
   - $W(N) = m(Kg) \cdot g$, where $g = 9.81$ m/s$^2$
   - $W(lbf) = m(slug) \cdot g$, where $g = 32.2$ ft/s$^2$
   - 1 N = 1 Kg $\times$ 1 m/s$^2$
   - 1 lbf = 1 slug $\times$ 1 ft/s$^2$
   - 1 slug = 32.2 lbm (weighs 32.2 lb under standard gravity)

4. Properties involving mass or weight of fluid
   - Specific weight $\gamma = \rho g$ (N/m$^3$)
   - Specific gravity $SG = \gamma/\gamma_{water}$

5. Viscosity
   - Newtonian fluid: $\tau = \mu \dot{\theta} = \mu \frac{du}{dy}$
     - $\tau$ Shear stress (N/m$^2$)
     - $\dot{\theta} = \frac{\delta \theta}{\delta t} = \frac{1}{\delta t} \left( \frac{\delta u \delta t}{\delta y} \right)$
     - $\mu$ Coefficient of viscosity (Ns/m$^2$)
     - $\nu = \mu/\rho$ Kinematic viscosity (m$^2$/s)
   - Non-Newtonian fluid: $\tau \propto \left( \frac{du}{dy} \right)^n$

   Ex) Couette flow
   $$ u(y) = \frac{U}{h} y, \tau = \mu \frac{du}{dy} = \mu \frac{U}{h} $$
6. Vapor pressure and cavitation
- When the pressure of a liquid falls below the vapor pressure it evaporates, i.e., changes to a gas.
- If the pressure drop is due to fluid velocity, the process is called cavitation.
- Cavitation number

\[ C_a = \frac{p - p_\infty}{1/2 \rho V_\infty^2} \]

- \( C_a < 0 \) implies cavitation

7. Surface tension
- Surface tension force

\[ F_\sigma = \sigma \cdot L \]

- \( F_\sigma \) = line force with direction normal to the cut
- \( \sigma \) = surface tension [N/m]
- \( L \) = length of cut through the interface

Chapter 2 PRESSURE AND FLUID STATICS

1. Absolute pressure, Gage pressure, and Vacuum

- \( p_A > p_a \)
- \( p_g = p_A - p_a \) = gage pressure
- \( p_A < p_a \)
- \( p_{vac} = -p_g = p_a - p_A \) = vacuum pressure

\( p_a = \text{atmospheric pressure} = 101.325 \text{ kPa} \)
2. Pressure variation with elevation

- For a static fluid, pressure varies only with elevation \( z \) and is constant in horizontal \( x, y \) planes.
  \[
  \frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = -\rho g = -\gamma
  \]

- If the density of fluid is constant,
  - \( p + \gamma z = \text{constant (piezometric pressure)} \)
  - \( \frac{p}{\gamma} + z = \text{constant (piezometric head)} \)
  - \( p_{z=0} = 0 \) gage, \( p = -\gamma z \) : increase linearly with depth, decrease linearly with height

3. Pressure measurements (Manometry)

1) U-tube manometer

- \( p_1 + \gamma_m \Delta h - \gamma \ell = p_4 \)
- \( p_4 = \gamma_m \Delta h - \gamma \ell \) \( gage \)
- \( = \gamma_{water} (SG_m \Delta h - SG \ell) \)

2) Differential U-tube manometer

- \( p_1 + \gamma_f \ell_1 - \gamma_m \Delta h - \gamma_f (\ell_2 - \Delta h) = p_2 \)
- \( p_1 - p_2 = \gamma_f (\ell_2 - \ell_1) + (\gamma_m - \gamma_f) \Delta h \)
- \( \left( \frac{p_1}{\gamma_f} + \ell_1 \right) - \left( \frac{p_2}{\gamma_f} + \ell_2 \right) = (\gamma_m/\gamma_f - 1) \Delta h \)

\( \text{difference in piezometric head} \)

- If fluid is a gas \( \gamma_f \ll \gamma_m : p_1 - p_2 = \gamma_m \Delta h \)
- If fluid is liquid & pipe horizontal \( \ell_1 = \ell_2 \):
  - \( p_1 - p_2 = (\gamma_m - \gamma_f) \Delta h \)
4. Hydrostatic forces on plane surfaces

1) Horizontal surfaces

- $F = pA$
- Line of action is through centroid of $A$, i.e., $(x_{cp}, y_{cp}) = (\bar{x}, \bar{y})$

2) Inclined surfaces

- $F = \bar{p}A$
  - $\bar{p} = \gamma \sin \alpha \bar{y}$: pressure at centroid of $A$
  - $\bar{y} = \frac{1}{A} \int y \, dA$ : 1st moment of area
- Magnitude of resultant hydrostatic force on plane surface is product of pressure at centroid of area and area of surface
- Center of pressure
  - $y_{cp} = \bar{y} + \frac{I}{\bar{y}A}$
  - $x_{cp} = \frac{I_{xy}}{\bar{y}A} + \bar{x}$

$I$ : moment of inertia with respect to horizontal centroidal axis
For plane surfaces with symmetry about an axis normal to 0-0, $I_{xy} = 0$ and $x_{cp} = \bar{x}$
5. Hydrostatic forces on curved surfaces

\[ p = \gamma h \]

\[ F = - \oint_{A} p n \, dA \]

\[ h \] - distance below free surface

- \( F_x = - \int_{A_x} p \, dA_x \) 
  \( (dA_x = n \cdot i \mathbf{A} : \text{projection of } ndA \text{ onto plane } \perp \text{ to } x\text{-direction}) \)

- \( F_y = - \int_{A_y} p \, dA_y \) 
  \( (dA_y = n \cdot j \mathbf{A} : \text{projection of } ndA \text{ onto plane } \perp \text{ to } y\text{-direction}) \)

- \( F_z = - \int_{A_z} p \, dA_z = \gamma \Psi = \text{weight of fluid above surface } A \)

6. Buoyancy

- \( F_B = F_{V_2} - F_{V_1} = \rho g \Psi \)
- Fluid weight equivalent to body volume \( \Psi \)
- Line of action is through centroid of \( \Psi = \text{center of buoyancy} \)

7. Stability

1) Immersed bodies

\[ \sum F_y = 0 \quad \text{and} \quad \sum M = 0. \]

\[ \sum M = 0 \] requires \( C = G \) and the body is neutrally stable

- If \( C \) is above \( G \): stable (righting moment when heeled)
- If \( G \) is above \( C \): unstable (heeling moment when heeled)
2) Floating bodies
- The center of buoyancy generally shifts when the body is rotated
- Metacenter M: The point of intersection of the lines of action of the buoyant force before and after heel

\[ GM = \frac{I_{oo}}{V} - CG \]
- GM: metacentric height
- \( I_{oo} \) = moment of inertia of waterplane area about centerplane axis
- GM > 0: stable (M is above G)
- GM < 0: unstable (G is above M)

8. Fluids in rigid-body motion
- If no relative motion between fluid particles

\[ \nabla p = \rho \left( g - a \right) \]
- For rigid body translation: \( a = a_x \hat{i} + a_z \hat{k} \)
  \[ \nabla p = -\rho \left[ a_x \hat{i} + (g + a_z) \hat{k} \right] \]
  \[ \frac{\partial p}{\partial x} = -\rho a_x \]
  - \( a_x < 0 \), \( p \) increase in +x
  - \( a_x > 0 \), \( p \) decrease in +x
  \[ \frac{\partial p}{\partial z} = -\rho (g + a_z) \]
  - \( a_z > 0 \), \( p \) decrease in +z
  - \( a_z < 0 \) and \( |a_z| < g \), \( p \) decrease in +z but slower than \( g \)
  - \( a_z < 0 \) and \( |a_z| > g \), \( p \) increase in +z

\[ p = \rho \dot{G} s + \text{constant} \Rightarrow p_{gage} = \rho \dot{G} s \]
- \( G = a_x^2 + (g + a_z)^2 \)
- \( \hat{s} \) = unit vector in direction normal of \( \nabla p \)
- For rigid body rotation: \( a = -r \Omega^2 \hat{e}_r \)
  \[ \nabla p = -\rho g \hat{k} + \rho r \Omega^2 \hat{e}_r \]
  \[ \frac{\partial p}{\partial r} = \rho r \Omega^2 \quad \frac{\partial p}{\partial z} = -\rho g \quad \frac{\partial p}{\partial \theta} = 0 \]
  \[ p = \frac{\rho}{2} r^2 \Omega^2 - \rho g z + \text{constant} \] or \( \frac{\rho}{\gamma} + z - \frac{\gamma^2}{2g} = \text{constant} (V = r \Omega) \)
  \[ z = \frac{p_0 - p}{\rho g} + \frac{r^2 \Omega^2}{2g} = a + br^2 : \text{curves of constant pressure} \ (p_0 : \text{pressure at } (r,z)=(0,0)) \]
Chapter 3 BERNOULLI EQUATION

1. Flow patterns
   - Streamline: a line that is everywhere tangent to the velocity vector at a given instant
   - Pathline: the actual path traveled by a given fluid particle
   - Streakline: the locus of particles which have earlier passed through a particular point

2. Streamline coordinates
   - Velocity: $\mathbf{V}(x, t) = v_s(x, t)\hat{s}$
   - Acceleration:
     \[ a = \left( \frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial s} \right) \hat{s} + \left( \frac{\partial v_n}{\partial t} + \frac{v_s^2}{R} \right) \hat{n} \]
     - $\frac{\partial v_s}{\partial t}$ = local $a_s$ in $\hat{s}$ direction
     - $\frac{\partial v_n}{\partial t}$ = local $a_n$ in $\hat{n}$ direction
     - $v_s \frac{\partial v_s}{\partial s}$ = convective $a_s$ due to spatial gradient of $\mathbf{V}$
     - $\frac{v_s^2}{R}$ = convective $a_n$ due to curvature $\psi$ : centrifugal acceleration
     - $R$ : the radius of curvature of the streamline

3. Bernoulli equation
   - Euler equation: $\rho a = \nabla (p + \gamma z)$
   - Along streamline
     \[ \frac{v_s^2}{2} + \frac{p_1}{\rho} + g z_1 = \text{constant} \]
     or
     \[ \frac{v_{s1}^2}{2} + \frac{p_1}{\rho} + g z_1 = \frac{v_{s2}^2}{2} + \frac{p_2}{\rho} + g z_2 \]
   - Across streamline
     \[ \int \frac{v_s^2}{R} \, dn + \frac{p}{\rho} + g z = \text{constant} \]
   - Assumptions
     - Inviscid flow
     - Steady flow
     - Incompressible flow
     - Flow along a streamline
4. Applications of Bernoulli equation

1) Stagnation tube
   - \[ p_1 + \rho \frac{v_1^2}{2} = p_2 + \rho \frac{v_2^2}{2} \]
     \[ z_1 = z_2, p_1 = \gamma d, V_2 = 0, p_2 = \gamma (l + d) \]
   - \[ V_1 = \sqrt{\frac{2}{\rho} (p_2 - p_1)} = \sqrt{\frac{2}{\rho} \gamma l} = \sqrt{2 gl} \]

2) Pitot tube
   - \[ \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 \]
     \[ V_1 = 0, h = \frac{p}{\gamma} + z \]
   - \[ V_2 = \sqrt{2g(h_1 - h_2)} \]
     \[ h_1 - h_2 \text{ from manometer or pressure gage} \]

3) Simplified continuity equation
   - Volume flow rate: \[ Q = VA \]
   - Mass flow rate: \[ \dot{m} = \rho Q = \rho VA \]
   - Conservation of mass: \[ \rho_1 V_1 A_1 = \rho_2 V_2 A_2 \]
   - For incompressible flow (\( \rho = \text{constant} \)): \[ V_1 A_1 = V_2 A_2 \text{ or } Q_1 = Q_2 \]

4) Flow rate measurement
   - If the flow is horizontal \( (z_1 = z_2) \), steady, inviscid, and incompressible, \[ p_1 + \frac{1}{2} \rho V_1^2 = p_2 + \frac{1}{2} \rho V_2^2 \]
   - If velocity profiles are uniform at sections (1) and (2), \[ Q = V_1 A_1 = V_2 A_2 \]
   - Flow rate is, \[ Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1-(A_2/A_1)^2]}} \]

Ex) Venturi meter
Chapter 4 FLUIDS KINEMATICS

1. Velocity and description Methods
   • Lagrangian: keep track of individual fluids particles
     \[ V_p = u_p \hat{i} + v_p \hat{j} + w_p \hat{k} \]
   • Eulerian: focus attention on a fixed point in space
     \[ V = V(x, t) = u \hat{i} + v \hat{j} + w \hat{k} \]

2. Acceleration and material derivatives
   • Lagrangian:
     \[ a_p = \frac{dV_p}{dt} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \]
     \[ a_x = \frac{du_p}{dt} \quad a_y = \frac{dv_p}{dt} \quad a_z = \frac{dw_p}{dt} \]
   • Eulerian:
     \[ a = \frac{DV}{Dt} = \frac{\partial V}{\partial t} + (V \cdot \nabla)V = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \]
     where,
     \[ \nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} : \text{ gradient operator} \]
     \[ a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \]
     \[ a_y = \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \]
     \[ a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \]
   • \( \frac{\partial v}{\partial t} \) = local or temporal acceleration. Velocity changes with respect to time at a given point.
   • \( (V \cdot \nabla)V \) = convective acceleration. Spatial gradients of velocity
   • Material (substantial) derivative
     \[ \frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \]
3. Flow classification
- One-, Two-, and Three-dimensional flow
- Steady vs. Unsteady flow
- Incompressible and Compressible flow
- Viscous and Inciscid flow
- Rotational vs. Irrotational flow
- Laminar vs. Trubulent viscous flow
- Internal vs. External flow
- Separated vs. Unseparated flow

4. Reynolds Transport Theorem (RTT)
\[
\frac{dB_{sys}}{dt} = \frac{\partial}{\partial t} \int_{CV} \beta \rho d\mathcal{V} + \int_{CS} \beta \rho V_R \cdot n dA
\]

Special Cases:
- Non-deforming CV moving at constant velocity: \( \frac{dB_{sys}}{dt} = \int_{CV} \frac{\partial}{\partial t} (\beta \rho) d\mathcal{V} + \int_{CS} \beta \rho V_R \cdot n dA \)
- Fixed CV: \( \frac{dB_{sys}}{dt} = \int_{CV} \left[ \frac{\partial}{\partial t} (\beta \rho) + \nabla \cdot (\beta \rho \mathbf{V}) \right] d\mathcal{V} \)
- Steady flow: \( \frac{\partial}{\partial t} = 0 \)
- Uniform flow across discrete CS (steady or unsteady): 
  \[ \int_{CS} \beta \rho V \cdot n dA = \sum_{CS} \beta \rho V \cdot n dA_{(-inlet,+outlet)} \]

5. Continuity equation
\[
\frac{dM}{dt} = 0 = \frac{\partial}{\partial t} \int_{CV} \rho d\mathcal{V} + \int_{CS} \rho V_R \cdot n dA
\]

Simplifications:
- Steady flow: \(- \frac{\partial}{\partial t} \int_{CV} \rho d\mathcal{V} = 0\)
- \( \mathcal{V} = \) constant over discrete \( dA \) (flow sections): 
  \( \int_{CS} \rho \mathcal{V} \cdot n dA = \sum_{CS} \rho \mathcal{V} \cdot A \)
- Incompressible fluid (\( \rho = \) constant): 
  \( \int_{CS} \rho \mathcal{V} \cdot A = - \frac{\partial}{\partial t} \int_{CV} \rho d\mathcal{V} \) (conservation of volume)
- Steady One-dimensional flow in a conduit: 
  \( \sum_{CS} \rho \mathcal{V} \cdot A = 0, -\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0, \) for \( \rho = \) const \( Q_1 = Q_2 \)