4.32 As a valve is opened, water flows through the diffuser shown in Fig. P4.32 at an increasing flowrate so that the velocity along the centerline is given by \( V = u_0(1 - e^{-\alpha t}) (1 - x/\ell) \), where \( u_0 \), \( \alpha \), and \( \ell \) are constants. Determine the acceleration as a function of \( x \) and \( t \). If \( V_0 = 10 \) ft/s and \( \ell = 5 \) ft, what value of \( \alpha \) (other than \( \alpha = 0 \)) is needed to make the acceleration zero for any \( x \) at \( t = 1 \) s? Explain how the acceleration can be zero if the flowrate is increasing with time.

\[
\ddot{a} = \frac{3\dot{V}}{\dot{t}} + \dot{V} \cdot \nabla V
\]

With \( \dot{u} = u(x, t), \dot{v} = 0, \) and \( \dot{w} = 0 \), this becomes
\[
\ddot{a} = \left( \frac{3\dot{u}}{3x} + u \frac{3\dot{u}}{3x} \right) \hat{x} = \alpha_x \hat{x}, \quad \text{where} \quad u = V_0 (1 - e^{-\alpha t})(1 - \frac{x}{\ell})
\]

Thus,
\[
\alpha_x = V_0 (1 - \frac{x}{\ell}) e^{-\alpha t} + V_0^2 (1 - e^{-2\alpha t})(1 - \frac{x}{\ell})(1 - \frac{x}{\ell})
\]

or
\[
\alpha_x = V_0 (1 - \frac{x}{\ell}) \left[ e^{-\alpha t} - \frac{V_0}{2} (1 - e^{-2\alpha t})^2 \right]
\]

If \( \alpha_x = 0 \) for any \( x \) at \( t = 1 \) s we must have
\[
\left[ e^{-\alpha t} - \frac{V_0}{2} (1 - e^{-2\alpha t})^2 \right] = 0 \quad \text{With} \quad V_0 = 10 \quad \text{and} \quad \ell = 5
\]

For the above conditions the local acceleration \( \frac{3\dot{u}}{3x} > 0 \) is precisely balanced by the convective deceleration \( \dot{u} \frac{3\dot{u}}{3x} < 0 \).

The flowrate increases with time, but the fluid flows to an area of lower velocity.

\[
\text{FIGURE P4.32}
\]