Problem 1: Shear stress (Chapter 1)

Information and assumptions

- \( \frac{u}{U} = 2 \frac{y}{h} - \frac{y^2}{h^2} \)
- \( \mu = 1.12 \times 10^{-3} \text{ N} \cdot \text{s/m}^2 \)
- \( U = 2 \text{ m/s} \)
- \( h = 0.1 \text{ m} \)

Find

- shearing stress that the water exerts on the fixed surface

Solution

\[
\tau = \mu \frac{du}{dy}
\]

(+4 points)

where

\[
\frac{du}{dy} = \frac{2U}{h} \left( 1 - \frac{y}{h} \right)
\]

(+3 points)

At the fixed surface \( (y = 0) \)

\[
\left( \frac{du}{dy} \right)_{y=0} = \frac{2U}{h}
\]

(+1 point)

So that

\[
\tau = \mu \left( \frac{2U}{h} \right) = \left( 1.12 \times 10^{-3} \text{ N} \cdot \text{s/m}^2 \right) \left( 2 \left( \frac{2 \text{ m}}{h} \right) \right) = 4.48 \times 10^{-2} \text{ N/m}^2
\]

(+2 points)
Problem 2: Hydrostatic force (Chapter 2)

Information and assumptions
- $\gamma = 9790 \text{ N/m}^3$
- A quarter-circle 50 m wide dam
- Water depth = 20 m

Find
- the horizontal and vertical components of hydrostatic force against the dam and the angle of the resultant force from the vertical

Solution
The horizontal force acts as if the dam were vertical and 20 m high:

$$F_H = \gamma h c A = \left(9790 \frac{\text{N}}{\text{m}^3}\right)(10 \text{ m})(20 \times 50 \text{ m}^2) = 97.9 \text{ MN}$$

(+4 points)

The vertical force is the weight of the fluid above the dam:

$$F_V = \gamma V = \left(9790 \frac{\text{N}}{\text{m}^3}\right)\left(\frac{\pi}{4}\right)(20 \text{ m})^2(50 \text{ m}) = 153.8 \text{ MN}$$

(+4 points)

The angle of the resultant force from the vertical is

$$\theta = \tan^{-1}\left(\frac{F_H}{F_V}\right) = 32.5^\circ$$

(+2 points)
**Problem 3: Bernoulli equation (Chapter 3)**

**Information and assumptions**
- Blood (SG = 1, \( \rho_{\text{water}} = 999 \text{ Kg/m}^3 \))
- \( V_1 = 0.5 \text{ m/s} \)
- \( A_2 = 1.8A_1 \)
- Steady and inviscid flow

**Find**
- pressure difference between the blood in the aneurysm and that in the artery

**Solution**

From Bernoulli equation,

\[
p_1 + \frac{1}{2} \rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \gamma z_2
\]

where \( z_1 = z_2 \). Or,

\[
p_2 - p_1 = \frac{1}{2} \rho (V_1^2 - V_2^2)
\]

However,

\[
\rho = \rho_{\text{water}} \cdot SG = (999 \text{ kg/m}^3)(1) = 999 \text{ kg/m}^3
\]

and from continuity,

\[
V_1A_1 = V_2A_2
\]

or

\[
V_2 = \frac{A_1}{A_2} V_1 = \left( \frac{1}{1.8} \right) V_1
\]

Thus,

\[
p_2 - p_1 = \frac{1}{2} \left( 999 \text{ kg/m}^3 \right) \left[ \left( 0.5 \text{ m/s} \right)^2 - \left( \frac{1}{1.8} \right)^2 \left( 0.5 \text{ m/s} \right)^2 \right] = 86.3 \text{ Pa}
\]
Problem 4: Acceleration (Chapter 4)

Information and assumptions
• Steady flow through a nozzle
• \( u = -V_0 \frac{x}{\ell} \) and \( v = V_0 \left[1 + \left(\frac{y}{\ell}\right)\right] \), where \( V_0 \) and \( \ell \) are constants
• water with \( V_0 = 1 \text{ m/s} \) and \( \ell = 0.1 \text{ m} \)
• Neglect viscous effects and use the Euler quation

Find
• pressure gradient along the \( y \)-axis at point (2), or at \((x,y) = (0 \text{ m}, 0 \text{ m})\)

Solution
Pressure gradient along the \( y \)-axis by using the Euler equation can be written as
\[
\frac{\partial p}{\partial y} = -\rho (a_y + g)
\]  
(+2 points)

Where,
\[
a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = 0 + \left(-V_0 \frac{x}{\ell}\right)(0) + V_0 \left(1 + \frac{y}{\ell}\right) \left(\frac{V_0}{\ell}\right) = \frac{V_0^2}{\ell} \left(1 + \frac{y}{\ell}\right)
\]  
(+5 points)

Thus,
\[
\frac{\partial p}{\partial y} = -\rho \left[\frac{V_0^2}{\ell} \left(1 + \frac{y}{\ell}\right) + g\right]
\]  
(+2 points)

At \((x,y) = (0 \text{ m}, 0 \text{ m})\)
\[
\frac{\partial p}{\partial y} = -\left(999 \frac{\text{kg}}{\text{m}^3}\right) \left[\left(\frac{1 \text{ m}}{\text{s}}\right)^2 0.1 \text{ m} \left(1 + \frac{0 \text{ m}}{0.1 \text{ m}}\right) + 9.81 \frac{\text{m}}{\text{s}^2}\right] = -19,790 \text{ Pa/m}
\]  
(+1 point)