6.14 The velocity components of an incompressible, two-dimensional velocity field are given by the equations

\[ u = y^2 - x(1 + x) \]
\[ v = y(2x + 1) \]

Show that the flow is irrotational and satisfies conservation of mass.

If the two-dimensional flow is irrotational,

\[ \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0 \]

For the velocity distribution given,

\[ \frac{\partial v}{\partial x} = 2y \quad \frac{\partial u}{\partial y} = 2y \]

Thus,

\[ \omega_z = \frac{1}{2} \left( 2y - 2y \right) = 0 \]

and the flow is irrotational.

To satisfy conservation of mass,

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

Since,

\[ \frac{\partial u}{\partial x} = -1 - 2x \quad \frac{\partial v}{\partial y} = 2x + 1 \]

then

\[ -1 - 2x + 2x + 1 = 0 \]

and

conservation of mass is satisfied.