3.52 Water flows through the pipe contraction shown in Fig. P3.52. For the given 0.2-m difference in the manometer level, determine the flow rate as a function of the diameter of the small pipe, D.

\[
\frac{\rho_1}{g} + \frac{V_1^2}{2g} + Z_1 = \frac{\rho_2}{g} + \frac{V_2^2}{2g} + Z_2 \quad \text{with} \quad A_1V_1 = A_2V_2
\]

Thus, with \( Z_1 = Z_2 \)

\[
\frac{\rho_1 - \rho_2}{g} = \frac{V_2^2 - V_1^2}{2g} = \frac{[(\frac{0.1}{D})^4 - 1]V_i^2}{2g}
\]

but

\( \rho = \frac{\rho_1 - \rho_2}{g} = \frac{\rho_1 - \rho_2}{g} = 0.2 \) \( \text{m} \)

Thus,

\[
0.2g = \frac{[(\frac{0.1}{D})^4 - 1]V_i^2}{2g} \quad \text{or} \quad V_i = \sqrt{\frac{0.2(2g)}{[(\frac{0.1}{D})^4 - 1]}}
\]

and

\[
Q = A_iV_i = \frac{\pi}{4}(0.1)^2 \sqrt{\frac{0.2(2.981)}{[(\frac{0.1}{D})^4 - 1]}}
\]

or

\[
Q = \frac{0.0156D^2}{\sqrt{(0.1)^4 - D^4}} \quad \text{m}^3/\text{s} \quad \text{when} \ D \sim m
\]