A cone and plate viscometer consists of a cone with a very small angle $\alpha$ which rotates above a flat surface as shown in Fig. 7.21. The torque, $\tau$, required to rotate the cone at an angular velocity, $\omega$, is a function of the radius, $R$, the cone angle, $\alpha$, and the fluid viscosity, $\mu$, in addition to $\omega$. With the aid of dimensional analysis, determine how the torque will change if both the viscosity and angular velocity are doubled.

\[
\tau = f (R, \alpha, \mu, \omega)
\]

\[
\tau = FR \quad R = L \quad \alpha = \frac{P \cdot L}{T^2} \quad \mu = \frac{F \cdot L}{T^3} \quad \omega = \frac{T}{T}
\]

From the pi theorem, $5 - 3 = 2$ pi terms required.

By inspection, for $\Pi_1$ (containing $T^2$):

\[
\Pi_1 = \frac{\tau}{\mu \omega R^3} = \frac{FL}{(FL^2T^2)(T^{-1})(L)^3} = \frac{P \cdot L^0 T^0}{L^2 T^3}
\]

Check using $MLT$:

\[
\frac{\tau}{\mu \omega R^3} = \frac{ML^2 T^{-2}}{(ML^2 T^2)(T^{-1})(L)^3} = \frac{M^0 L^0 T^0}{L^2 T^3} \quad \therefore \text{ok}
\]

The angle, $\alpha$, can be used as $\Pi_2$ since it is dimensionless. Thus,

\[
\frac{\tau}{\mu \omega R^3} = \phi (\alpha)
\]

or

\[
\tau = \mu \omega R^3 \phi (\alpha)
\]

It follows that if both $\mu$ and $\omega$ are doubled $\tau$ will increase by a factor of 4.