5.57 A horizontal circular jet of air strikes a stationary flat plate as indicated in Fig. 5.57. The jet velocity is 40 m/s and the jet diameter is 30 mm. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine: (a) the magnitude of \( F_A \), the anchoring force required to hold the plate stationary; (b) the fraction of mass flow along the plate surface in each of the two directions shown; (c) the magnitude of \( F_A \), the anchoring force required to allow the plate to move to the right at a constant speed of 10 m/s.

![Figure P5.57](image)

The non-deforming control volume shown in the sketch above is used. (a) To determine the magnitude of \( F_A \), we apply the component of the linear momentum equation (Eq. 5.22) along the direction of \( F_A \). Thus,

\[
\int_{CS} \rho \vec{V} \cdot \hat{n} \, dA = \sum F_y,
\]

or

\[
F_A = m \vec{V}_j \sin 30^\circ = \rho A_j V_j \vec{V}_j \sin 30^\circ = \rho D_j^2 V_j^2 \sin 30^\circ
\]

or

\[
F_A = (1.23 \text{ kg/m}^3) \frac{\pi (0.030 \text{ m})^2 (40 \text{ m/s})^2 (\sin 30^\circ)}{4} \left( \frac{1 \text{ N}}{\text{kg-m/s}^2} \right) = 0.696 \text{ N}
\]

(b) To determine the fraction of mass flow along the plate surface in each of the 2 directions shown in the sketch above, we apply the component of the linear momentum equation parallel to the surface of the plate, \( \int_{CS} \rho \vec{V} \cdot \hat{n} \, dA = \sum F_x \), to obtain

\[
R_{\text{along plate}} = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_1 V_j \cos 30^\circ
\]

(1)
Since the air velocity magnitude remains constant, the value of $R_{\text{along plate surface}}$ is zero.* Thus from Eq. 1 we obtain

$$\dot{m}_3 v_3 = \dot{m}_2 v_2 - \dot{m}_j v_j \cos 30^\circ$$

(2)

Since $v_3 = v_2 = v_j$, Eq. 2 becomes

$$\dot{m}_3 = \dot{m}_2 - \dot{m}_j \cos 30^\circ$$

(3)

From conservation of mass we conclude that

$$\dot{m}_j = \dot{m}_2 + \dot{m}_3$$

(4)

Combining Eqs. 3 and 4 we get

$$\dot{m}_3 = \dot{m}_j - \dot{m}_3 - \dot{m}_j \cos 30^\circ$$

or

$$\dot{m}_3 = \dot{m}_j \left(1 - \cos 30^\circ \right) = \dot{m}_j \left(0.0670 \right)$$

And

$$\dot{m}_2 = \dot{m}_j \left(1 - 0.067 \right) = \dot{m}_j \left(0.933 \right)$$

Thus, $\dot{m}_2$ involves 93.3% of $\dot{m}_j$ and $\dot{m}_3$ involves 6.7% of $\dot{m}_j$.

(c) To determine the magnitude of $F_A$ required to allow the plate to move to the right at a constant speed of $10 \text{ m/s}$, we use a non-deforming control volume like the one in the sketch above that moves to the right with a speed of $10 \text{ m/s}$. The translating control volume linear momentum equation (Eq. 5.29) leads to

$$F_A = \rho \frac{\pi D_s^2}{4} \left( v_j - \frac{10 \text{ m/s}}{5} \right)^2 \sin 30^\circ$$

or

$$F_A = \left(1.23 \text{ kg/m}^3 \right) \pi \left(0.030 \text{ m}^2 \right) \left( \frac{40 \text{ m/s} - 10 \text{ m/s}}{5} \right)^2 \left( \sin 30^\circ \right) \left( \frac{1 \text{ N}}{\text{kg} \cdot \text{m/s}^2} \right)$$

and

$$F_A = 0.391 \text{ N}$$

* Since $v_1 = v_2 = v_3$ and $\rho_1 = \rho_2 = \rho_3$ and $x_1 = x_2 = x_3$ if it follows that the Bernoulli equation is valid from $1 \rightarrow 2$ and $1 \rightarrow 3$. Thus, there are no viscous effects (Bernoulli equation is valid only for inviscid flow) so that $\tau = 0$. Hence, $R_{\text{along plate}} = 0.$