Quiz 9. The drag force, $R$, on a sphere located in a pipe through which a fluid is flowing is to be determined experimentally. Assume that the drag is a function of the sphere diameter, $d$, the pipe diameter, $D$, the fluid velocity, $V$, and the fluid density, $\rho$.

(a) What dimensionless parameters would you use for this problem? (b) Some experiments using water indicate that for $d = 0.2$ in., $D = 0.5$ in., and $V = 2$ ft/s, the drag is $1.5 \times 10^{-3}$ lb. Estimate the drag on a sphere located in a 2-ft-diameter pipe through which water is flowing with a velocity of 6 ft/s. The sphere diameter is such that geometric similarity is maintained.

Solution:

(a) Given $R = f(d, D, V, \rho)$, where $R \equiv F$, $d \equiv L$, $D \equiv L$, $V \equiv LT^{-1}$, and $\rho \equiv FL^{-4}T^2$, $5 - 3 = 2 \pi$ terms required. By inspection

$$\Pi_1 = \frac{d}{D}$$

and by using the exponent method

$$\Pi_2 = R \rho^a V^b D^c = (F)(FL^{-4}T^2)^a(LT^{-1})^b(L)^c = F^0 L^0 T^0$$

or

$$\Pi_2 = \frac{R}{\rho V^2 D^2}$$

(+5 points)

(b) The similarity requirement is

$$\frac{d_m}{D_m} = \frac{d}{D}$$

so that

$$d = \frac{d_m}{D_m} \cdot D = \frac{0.2 \text{ in}}{0.5 \text{ in}} \cdot (2 \text{ ft}) = 0.8 \text{ ft}$$

(+2 points)

And,

$$\frac{R}{\rho V^2 D^2} = \frac{R_m}{\rho_m V_m^2 D_m^2}$$

so that (with $\rho = \rho_m$)

$$R = \frac{\rho}{\rho_m} \left(\frac{V}{V_m}\right)^2 \left(\frac{D}{D_m}\right)^2 R_m = \left(\frac{6 \text{ ft/s}}{2 \text{ ft/s}}\right)^2 \left(\frac{2 \text{ ft}}{0.5/12 \text{ ft}}\right)^2 (1.5 \times 10^{-3} \text{ lb}) = 31.1 \text{ lb}$$

(+3 points)