2.104 A 1-m-diameter cylindrical mass, \( M \), is connected to a 2-m-wide rectangular gate as shown in Fig. P2.104. The gate is to open when the water level, \( h \), drops below 2.5 m. Determine the required value for \( M \). Neglect friction at the gate hinge and the pulley.

\[
F_R = \gamma h^2 A = \gamma \left( \frac{h}{2} \right)^2 \frac{\pi d^2}{4}
\]

where all lengths are in m.

For equilibrium, \( \sum M_0 = 0 \)

so that

\[
4T = \left( \frac{h}{3} \right) F_R = \gamma \frac{h^3}{3}
\]

and

\[
T = \frac{\gamma h^3}{12}
\]

For the cylindrical mass \( \sum F_{\text{vertical}} = 0 \) and

\[
T = Mg - F_R = Mg - \gamma \frac{h^3}{3}
\]

Thus,

\[
M = \frac{T + \gamma \frac{h^3}{3}}{g} = \frac{\gamma h^3}{12} + \gamma \left( \frac{\pi}{4} \right) (1)^2 (2.5 - 1)
\]

and for \( h = 2.5 \) m

\[
M = \left( 9.80 \times 10^3 \frac{N}{m^3} \right) \left[ \frac{(2.5m)^3}{12} + \frac{\pi}{4} \left( \frac{1}{1} \right)^2 (2.5m - 1.0m) \right]
\]

\[
= 2.480 \text{ kg}
\]