2.88 The homogeneous gate shown in Fig. P2.88 consists of one quarter of a circular cylinder and is used to maintain a water depth of 4 m. That is, when the water depth exceeds 4 m, the gate opens slightly and lets the water flow under it. Determine the weight of the gate per meter of length.

![Figure P2.88]

Consider the free body diagram of the gate and a portion of the water as shown.

\[ \Sigma M_o = 0 \text{, or} \]

\[ l_2 W + l_1 W_i - F_H l_3 - F_V l_4 = 0, \text{ where} \]

(1) \[ F_H = \delta h_c A = 9.8 \times 10^3 \frac{N}{m^2} (3.5 \text{ m})(1 \text{ m})(1 \text{ m}) = 34.3 \text{ kN} \]

since for the vertical side, \( h_c = 4 \text{ m} - 0.5 \text{ m} = 3.5 \text{ m} \)

Also,

(2) \[ F_V = \delta h_c A = 9.8 \times 10^3 \frac{N}{m^2} (4 \text{ m})(1 \text{ m})(1 \text{ m}) = 39.2 \text{ kN} \]

Also,

(3) \[ W_i = \delta (1 \text{ m})^3 - \delta \frac{(\pi)}{4} (1 \text{ m})^2 (1 \text{ m}) = 9.8 \times 10^3 \frac{N}{m^2} [1 - \frac{\pi}{4}] \text{ m}^2 = 2.10 \text{ kN} \]

(4) \[ l_4 = 0.5 \text{ m} \text{ and} \]

(5) \[ l_3 = 0.5 \text{ m} + (y_A - y_c) = 0.5 \text{ m} + \frac{I_{ac}}{Y_c h} = 0.5 \text{ m} + \frac{\frac{1}{3}(1 \text{ m})(1 \text{ m})^3}{3.5 \text{ m}(1 \text{ m})(1 \text{ m})} = 0.524 \text{ m} \]

(6) \[ l_2 = 1 \text{ m} - \frac{4R}{3 \pi} = 1 - \frac{4(1 \text{ m})}{3 \pi} = 0.576 \text{ m} \]

To determine \( l_1 \), consider a unit square that consists of a quarter circle and the remainder as shown in the figure. The centroids of areas (1) and (2) are as indicated.

Thus,

\[ (0.5 - l_1) A_2 = (0.5 - l_1) A_1, \]

(con't)
so that with \( A_2 = \frac{A}{W} (1)^2 = \frac{A}{W} \) and \( A_1 = 1 - \frac{A}{W} \) this gives

\[
(0.5 - \frac{4}{3W}) \frac{W}{4} = (0.5 - \lambda_1) (1 - \frac{W}{4})
\]

or

\( \lambda_1 = 0.223 \) m

Hence, by combining Eqs (1) through (8):

\[
(0.576 m) W + (0.223 m)(2.10 kN) - (34.3 kN)(0.524 m) - (39.2 kN)(0.5 m) = 0
\]

or

\[
W = 64.4 \text{ kN}
\]