Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

\[ \rho = 1260 \text{ kg/m}^3 \]
\[ \mu = 1.50 \text{ N s/m}^2 \]
\[ \ell = 10\text{ m} \]

\[ D = 0.075\text{ m} \]

For laminar flow in a pipe,

\[ V = \text{average velocity} = \frac{1}{2} V_{\text{max}} = \frac{1}{2} (1\text{ m}) = 0.5\text{ m/s} \]

Thus,

\[ Re = \frac{\rho V D}{\mu} = \frac{(1260 \text{ kg/m}^3)(0.5 \text{ m})(0.075 \text{ m})}{1.50 \text{ N s/m}^2} = 31.5 < 2100 \]

The flow is laminar so that

\[ V = \frac{(\Delta P - \rho g \sin \theta)D^2}{32 \mu L} \text{, where } \theta = 90^\circ \]

Thus,

\[ \Delta P = \frac{32 \mu L V^2}{D^2} + \rho g (\Delta P - \rho g \sin \theta)D^2 \frac{L}{32 \mu L} \]

\[ = 1.66 \times 10^5 \text{ N/m}^2 \text{, or } \Delta P = 166 \text{ kPa} \]

Also,

\[ \frac{r_1^2}{r_2^2} + z_1 + \frac{V_1^2}{2g} = \frac{r_2^2}{r_2^2} + z_2 + \frac{V_2^2}{2g} + h_L \text{, or with } V_1 = V_2, \quad z_2 - z_1 = L, \text{ and} \]

\[ \rho_1 + \Delta P \text{ this gives} \]

\[ h_L = \frac{\Delta P}{\rho} - L = \frac{1.66 \times 10^5 \text{ N/m}^2}{(9.81 \text{ m/s}^2)(1260 \text{ kg/m}^3)} - 10\text{ m} = 3.43\text{ m} \]