5.124 If a 1/2-hp motor is required by a ventilating fan to produce a 24-in. stream of air having a velocity of 40 ft/s as shown in Fig. P5.124, estimate (a) the efficiency of the fan and (b) the thrust of the supporting member on the conduit enclosing the fan.

\begin{figure}[h]
\centering
\includegraphics[width=0.4	extwidth]{image}
\caption{Figure P5.124}
\end{figure}

(a) The solution to this part of the problem is like Example 5.24. We use
\[ \eta = \frac{\dot{w}_{\text{shaft}} - \text{loss}}{\dot{w}_{\text{shaft}}} \]
to calculate the fan efficiency.

We use the energy equation (eq. 5.82) for flow through the control volume sketched above to calculate the loss as follows
\[ \frac{P_1}{P} \cdot \frac{V_1^2}{2} + g \zeta_1 = \frac{P_2}{P} \cdot \frac{V_2^2}{2} + g \zeta_2 + \dot{w}_{\text{shaft}} = \text{loss} \]
But \[ P_2 = P_1 \text{ and } \zeta_2 = \zeta_1 \text{, } V_1 \approx 0 \text{, } \dot{w}_{\text{shaft}} = \frac{hp}{\eta} \text{ net in} \]
Also \[ \dot{m} = \rho A_2 V_2 = \frac{P}{RT} \frac{\pi d_2^2}{4} V_2 \]
So \[ \text{loss} = \frac{\dot{w}_{\text{shaft}}}{\text{net in}} - \frac{V_2^2}{2} = \frac{hp}{\frac{P}{RT} \frac{\pi d_2^2}{4} V_2} - \frac{V_2^2}{2} \]
\[ \text{loss} = \left( \frac{\frac{\eta}{4}}{\frac{hp}{\frac{P}{RT} \frac{\pi d_2^2}{4} V_2}} \right) \left( \frac{550 \text{ ft-lb}}{5 \text{ hp}} \right) \left( \frac{\frac{24^3 \text{ in}}{12 \text{ in}^2}}{\pi} \right) \left( \frac{40 \text{ ft}}{5 \text{s}} \right) \]
\[ \left( \frac{53.3 \text{ ft-lb}}{16 \text{ in} \cdot \text{s}^2 \cdot \text{R}} \right) \left( 530 \text{R} \right) \]

\[(\text{cont'd})\]