1.24 The specific gravity of mercury at 80 °C is 13.4. Determine its density and specific weight at this temperature. Express your answer in both BG and SI units.

\[ \rho = SG \times \rho_{H_2O @ 4°C} \]

\[ \gamma = \rho g \]

In BG units:

\[ \rho = 13.4 \left( \frac{1.94 \text{ slugs}}{ft^3} \right) = 26.0 \text{ slugs} \frac{ft}{ft^3} \]

\[ \gamma = \left( \frac{26.0 \text{ slugs}}{ft^3} \right) \left( \frac{32.2 \frac{ft}{s^2}}{s^2} \right) = 837 \frac{lb}{ft^3} \]

In SI units:

\[ \rho = 13.4 \left( 1000 \frac{kg}{m^3} \right) = 13.4 \times 10^3 \frac{kg}{m^3} \]

\[ \gamma = \left( 13.4 \times 10^3 \frac{kg}{m^3} \right) \left( 9.81 \frac{m}{s^2} \right) = 131 \frac{KN}{m^3} \]

1.25 A hydrometer is used to measure the specific gravity of liquids. (See Video V2.6.) For a certain liquid a hydrometer reading indicates a specific gravity of 1.15. What is the liquid’s density and specific weight? Express your answer in SI units.

\[ SG = \frac{\rho}{\rho_{H_2O @ 4°C}} \]

1.15 = \[ \frac{\rho}{1000 \frac{kg}{m^3}} \]

\[ \rho = (1.15) (1000 \frac{kg}{m^3}) = 1150 \frac{kg}{m^3} \]

\[ \gamma = \rho g = (1150 \frac{kg}{m^3}) (9.81 \frac{m}{s^2}) = 11.3 \frac{KN}{m^3} \]
A tire having a volume of 2.5 ft$^3$ contains air at a gage pressure of 30 psi and a temperature of 70 °F. Determine the density of the air and the weight of the air contained in the tire.

\[
\rho = \frac{p}{RT} = \frac{(30 \text{ ft}^2 \text{ in}^{-2} + 14.7 \text{ ft}^2 \text{ in}^{-2}) (144 \text{ in}^2 \text{ ft}^{-2})}{(1716 \text{ ft}^2 \text{ lb} \text{ slug}^{-1} \text{ °R})(70°\text{F} + 460°\text{R})} = 7.08 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}
\]

\[
\text{weight} = \rho g \times \text{volume} = (7.08 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}) (32.2 \frac{\text{ft}}{\text{s}^2}) (2.5 \text{ ft}^3)
\]
\[
= 0.570 \text{ lb}
\]
A layer of water flows down an inclined fixed surface with the velocity profile shown in Fig. P1.59. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for $U = 2 \text{ m/s}$ and $h = 0.1 \text{ m}$.

\[
\tau = \mu \frac{du}{dy}
\]

\[
\frac{du}{dy} = U \left( \frac{y}{h} - \frac{y^2}{h^2} \right)
\]

Thus, at the fixed surface ($y=0$)

\[
(\frac{du}{dy})_{y=0} = \frac{2U}{h}
\]

So that

\[
\tau = \mu \left( \frac{2U}{h} \right) = \left( 1.12 \times 10^{-3} \frac{\text{N.s}}{\text{m}^2} \right) (2) \left( \frac{2 \text{ m}}{0.1 \text{ m}} \right)
\]

\[
= 4.48 \times 10^{-2} \frac{\text{N}}{\text{m}^2} \text{ acting in direction of flow}
\]
1.67 Estimate the increase in pressure (in psi) required to decrease a unit volume of mercury by 0.1%.

\[ E_V = -\frac{d\rho}{dV/V} \]

(Eq. 1.12)

Thus,

\[ \Delta \rho \approx -\frac{E_V \Delta V}{V} = -\left(4.14 \times 10^4 \text{ lb/in}^2\right)(-0.001) \]

\[ \Delta \rho \approx 4.14 \times 10^3 \text{ psi} \]

1.68 A 1-m³ volume of water is contained in a rigid container. Estimate the change in the volume of the water when a piston applies a pressure of 35 MPa.

\[ E_V = -\frac{d\rho}{dV/V} \]

(Eq. 1.12)

Thus,

\[ \Delta V \approx -\frac{V \Delta \rho}{E_V} = -\left(1 \text{ m}^3 \right) \left(35 \times 10^6 \frac{N}{m^2}\right) \frac{-1 \text{ m}^3}{2.15 \times 10^9 \frac{N}{m^2}} \]

\[ \Delta V \approx 0.0163 \text{ m}^3 \]

or

decrease in volume \approx 0.0163 \text{ m}^3
1.78 When a fluid flows through a sharp bend, low pressures may develop in localized regions of the bend. Estimate the minimum absolute pressure (in psi) that can develop without causing cavitation if the fluid is water at 160 °F.

Cavitation may occur when the local pressure equals the vapor pressure. For water at 160 °F (from Table B.1 in Appendix B)

\[ \bar{P}_v = 47.4 \text{ psi (abs)} \]

Thus, minimum pressure = 47.4 psi (abs)

1.79 Estimate the minimum absolute pressure (in pascals) that can be developed at the inlet of a pump to avoid cavitation if the fluid is carbon tetrachloride at 20 °C.

Cavitation may occur when the suction pressure at the pump inlet equals the vapor pressure.

For carbon tetrachloride at 20 °C \( \bar{P}_v = 13 \text{ kPa (abs)} \).

Thus, minimum pressure = 13 kPa (abs)
1.86 Under the right conditions, it is possible, due to surface tension, to have metal objects float on water. (See Video V1.5.) Consider placing a short length of a small diameter steel (sp. wt. = 490 lb/ft³) rod on a surface of water. What is the maximum diameter that the rod can have before it will sink?
Assume that the surface tension forces act vertically upward.
**Note:** A standard paper clip has a diameter of 0.036 in. Partially unfold a paper clip and see if you can get it to float on water. Do the results of this experiment support your analysis?

In order for rod to float (see figure), it follows that

\[ 2\sigma l \geq W \geq \left( \frac{\pi}{4} \right) (D^2) l \gamma_{steel} \]

Thus, for the limiting case

\[ D_{max} = \frac{2\sigma l}{(\pi/4) l \gamma_{steel}} = \frac{8\sigma}{\pi \gamma_{steel}} \]

so that

\[ D_{max} = \left[ \frac{8 (5.03 \times 10^{-3} \frac{lb}{ft})}{\pi (490 \frac{lb}{ft^3})} \right]^{1/2} = 5.11 \times 10^{-3} ft \]

\[ = 0.0614 \text{ in.} \]

Since a standard steel paper clip has a diameter of 0.036 in, which is less than 0.0614 in, it should float. A simple experiment will verify this. **Yes.**
2.12 The basic elements of a hydraulic press are shown in Fig. P2.12. The plunger has an area of 1 in.², and a force, \( F_1 \), can be applied to the plunger through a lever mechanism having a mechanical advantage of 8 to 1. If the large piston has an area of 150 in.², what load, \( F_2 \), can be raised by a force of 30 lb applied to the lever? Neglect the hydrostatic pressure variation.

A force of 30 lb applied to the lever results in a plunger force, \( F_1 \), of \( F_1 = (8)(30) = 240 \) lb.

Since \( F_1 = pA_1 \) and \( F_2 = pA_2 \) where \( p \) is the pressure and \( A_1 \) and \( A_2 \) are the areas of the plunger and piston, respectively. Since \( p \) is constant throughout the chamber,

\[
\frac{F_1}{A_1} = \frac{F_2}{A_2}
\]

so that

\[
F_2 = \frac{A_2}{A_1} F_1 = \left(\frac{150 \text{ in.}^2}{1 \text{ in.}^2}\right)(240 \text{ lb}) = 36,000 \text{ lb}
\]

2.13 A 0.3-m-diameter pipe is connected to a 0.02-m-diameter pipe and both are rigidly held in place. Both pipes are horizontal with pistons at each end. If the space between the pistons is filled with water, what force will have to be applied to the larger piston to balance a force of 80 N applied to the smaller piston? Neglect friction.

\[
F_1 = pA_1 \\
F_2 = pA_2
\]

Thus,

\[
\frac{F_1}{A_1} = \frac{F_2}{A_2}
\]

or

\[
F_1 = \frac{A_1}{A_2} F_2 = \left(\frac{0.3m}{0.02m}\right)^2 (80 N) = 18,000 \text{ N}
\]
As shown in Fig. 2.6 for the U.S. standard atmosphere, the troposphere extends to an altitude of 11 km where the pressure is 22.6 kPa (abs). In the next layer, called the stratosphere, the temperature remains constant at \(-56.5^\circ C\). Determine the pressure and density in this layer at an altitude of 15 km. Assume \(g = 9.77 \text{ m/s}^2\) in your calculations. Compare your results with those given in Table C.2 in Appendix C.

For isothermal conditions,

\[
\bar{p}_2 = \bar{p}_1 \, e^{-\frac{g}{R} \frac{(z_2 - z_1)}{T_0}} \tag{Eq. 2.10}
\]

Let \(z_1 = 11 \text{ km} \), \(\bar{p}_1 = 22.6 \text{ kPa} \), \(R = 287 \frac{J}{\text{kg} \cdot \text{K}} \), \(g = 9.77 \frac{\text{m}}{\text{s}^2} \), and \(T_0 = -56.5^\circ C + 273.15 = 216.65 \text{ K} \).

Thus,

\[
\bar{p}_2 = (22.6 \text{ kPa}) \, e^{-\frac{9.77 \frac{\text{m}}{\text{s}^2}}{287 \frac{J}{\text{kg} \cdot \text{K}}} \left(15 \times 10^3 \text{ m} - 11 \times 10^3 \text{ m} \right)}
\]

\[
\bar{p}_2 = \frac{12.1 \text{ kPa}}{\text{m}}
\]

Also,

\[
\bar{p} = \frac{\bar{p}}{kT} = \frac{12.1 \times 10^3 \frac{N}{\text{m}^2}}{(287 \frac{J}{\text{kg} \cdot \text{K}})(216.65 \text{ K})} = 0.195 \frac{\text{kg}}{\text{m}^3}
\]

(From Table C.2 in Appendix C, \(\bar{p}_2 = 12.11 \text{ kPa} \) and \(\bar{p}_2 = 0.1948 \frac{\text{kg}}{\text{m}^3} \).)
2.24 A U-tube manometer is connected to a closed tank containing air and water as shown in Fig. P2.24. At the closed end of the manometer the air pressure is 16 psia. Determine the reading on the pressure gage for a differential reading of 4 ft on the manometer. Express your answer in psi (gage). Assume standard atmospheric pressure, and neglect the weight of the air columns in the manometer.

\[ P_i + \gamma_{gf} (4 \text{ ft}) + \gamma_{h_0} (2 \text{ ft}) = P_{gage} \]

Thus,

\[ P_{gage} = \left( 16 \frac{lb}{in^2} - 14.7 \frac{lb}{in^2} \right) (144 \frac{in^2}{ft^2}) + (90 \frac{lb}{ft^3})(4 \text{ ft}) \]
\[ + \left( 62.4 \frac{lb}{ft^3} \right) (2 \text{ ft}) \]
\[ = 672 \frac{lb}{ft^2} = \left( 672 \frac{lb}{ft^2} \right) \left( \frac{1 ft^2}{144 in^2} \right) = 4.67 \text{ psi} \]
The differential mercury manometer of Fig. P2.40 is connected to pipe A containing gasoline (SG = 0.65), and to pipe B containing water. Determine the differential reading, $h$, corresponding to a pressure in A of 20 kPa and a vacuum of 150 mm Hg in B.

Thus,

$$h = \frac{P_A - P_B + \gamma_{gas} (0.3m + h) + \gamma_{H_2O} (0.3m + h)}{\gamma_{Hg} - \gamma_{gas} - \gamma_{H_2O}}$$

where

$$P_B = -\frac{\gamma_{Hg} (0.150m)}{\rho}$$

$$h = \frac{20 \, kPa - \left[ -\left(\frac{133 \, kN}{m^3}\right) (0.150m) \right] + (0.65) (4.81 \, \frac{kN}{m^3}) (0.3m) + (9.80 \, \frac{kN}{m^3}) (0.3m)}{133 \, \frac{kN}{m^3} - (0.65) (4.81 \, \frac{kN}{m^3}) - 9.80 \, \frac{kN}{m^3}}$$

$$= 0.384 \, m$$
2.51 A large, open tank contains water and is connected to a 6-ft diameter conduit as shown in Fig. P2.51. A circular plug is used to seal the conduit. Determine the magnitude, direction, and location of the force of the water on the plug.

\[ F_R = \gamma h_c A = (62.4 \text{ lb/ft}^3)(12 \text{ ft})(\frac{\pi}{4})(6 \text{ ft})^2 = 21,200 \text{ lb} \]

\[ y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad I_{xc} = \frac{\pi}{4} (3 \text{ ft})^4 = 63.6 \text{ ft}^4 \]

Thus,

\[ y_R = \frac{\frac{\pi}{4} (3 \text{ ft})^4}{(12 \text{ ft}) \pi (3 \text{ ft})^2} + 12 \text{ ft} = 12.19 \text{ ft} \]

The force of 21,200 lb acts 12.19 ft below the water surface and is perpendicular to the plug surface as shown.
A homogeneous, 4-ft.-wide, 8-ft-long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in Fig. P2.52. Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.

\[ F_R = \gamma h_c A \quad \text{where} \quad h_c = \left(\frac{6\text{ft}}{\pi}\right) \sin 60^\circ \]

Thus,

\[ F_R = (62.4 \frac{\text{lb}}{\text{ft}^3}) \left(\frac{4\text{ft}}{\pi}\right) \sin 60^\circ (6\text{ft} \times 4\text{ft}) \]

\[ = 8890 \text{ lb} \]

To locate \( F_R \),

\[ y_R = \frac{I_x c}{y_c A} + y_c \quad \text{where} \quad y_c = 3\text{ ft} \]

so that

\[ y_R = \frac{\frac{1}{2} (4\text{ft})(4\text{ft})^3}{(3\text{ ft})(6\text{ft} \times 4\text{ft})} + 3\text{ ft} = 4.0 \text{ ft} \]

For equilibrium,

\[ \Sigma M_H = 0 \]

and

\[ T (8\text{ft})(\sin 60^\circ) = W (4\text{ft})(\cos 60^\circ) + F_R (2\text{ft}) \]

\[ T = \frac{(800 \text{lb})(4\text{ft})(\cos 60^\circ) + 8890 \text{lb})(2\text{ft})}{(8 \text{ft})(\sin 60^\circ)} \]

\[ = 1350 \text{ lb} \]
2.70 A 4-m-long curved gate is located in the side of a reservoir containing water as shown in Fig. P2.70. Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point A? Explain.

For equilibrium,
\[ \Sigma F_x = 0 \]
or
\[ F_H = F_2 = \gamma h_{22} A_2 = \gamma (6 m + 1.5 m)(3 m)(4 m) \]
so that
\[ F_H = (9.80 \frac{kN}{m^3})(95 m)(12 m^2) = 882 kN \]

Similarly,
\[ \Sigma F_y = 0 \]
\[ F_V = F_1 + q_w \]
where:
\[ F_1 = \gamma (6 m)(3 m \times 4 m) = (9.80 \frac{kN}{m^3})(6 m)(12 m^2) \]
\[ q_w = \gamma V = (9.80 \frac{kN}{m^3})(9 \pi m^3) \]

Thus,
\[ F_V = (9.80 \frac{kN}{m^3}) \left[ 72 m^3 + 9 \pi m^3 \right] = 983 kN \]

(Note: Force of water on gate will be opposite in direction to that shown on figure.)

The direction of all differential forces acting on the curved surface is perpendicular to surface, and therefore, the resultant must pass through the intersection of all these forces which is at point A. Yes.
2.78 An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in Fig. P2.78. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge.

\[ F_H \sim \text{horizontal force of wall on fluid} \]
\[ F_V \sim \text{vertical force of wall on fluid} \]

\[ W = \gamma h o V \]
\[ = (62.4 \frac{lb}{ft^3})(\frac{\pi (6\text{ft})^2}{2})(1\text{ ft}) \]
\[ = 882\text{ lb} \]

\[ F_I = \gamma b A = (62.4 \frac{lb}{ft^3})(6\text{ ft} \times 3\text{ ft})(1\text{ ft} \times 1\text{ ft}) \]
\[ = 3370\text{ lb} \]

For equilibrium, \( F_V = W = 882\text{ lb} \uparrow \)
and \( F_H = F_I = 3370\text{ lb} \downarrow \)

The force the water exerts on the bulge is equal to, but opposite in direction to \( F_V \) and \( F_H \) above. Thus,

\[ (F_{H})_{\text{wall}} = 3370\text{ lb} \rightarrow \]
\[ (F_{V})_{\text{wall}} = 882\text{ lb} \downarrow \]
2.84 When the Tucuruí dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.

For equilibrium,

$$\sum F_{\text{vertical}} = 0$$

so that

$$T = F_B - 9W$$

(1)

For a truncated cone,

$$\text{Volume} = \frac{\pi h}{3} \left( r_1^2 + r_1 r_2 + r_2^2 \right)$$

where:

- $r_1$ = base radius
- $r_2$ = top radius
- $h$ = height

Thus,

$$V_{\text{tree}} = \frac{(\pi)(100\text{ ft})}{3} \left[ (4\text{ ft})^2 + (4\text{ ft} \times 1\text{ ft}) + (1\text{ ft})^2 \right] = 2200\text{ ft}^3$$

For buoyant force,

$$F_B = \rho \cdot V_{\text{tree}} = (62.4 \frac{\text{lb}}{\text{ft}^3})(2200\text{ ft}^3) = 137,000\text{ lb}$$

For weight,

$$W = \rho \cdot V_{\text{tree}} = (0.6)(62.4 \frac{\text{lb}}{\text{ft}^3})(2200\text{ ft}^3) = 82,400\text{ lb}$$

From Eq. (1)

$$T = 137,000\text{ lb} - 82,400\text{ lb} = 54,600\text{ lb}$$
2.89 When a hydrometer (see Fig. P2.89 and Video V2.6) having a stem diameter of 0.30 in. is placed in water, the stem protrudes 3.15 in. above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs 0.042 lb.

**Figure P2.89**

When the hydrometer is floating, its weight, \( W \), is balanced by the buoyant force, \( F_B \). For equilibrium,

\[
\sum F_{\text{vertical}} = 0
\]

Thus, for water

\[
F_B = W = (\rho_{H_2O}) V_1^o = W
\]

\[ (1) \]

where \( V_1^o \) is the submerged volume. With the new liquid

\[
(SG)(\rho_{H_2O}) V_2^o = W
\]

(2)

Combining Eqs. (1) and (2) with \( W \) constant

\[
(\rho_{H_2O}) V_1^o = (SG)(\rho_{H_2O}) V_2^o
\]

and

\[
V_2^o = \frac{V_1^o}{SG}
\]

\[ (3) \]

\[ (\text{cont'\text{t}}) \]
From Eq. (1)
\[ V_1 = \frac{2 \omega}{d_{420}} = \frac{0.042 \text{ lb}}{62.4 \frac{1}{420}} = 6.73 \times 10^{-4} \text{ ft}^3 \]
so that from Eq. (3)
\[ V_2 = \frac{6.73 \times 10^{-4} \text{ ft}^3}{1.10} = 6.12 \times 10^{-4} \text{ ft}^3 \]
Thus,
\[ V_1 - V_2 = (6.73 - 6.12) \times 10^{-4} \text{ ft}^3 = 0.61 \times 10^{-4} \text{ ft}^3 \]
To obtain the difference the change in length, \( \Delta l \), is
\[ \left( \frac{T}{4} \right) (0.30 \text{ in.})^3 \Delta l = (0.61 \times 10^{-4} \text{ ft}^3) (1728 \text{ in.}^3) \]
\[ \Delta l = 1.49 \text{ in.} \]
With the new liquid, the stem would protrude
3.15 in. + 1.49 in. = \underline{4.64 in.} above the surface
3.148 Information and assumptions

provided in problem statement

Find
location of water line for stability and specific gravity of material

Solution
For neutral stability, the distance to the metacenter is zero. In other words

\[ GM = \frac{I_{oo}}{V} - GC = 0 \]

where \( GC \) is the distance from the center of gravity to the center of buoyancy. The moment of inertia at the waterline is

\[ I_{oo} = \frac{w^3L}{12} \]

where \( L \) is the length of the body. The volume of liquid displaced is \( hwL \) so

\[ GC = \frac{w^3L}{12hwL} = \frac{w^2}{12h} \]

The value for \( GC \) is the distance from the center of buoyancy to the center of gravity, or

\[ GC = \frac{w}{2} - \frac{h}{2} \]

So

\[ \frac{w}{2} - \frac{h}{2} = \frac{w^2}{12h} \]

or

\[ \left( \frac{h}{w} \right)^2 - \frac{h}{w} + \frac{1}{6} = 0 \]

Solving for \( h/w \) gives 0.789 and 0.211. The first root gives a physically unreasonable solution. Therefore

\[ \frac{h}{w} = 0.211 \]

The weight of the body is equal to the weight of water displaced.

\[ \gamma_b V_b = \gamma_f V \]

Therefore

\[ S = \frac{\gamma_b}{\gamma_f} = \frac{whL}{w^2L} = \frac{h}{w} = 0.211 \]

The specific gravity is smaller than this value, the body will be unstable (floats too high).
3.152 Information and assumptions

Provided in problem statement

Find
Stability

Solution

\[ GM = \frac{I_{00}}{\forall - CG} \]
\[ = \left( \frac{3H \left( 2H \right)^3}{12 \times H \times 2H \times 3H} \right) - \frac{H}{2} \]
\[ = -\frac{H}{6} \]
Not stable about longitudinal axis

\[ GM = \frac{I_{00}}{\forall - CG} \]
\[ = \left( \frac{2H \left( 3H \right)^3}{12 \times H \times 2H \times 3H} \right) - \frac{H}{2} \]
\[ = \frac{H}{4} \]
Stable about transverse axis

Not stable.
An open rectangular tank 1 m wide and 2 m long contains gasoline to a depth of 1 m. If the height of the tank sides is 1.5 m, what is the maximum horizontal acceleration (along the long axis of the tank) that can develop before the gasoline would begin to spill?

To prevent spilling,
\[
\frac{d\xi}{dy} < -\frac{1.5\text{m} - 1.0\text{m}}{1\text{m}} = -0.50
\]
(see figure).

Since,
\[
\frac{d\xi}{dy} = -\frac{a_y}{g + a_z}
\]
(\text{Eq. 2.20})
or, with \(a_z = 0\),
\[
a_y = -\left(\frac{d\xi}{dy}\right)g
\]
so that
\[
(a_y)_{\text{max}} = -(-0.50)(9.81 \frac{\text{m}}{\text{s}^2}) = 4.91 \frac{\text{m}}{\text{s}^2}
\]
(Note: Acceleration could be either to the right or the left.)
2.101 A closed, 0.4-m-diameter cylindrical tank is completely filled with oil (SG = 0.9) and rotates about its vertical longitudinal axis with an angular velocity of 40 rad/s. Determine the difference in pressure just under the vessel cover between a point on the circumference and a point on the axis.

Pressure in a rotating fluid varies in accordance with the equation,

\[ p = \rho \frac{\omega^2 r^2}{2} - yz + \text{constant} \quad (\text{Eq. 2.33}) \]

Since \( z_A = z_B \),

\[ p_B - p_A = \frac{\rho \omega^2}{2} (r_B^2 - r_A^2) \]

\[ = \frac{(0.9)(10^3 \text{ kg/m}^3)(40 \text{ rad/s})^2}{2} \left[(0.2 \text{ m})^2 - 0 \right] \]

\[ = 28.8 \text{ kPa} \]
3.1 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.1. The velocity is given by \( V = 10(1 + x) \) ft/s, where \( x \) is in feet. Viscous effects are neglected. (a) Determine the pressure gradient, \( \frac{dp}{dx} \), (as a function of \( x \)) needed to produce this flow. (b) If the pressure at section (1) is 50 psi, determine the pressure at section (2) by: (i) integration of the pressure gradient obtained in (a); (ii) application of the Bernoulli equation.

\[
\begin{align*}
(a) \quad & -8 \sin \theta - \frac{4x}{s} = \rho V \frac{dV}{ds} \quad \text{but} \quad \theta = 0 \quad \text{and} \quad V = 10(1 + x) \text{ ft/s} \\
& \frac{dV}{ds} = -\rho \frac{dV}{dx} \quad \text{or} \quad \frac{dP}{dx} = -\rho \frac{dV}{dx} = -\rho (10(1 + x)) (10) \\
& \text{Thus,} \quad \frac{dP}{dx} = -194 \frac{\text{lbf}}{\text{ft}^2} \left( \frac{(10 \text{ ft})^2}{\text{ft}^2} \right) (1 + x), \text{with} \ x \ \text{in feet} \\
& \quad = -194(1 + x) \frac{16}{\text{ft}^2}
\end{align*}
\]

(b)(i) \( \frac{dp}{dx} = -194(1 + x) \) so that \( \int_{\rho_1}^{\rho_2} dp = -194 \int_{x_1}^{x_2} (1 + x) \, dx \\
\rho_1 = 50 \text{ psi} \quad x_1 = 0 \quad \rho_2 = -194(3 + \frac{3^2}{2}) \frac{16}{\text{ft}^2} \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \quad = 50 - 10.1 = 39.9 \text{ psi}

or \( \rho_2 = 50 \text{ psi} - 194(3 + \frac{3^2}{2}) \frac{16}{\text{ft}^2} \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = 39.9 \text{ psi}

(ii) \( \rho_1 + \frac{1}{2} \rho V_1^2 = \rho_2 + \frac{1}{2} \rho V_2^2 + \Delta Z_1 \) or with \( Z_1 = Z_2 \)

\( \rho_2 = \rho_1 + \frac{1}{2} \rho (V_1^2 - V_2^2) \) where \( V_1 = 10(1 + 0) = 10 \text{ ft/s} \)

\( V_2 = 10(1 + 3) = 40 \text{ ft/s} \)

Thus,

\[
\rho_2 = 50 \text{ psi} + \frac{1}{2} (194 \frac{\text{lbf}}{\text{ft}^2}) \left( 10^2 - 40^2 \right) \frac{16}{\text{ft}^2} \left( \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) = 39.9 \text{ psi}
\]
3.10 Water flows around the vertical two-dimensional bend with circular streamlines and constant velocity as shown in Fig. P3.10. If the pressure is 40 kPa at point (1), determine the pressures at points (2) and (3). Assume that the velocity profile is uniform as indicated.

\[-g \frac{dz}{dn} - \frac{\partial P}{\partial n} = \frac{\rho V^2}{R} \quad \text{with} \quad \frac{dz}{dn} = 1 \quad \text{and} \quad V = 10 \text{m/s}
\]

Thus, with \( R = 6 - n \)

\[\frac{dP}{dn} = -g - \frac{\rho V^2}{6 - n} \quad \text{or} \]

\[\int_{n=0}^{n} \frac{dP}{dn} \, dn = -\int_{n=0}^{n} g \, dn - \int_{n=0}^{n} \frac{\rho V^2}{6 - n} \, dn \]

so that since \( g \) and \( V \) are constants

\[P - P_1 = -g n - \rho V^2 \int_{n=0}^{n} \frac{dn}{6 - n} \]

Thus,

\[P = P_1 - g n - \rho V^2 \ln \left( \frac{6}{6 - n} \right) \]

With \( P_1 = 40 \text{ kPa} \) and \( n_2 = 1 \text{ m} \):

\[P_2 = 40 \text{ kPa} - 9.8 \times 10^3 \frac{N}{m^2} (1 \text{ m}) \times \frac{-999 \text{ kPa}}{10^3 \text{ m}^2} \ln \left( \frac{6}{5} \right) \]

or

\[P_2 = 12.0 \text{ kPa} \]

and

With \( P_1 = 40 \text{ kPa} \) and \( n_3 = 2 \text{ m} \):

\[P_3 = 40 \text{ kPa} - 9.8 \times 10^3 \frac{N}{m^2} (2 \text{ m}) \times \frac{-999 \text{ kPa}}{10^3 \text{ m}^2} \ln \left( \frac{6}{4} \right) \]

or

\[P_3 = -20.1 \text{ kPa} \]
3.18 A fire hose nozzle has a diameter of 1 1/4 in. According to some fire codes, the nozzle must be capable of delivering at least 250 gal/min. If the nozzle is attached to a 3-in.-diameter hose, what pressure must be maintained just upstream of the nozzle to deliver this flow rate?

\[
\frac{\rho_1}{g} + \frac{V_1^2}{2g} + z_1 = \frac{\rho_2}{g} + \frac{V_2^2}{2g} + z_2
\]

with \( z_1 = z_2 \) and \( \rho_2 = 0 \)

and \( Q = (250 \text{ gal/min})(2.31 \text{ in}^3/\text{gal})(\frac{1 \text{ ft}^3}{1728 \text{ in}^3})(\frac{1 \text{ min}}{60 \text{ s}}) = 0.557 \text{ ft}^3/\text{s} \)

Thus,

\[
\frac{\rho_1}{g} = \frac{x}{g} \left[ V_2^2 - V_1^2 \right] \quad \text{where} \quad V_2 = \frac{Q}{A_2} = \frac{0.557 \text{ ft}^3/\text{s}}{\frac{\pi}{4} \left( \frac{3}{12} \right)^2} \text{ ft/s} = 80.7 \text{ ft/s}
\]

and

\[
V_1 = \frac{Q}{A_1} = \frac{0.557 \text{ ft}^3/\text{s}}{\frac{\pi}{4} \left( \frac{1.125}{12} \right)^2} \text{ ft/s} = 11.34 \text{ ft/s}
\]

so that with \( \frac{\mu}{g} = \rho \)

\[
\rho_1 = \frac{1}{2} \left( 1.94 \frac{\text{slug}}{\text{ft}^3} \right) \left[ 80.7^2 - 11.34^2 \right] \frac{\text{ft}^2}{\text{s}^2}
\]

\[
= 6190 \frac{\text{lb}}{\text{ft}^2} = 43.0 \text{ psi}
\]
3.30 Water flows through the pipe contraction shown in Fig. P3.30. For the given 0.2-m difference in manometer level, determine the flow rate as a function of the diameter of the small pipe, $D$.

\[
\frac{L}{g} + \frac{V_1^2}{2g} + z_1 = \frac{L}{g} + \frac{V_2^2}{2g} + z_2 \quad \text{or with } z_1 = z_2 \text{ and } V_1 = 0
\]

\[
V_2 = \sqrt{2g \left( \frac{\rho_1 - \rho_2}{\rho} \right)}
\]

but $\rho_1 = \delta h_1$ and $\rho_2 = \delta h_2$ so that $\rho_1 - \rho_2 = \delta (h_1 - h_2) = 0.2\delta$

Thus,

\[
V_2 = \sqrt{2g \frac{0.2\delta}{\delta}} = \sqrt{2g (0.2)}
\]

or

\[
Q = A_2 V_2 = \frac{\pi}{4} D^2 V_2 = \frac{\pi}{4} D^2 \sqrt{2(9.81)(0.2)} = 1.56 D^2 \frac{m^3}{s} \quad \text{when } D = \text{m}
\]
3.60 Water flows from a large tank as shown in Fig. P3.60. Atmospheric pressure is 14.5 psia and the vapor pressure is 1.60 psia. If viscous effects are neglected, at what height, \( h \), will cavitation begin? To avoid cavitation, should the value of \( D_1 \) be increased or decreased? To avoid cavitation, should the value of \( D_2 \) be increased or decreased? Explain.

\[
\frac{P_0}{\gamma} + \frac{V_0^2}{2g} + Z_0 = \frac{P_i}{\gamma} + \frac{V_i^2}{2g} + Z_1
\]

Thus,
\[
h = \frac{P_i - P_0}{\gamma} + \frac{V_i^2}{2g}
\]

However,
\[
A_i V_i = A_2 V_2 \quad \text{or} \quad V_i = (\frac{D_2}{D_1})^2 V_2
\]

where
\[
\frac{P_0}{\gamma} + \frac{V_0^2}{2g} + Z_0 = \frac{P_0}{\gamma} + \frac{V_0^2}{2g} + Z_2 \quad \text{with} \quad P_0 = P_2 \quad \text{and} \quad Z_2 = 0
\]

Thus,
\[
\frac{V_2^2}{2g} = h
\]

so that
\[
\frac{V_2^2}{2g} = (\frac{D_2}{D_1})^4 \frac{V_2^2}{2g} = (\frac{D_2}{D_1})^4 h
\]

Combine Eqs. (1) and (2) to obtain
\[
h = \frac{P_i - P_0}{\gamma} + (\frac{D_2}{D_1})^4 h
\]

or
\[
h = \frac{P_0 - P_1}{\gamma} \left[ (\frac{D_2}{D_1})^4 - 1 \right] = \frac{(14.5 - 1.60) \frac{16}{in^2}}{62.4 \frac{16}{in^2} \left( \frac{2-in}{1-in} \right)^4 - 1} = 1.98 \, ft
\]

From Eq. (3) it is seen that \( h \) increases in increasing \( D_1 \) and decreasing \( D_2 \). Thus, to avoid cavitation (i.e., to have \( h \) small enough) \( D_1 \) should be increased and \( D_2 \) decreased.
3.94 Water flows in a rectangular channel that is 2.0 m wide as shown in Fig. P3.94. The upstream depth is 70 mm. The water surface rises 40 mm as it passes over a portion where the channel bottom rises 10 mm. If viscous effects are negligible, what is the flowrate?

\[
\frac{\rho_1}{g} + \frac{V_1^2}{2g} + Z_1 = \frac{\rho_2}{g} + \frac{V_2^2}{2g} + Z_2
\]

where \( \rho_1 = 0 \), \( \rho_2 = 0 \), \( Z_1 = 0.07m \), \( Z_2 = (0.01 + 0.10)m = 0.11m \)

Also, \( A_1 V_1 = A_2 V_2 \)

or

\[ V_2 = \frac{h_1}{h_2} V_1 = \frac{0.07m}{0.10m} V_1 = 0.7V_1 \]

Thus, Eq. (1) becomes

\[
[1 - 0.7^2] V_1^2 = 2 \left( 9.81 \frac{m^2}{s^2} \right) (0.11 - 0.07)m \text{ or } V_1 = 1.24 \frac{m}{s}
\]

Hence,

\[
Q = A_1 V_1 = (0.07m)(2.0m)(1.24 \frac{m}{s}) = 0.174 \frac{m^3}{s}
\]
4.4 The components of a velocity field are given by \( u = x + y \), 
\( v = xy^3 + 16 \), and \( w = 0 \). Determine the location of any stagnation points \( (V = 0) \) in the flow field.

\[
V = \sqrt{u^2 + v^2 + w^2} = \sqrt{(x + y)^2 + (xy^3 + 16)^2} = 0
\]

or

\( u = x + y = 0 \) so that \( x = -y \)

and

\( v = xy^3 + 16 = 0 \) so that \( xy^3 = -16 \)

Hence, \( (-y)y^3 = -16 \), or \( y = 2 \)

Therefore, \( V = 0 \) at \( x = -2, y = 2 \)
4.14 A velocity field is given by \( u = cx^2 \) and \( v = cy^2 \), where \( c \) is a constant. Determine the \( x \) and \( y \) components of the acceleration. At what point (points) in the flow field is the acceleration zero?

\[
Q_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (cx^2)(2cx) = 2c^2 x^3
\]
and
\[
a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = (cy^2)(2cy) = 2c^2 y^3
\]

Thus, \( \vec{a} = a_x \hat{i} + a_y \hat{j} = 0 \) at \((x, y) = (0, 0)\)

4.15 Determine the acceleration field for a three-dimensional flow with velocity components \( u = -x \), \( v = 4x^2y^2 \), and \( w = x - y \).

\[
u = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial z} = \]

\[
= 0 + (-x)(-1) + 4x^2y^2(0) + (x - y)(0) = x
\]

\[
a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial z} = \]

\[
= 0 + (-x)(8xy^2) + (4x^2y^2)(8x^2y) + (x - y)(0) = -8x^2y^2 + 32x^4y^3 = 8x^2y^2(4x^2y - 1)
\]

and

\[
a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + \omega \frac{\partial w}{\partial z} = \]

\[
= 0 + (-x)(1) + (4x^2y^2)(-1) + (x - y)(0) = -x - 4x^2y^2
\]

Thus,

\[
\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}
\]

\[
= x \hat{i} + 8x^2y^2(4x^2y - 1) \hat{j} - (x + 4x^2y^2) \hat{k}
\]
4.21 The fluid velocity along the x axis shown in Fig. P4.21 changes from 6 m/s at point A to 18 m/s at point B. It is also known that the velocity is a linear function of distance along the streamline. Determine the acceleration at points A, B, and C. Assume steady flow.

\[
\vec{a} = \frac{\partial \vec{v}}{\partial t} + \vec{V} \cdot \nabla \vec{V}
\]
With \( u = u(x), \ v = 0, \) and \( w = 0 \)

This becomes

\[
\vec{a} = \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \hat{\text{i}} = u \frac{\partial u}{\partial x} \hat{\text{i}}
\]

Since \( u \) is a linear function of \( x \), \( u = c_1 x + c_2 \) where the constants \( c_1, c_2 \) are given as: \( u_A = 6 = c_2 \)

and \( u_B = 18 - 0.1 c_1 + c_2 \)

Thus, \( u = (120 x + 6) \frac{m}{s} \) with \( \text{x in m} \)

From Eq. (1)

\[
\vec{a} = u \frac{\partial u}{\partial x} \hat{\text{i}} = (120 x + 6) \frac{m}{s} \left( 120 \frac{m}{s^2} \right) \hat{\text{i}}
\]

or

for \( x_A = 0 \), \( \vec{a}_A = 720 \ \hat{\text{i}} \ \frac{m}{s^2} \)

for \( x_B = 0.05 \text{ m} \), \( \vec{a}_B = 1440 \ \hat{\text{i}} \ \frac{m}{s^2} \)

and

for \( x_C = 0.1 \text{ m} \), \( \vec{a}_C = 2160 \ \hat{\text{i}} \ \frac{m}{s^2} \)
4.25 A hydraulic jump is a rather sudden change in depth of a liquid layer as it flows in an open channel as shown in Fig. P4.25 and Video V10.6. In a relatively short distance (thickness = \( \ell \)) the liquid depth changes from \( z_1 \) to \( z_2 \), with a corresponding change in velocity from \( V_1 \) to \( V_2 \). If \( V_1 = 1.20 \text{ ft/s} \), \( V_2 = 0.30 \text{ ft/s} \), and \( \ell = 0.02 \text{ ft} \), estimate the average deceleration of the liquid as it flows across the hydraulic jump. How many \( g \)'s deceleration does this represent?

\[
\vec{a} = \frac{d\vec{V}}{dt} + \vec{V} \cdot \nabla \vec{V} \quad \text{so with} \quad \vec{V} = u(x) \hat{i}, \quad \vec{a} = a_x \hat{i} = u \frac{du}{dx} \hat{i}
\]

Without knowing the actual velocity distribution, \( u = u(x) \), the acceleration can be approximated as

\[
a_x = u \frac{du}{dx} \approx \frac{1}{2} (V_1 + V_2) \left( \frac{V_2 - V_1}{\ell} \right) = \frac{1}{2} (1.20 + 0.30) \frac{\text{ft}}{s} \left( \frac{0.30 - 1.20}{0.02} \right) \frac{\text{ft}}{s^2}
\]

\[
= -33.8 \frac{\text{ft}}{s^2}
\]

Thus,

\[
\frac{|a_x|}{g} = \frac{33.8 \frac{\text{ft}}{s^2}}{32.2 \frac{\text{ft}}{s^2}} = 1.05
\]
4.60 Water flows through the 2-m-wide rectangular channel shown in Fig. P4.60 with a uniform velocity of 3 m/s. (a) Directly integrate Eq. 4.16 with \( b = 1 \) to determine the mass flowrate (kg/s) across section CD of the control volume. (b) Repeat part (a) with \( b = 1/\rho \), where \( \rho \) is the density. Explain the physical interpretation of the answer to part (b).

\[ \dot{B}_{out} = \int_{CD} \rho b \mathbf{V} \cdot \mathbf{n} \, dA \]  

\[ \text{With } b = 1 \text{ and } \mathbf{V} \cdot \mathbf{n} = V \cos \theta \text{ this becomes} \]

\[ \dot{B}_{out} = \int_{CD} \rho V \cos \theta \, dA = \rho V \cos \theta \int_{CD} dA \]

\[ = \rho V \cos \theta A_{CD}, \text{ where } A_{CD} = l \cdot (2m) \]

\[ = \left( \frac{0.5m}{\cos \theta} \right) (2m) \]

\[ = \left( \frac{1}{\cos \theta} \right) m^2 \]

Thus, with \( V = 3 \text{ m/s} \),

\[ \dot{B}_{out} = (3 \text{ m/s}) \cos \theta \left( \frac{1}{\cos \theta} \right) m^2 (999 \text{ kg/m}^3) = 3000 \text{ kg/s} \]

(b) With \( b = 1/\rho \) Eq. (1) becomes

\[ \dot{B}_{out} = \int_{CD} \mathbf{V} \cdot \mathbf{n} \, dA = \int_{CD} V \cos \theta \, dA = V \cos \theta A_{CD} \]

\[ = (3 \text{ m/s}) \cos \theta \left( \frac{1}{\cos \theta} \right) m^2 = 3.00 \text{ m}^3 \]

With \( b = 1/\rho = \frac{1}{\text{mass/vol}} = \frac{\text{vol}}{\text{mass}} \) so that \( \int \mathbf{V} \cdot \mathbf{n} \, dA = \dot{B}_{out} \) represents the volume flowrate (m³/s) from the control volume.
The wind blows across a field with an approximate velocity profile as shown in Fig. P4.61. Use Eq. 4.16 with the parameter $b$ equal to the velocity to determine the momentum flowrate across the vertical surface $A-B$, which is of unit depth into the paper.

\[
\vec{B}_{AB} = \int_{AB} \rho \vec{V} \cdot \hat{n} \, dA = \int_{AB} \rho \vec{V} \cdot \hat{n} \, dA = \rho \int_{0}^{10} \left[ V_{2} \right] \left[ (V_{2}) \cdot \hat{n} \right] (1 \, ft) \, dy
\]

\[
= \rho \hat{\epsilon} \int_{0}^{20} V_{2} \, dy
\]

But, $V = \frac{15}{10} y \, \text{ft/s}$ for $0 \leq y \leq 10 \, \text{ft}$ (i.e., $V = 0$ at $y = 0$; $V = 15 \, \text{ft/s}$ at $y = 10$) and $V = 15 \, \text{ft/s}$ for $y \geq 10 \, \text{ft}$

Thus,

\[
\vec{B}_{AB} = \rho \hat{\epsilon} \left[ \int_{0}^{10} \left( \frac{15}{10} y \right)^{2} \, dy + \int_{10}^{20} (15)^{2} \, dy \right] = \rho \hat{\epsilon} \left[ \frac{2.25 \, y^{3}}{3} + 225 \, y \right]_{0}^{10}
\]

\[
= 0.00238 \, \text{s lbf/s ft} \left[ \frac{750 \, \text{ft}^{4}}{\text{s}^{2}} + 2250 \, \text{ft}^{4} \right] \hat{\epsilon}
\]

\[
= 7.14 \, \text{s lbf/s ft} \frac{\text{ft}}{\text{s}^{2}}
\]
As shown in Fig. P5.19, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity $V$. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where $u$ is in ft/s and $y$ is in ft. Determine the value of $V$.

![FIGURE P5.19](image)

Use the control volume indicated by the broken lines in the sketch above.

From the conservation of mass principle

$$Q_1 = Q_2$$

$$V_1A_1 = \int u \, dA$$

$$V(0.75 \text{ ft}) b = \int_0^1 (4y - 2y^2) b \, dy$$

$$V = \frac{4}{3(0.75)} = 1.78 \text{ ft/s}$$
5.22 Estimate the time required to fill with water a cone-shaped container (see Fig. P5.22) 5 ft high and 5 ft across at the top if the filling rate is 20 gal/min.

From application of the conservation of mass principle to the control volume shown in the figure, we have

$$\frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho \hat{V} \cdot \hat{n} \, dA = 0$$

For incompressible flow

$$\frac{\partial}{\partial t} \int_{CV} \rho \, dV = 0$$

or

$$\int_0^t \frac{dV}{Q} = Q \int_0^t dt$$

Thus

$$t = \frac{V}{Q} = \frac{\pi D^2 h}{12 Q} = \frac{\pi \left(5 \text{ ft}\right)^2 \left(5 \text{ ft}\right) \left(1728 \text{ in}^3\right)}{(12) (20 \text{ gal/min}) (231 \text{ in}^3 \text{ gal})}$$

and

$$t = 12.2 \text{ min}$$
5.66 A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.66 and Video V5.4. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the x direction component of anchoring force required to (a) hold the vane stationary, (b) confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.

![Diagram of Pelton wheel vane](image)

**FIGURE P5.66**

(a) To determine the x-direction component of anchoring force required to hold the vane stationary we use the stationary control volume shown above and the x-direction component of the linear momentum equation (Eq. 5.22). Thus,

\[ F_A = \rho A V_x (V_y + V_z \cos \theta) = \rho A V_x (V_y + V_z \cos \theta) \]

or

\[ F_A = (1.94 \text{ slugs/ft}^3) (1 \text{ in.}^2) (100 \text{ ft/s}) \left[ (100 \text{ ft/s}) (100 \text{ ft/s}) \cos \theta \right] \left( \frac{1 \text{ lb}}{\text{slug \cdot ft}} \right) \]

and

\[ F_A = 181 \text{ lb} \]

(b) To determine the x-direction component of anchoring force required to confine the vane to a constant speed of 10 ft/s to the right we use a control volume moving to the right with a speed of 10 ft/s and the x-direction component of the linear momentum equation for a translating control volume (Eq. 5.29). Thus,

\[ F_A = \rho A W_x (W_y + W_z \cos \theta) = \rho A W_x (W_y + W_z \cos \theta) \]

We note that

\[ W_x = V_x - 10 \text{ ft/s} = 100 \text{ ft/s} - 10 \text{ ft/s} = 90 \text{ ft/s} \]

Thus, Eq. 1 leads to

\[ F_A = (1.94 \text{ slugs/ft}^3) (1 \text{ in.}^2) (90 \text{ ft/s}) (90 \text{ ft/s}) (100 \text{ ft/s}) \cos \theta \left( \frac{1 \text{ lb}}{\text{slug \cdot ft}} \right) \]

or

\[ F_A = 144 \text{ lb} \]
5.67 How much power is transferred to the moving vane of Problem 5.66?

\[
\text{Power} = F_A V, \text{ where from Problem 5.66 } F_A = 146 \text{ lb}
\]

Thus,

\[
\text{Power} = \frac{(146 \text{ lb})(10 \text{ ft/s})}{(550 \frac{\text{ft} \cdot \text{lb}}{\text{s} \cdot \text{hp}})} = 2.65 \text{ hp}
\]
Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.33. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.

**FIGURE P5.33**

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee, we use the x-component of the linear momentum equation:

\[-V_1pV_1A_1 + V_2pV_2A_2 = pA_1 - pA_2 - p_{atm}(A_1-A_2) + F_x\]

\[= (p + p_{gase})A_1 - (p + p_{gase})A_2 - p_{atm}(A_1-A_2) + F_x\]

\[= p_{gase}A_1 + F_x \]  \hspace{1cm} (1)

To get \(v_1\), we use conservation of mass

\[Q_1 = Q_2 + Q_3\]

or

\[A_1v_1 = A_2v_2 + A_3v_3\]

so

\[v_1 = \frac{A_2v_2 + A_3v_3}{A_1}\]

To estimate \(p_{gase}\), we use Bernoulli's equation for flow between (1) and (2)

\[\frac{p_{gase} + \frac{V_1^2}{2}}{p} = \frac{p_{gase} + \frac{V_2^2}{2}}{p}\]

\[p_{gase} = p \left(\frac{\frac{V_2^2}{2}}{\frac{V_1^2}{2}}\right) = (999 \text{ kg/m}^3) \left[\frac{(15 \text{ m/s})^2 - (12 \text{ m/s})^2}{2}\right] \left(1 \text{ N.s}^2/\text{kg.m}\right)\]

\[p_{gase} = 40,500 \text{ N/m}^2\]

Now using Eq. (1) we get:

\[-(\frac{A_1}{5})(999 \text{ kg/m}^3)\left(\frac{12 \text{ m}}{5}\right)(1 \text{ m}^2) + (\frac{A_2}{5})(999 \text{ kg/m}^3)\left(\frac{15 \text{ m}}{5}\right)(0.3 \text{ m}^2) \left(1 \text{ N.s}^2/\text{kg.m}\right) = (40,500 \text{ N/m}^2)(1 \text{ m}^2) + F_x\]

or

\[-72,000N = F_x\]

so

\[F_x = 72,000N\]

For the y component of the force exerted by the pipe on the tee, we use the y component of the linear momentum equation to get

\[V_2pV_2A_2 = F_y\]

\[\left(\frac{15 \text{ m}}{5}\right)(999 \text{ kg/m}^3)\left(\frac{0.3 \text{ m}}{5}\right) = 67400N \uparrow = F_y\]
5.32 Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown in Fig. P5.32 in place. Atmospheric pressure is 100 kPa

\[ p_1 = 100 \text{kPa} \]
\[ V_1 = 2 \text{m/s} \]

**FIGURE P5.32**

The control volume shown in the sketch above is used. Application of the y direction component of the linear momentum equation yields

\[ R_y = 0 \]

Application of the x direction linear momentum equation leads to

\[-u_1 \rho u_1 A_x - u_2 \rho u_2 A_x = p_x A_x - R_x + p_x A_x \]

From the conservation of mass equation

\[ m = \rho u_1 A_1 = \rho u_2 A_2 \]

Thus

\[ R_x = \rho u_1 A_1 (u_1 + u_2) + p_x A_x + p_x A_x = \rho u_1 \frac{\pi D_1^2}{4} (u_1 + \frac{D_1^2}{4} u_1) + p_x \frac{\pi D_1^2}{4} (0) A_x \]

or

\[ R_x = \left( \frac{999 \text{ kg}}{\text{m}^3} \right) \left( \frac{2}{5} \right)^2 \left( \frac{300 \text{ mm}}{1000 \text{ mm/m}} \right)^2 \left[ \left( \frac{2}{5} \right) + \left( \frac{300 \text{ mm}}{160} \right)^2 \left( \frac{1000 \text{ mm}}{160} \right) \right] \]

\[ + (100 \text{kPa}) \left( \frac{300 \text{ mm}}{1000 \text{ mm/m}} \right)^2 \left( \frac{1000 \text{ N}}{m^2 \text{kPa}} \right) \]

and

\[ R_x = 8340 \text{ N} \]
The four devices shown in Fig. P5.58 rest on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.

We apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force $F_A$.

If $F_A$ is in the direction shown in the sketches, motion will be to the left. If $F_A$ is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$, there is no horizontal motion.

For sketch (a)

$$-V_1 p V_1 A_1 - V_2 p V_2 A_2 = F_A$$

Since $F_A$ is to the left, motion is to the right.

For sketch (b)

$$-V_1 p V_1 A_1 + V_2 p V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$, then $F_A$ is to the left and motion is to the right.

For sketch (c) (note: flow is into CV at (1))

$$-V_1 p V_1 A_1 = F_A$$

and $F_A$ is to the left so motion is to the right.

For sketch (d)

$$-V_1 p V_1 A_1 + V_2 p V_2 A_2 = F_A$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and $V_1 < V_2$

so $F_A$ is to the right and motion is to the left.
An incompressible liquid flows steadily along the pipe shown in Fig. 5.91. Determine the direction of flow and the head loss over the 6-m length of pipe.

Assume flow from (1) to (2) and use the energy equation (Eq. 5.84) to get for the contents of the control volume shown:

\[ \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h - h_1 \]

Thus

\( h_1 = \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2 = 3m - 1.0m - 1.5m = 0.5m \)

and since \( h_1 > 0 \), the assumed direction of flow is correct.

The flow is uphill.
5.95 Water flows through a vertical pipe, as is indicated in Fig. P5.95. Is the flow up or down in the pipe? Explain.

The control volume shown in the sketch above is used. For steady, incompressible flow downward from (A) to (B) we obtain from Eq. 5.79

\[ \frac{P_A}{\rho} + \frac{V_A^2}{2} + g z_A = \frac{P_B}{\rho} + \frac{V_B^2}{2} + g z_B - \text{loss}_B \]

From conservation of mass we conclude that

\[ V_A = V_B \]

Thus from Eq. 1

\[ \text{loss}_B = g H + \frac{P_A - P_B}{\rho} \]

However the manometer equation (see Section 2.6) yields

\[ \frac{P_A - P_B}{\rho} = g \left[ h \left( 1 - \frac{S_{Hg}}{S_{Hg}} \right) - H \right] \]

and

\[ \text{loss}_B = gh \left( 1 - \frac{S_{Hg}}{S_{Hg}} \right) \]

which is a negative quantity since \( S_{Hg} = 13.6 \). A negative loss is not physically possible so the flow must be upward from B to A. For upward flow the above analysis leads to

\[ \text{loss}_A = gh \left( \frac{S_{Hg}}{S_{Hg}} - 1 \right) \]

which is positive and therefore physically reasonable.
5.108 The hydroelectric turbine shown in Fig. P5.108 passes 8 million gal/min across a head of 600 ft. What is the maximum amount of power output possible? Why will the actual amount be less?

From the energy equation
\[ \frac{\rho_1}{g} + z_1 + \frac{V_1^2}{2g} + h_s - h_2 = \frac{\rho_2}{g} + z_2 + \frac{V_2^2}{2g} \]

where \( \rho_1 = 0 \), \( \rho_2 = 0 \), and \( V_1 = 0 \).

Thus,
\[ h_s = (z_2 - z_1) + h_2 + \frac{V_2^2}{2g} \]

Note: Since this is a turbine, \( h_s < 0 \). Let \( h_T = -h_s \), where \( h_T > 0 \) and from the above,
\[ h_T = (z_1 - z_2) - h_L - \frac{V_2^2}{2g} \]

Also, the power is given by
\[ W_{\text{turb}} = \gamma Q h_T = \gamma Q [(z_1 - z_2) - h_L - \frac{V_2^2}{2g}] \]

The maximum power would occur if there were no losses (\( h_L = 0 \)) and negligible kinetic energy at the exit (\( V_2 \approx 0 \); large diameter outlet).

Thus,
\[ W_{\text{turb}} = \frac{\gamma Q (z_1 - z_2)}{\text{max}} = 62.4 \frac{\text{lb}}{\text{ft}} (8 \times 10^6 \frac{\text{gal}}{\text{min}}) \left( \frac{1}{60 \text{sec}} \right) (1 \frac{\text{ft}^3}{\text{lb} \cdot \text{sec}}) (600 \text{ ft}) \]
\[ = 6.67 \times 10^8 \text{ ft} \cdot \text{lb} \frac{s}{\text{sec} \cdot \text{lb}} = 1.21 \times 10^6 \text{ hp} \]
5.111 Gasoline ($SG = 0.68$) flows through a pump at 0.12 m\(^3\)/s as indicated in Fig. P5.111. The loss between sections (1) and (2) is loss = $h_L \cdot g = 0.3 \frac{V_2^2}{2}$. What will be the difference in pressures between sections (1) and (2) if 20 kW is delivered by the pump to the fluid?

From Eq. 5.82 we get for the flow from section (1) to section (2)

$$P_1 - P_2 = \rho \left[ \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) - \frac{\nu_{shaft}}{\text{net in}} + \text{loss} \right] \tag{1}$$

From the volume flowrate we obtain

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(0.12 \text{ m}^3/\text{s})^2}{\pi (0.2 \text{ m})^2} = 3.87 \text{ m}^3/\text{s}$$

and from conservation of mass (Eq. 5.13) it follows that

$$V_2 = V_1 \frac{A_2}{A_1} = \frac{V_1 D_2^2}{D_1^2} = \left(3.82 \text{ m}^3/\text{s}\right) \left(\frac{0.2 \text{ m}}{0.1 \text{ m}}\right)^2 = 15.28 \text{ m}^3/\text{s}$$

Also

$$\nu_{shaft} = \frac{\nu_{shaft}}{\text{net in}} = \frac{(20,000 \text{ N.m}/\text{s})}{(0.68)(999 \text{ kg/m}^3)(0.12 \text{ m}^3/\text{s})} = 245.3 \text{ N.m/kg}$$

and

$$\text{loss} = 0.3 \frac{V_2^2}{2} = (0.3) \left(15.28 \text{ m}^3/\text{s}\right)^2 \frac{1 \text{ N} \cdot \text{m}}{\text{kg} \cdot \text{m}^2} = 35.02 \text{ N.m/kg}$$

From Eq. 1 then

$$P_1 - P_2 = (0.68)(999 \text{ kg/m}^3) \left\{ \left[\frac{(3.82 \text{ m}^3/\text{s})^2}{2} \left(15.28 \text{ m}^3/\text{s}\right)^2 \right] + (9.81 \text{ m/s}^2)(3 \text{ m}) \right\} \left(\frac{1 \text{ N}}{\text{kg} \cdot \text{m}^2} \right)$$

or

$$P_1 - P_2 = -197,000 \frac{\text{N}}{\text{m}^2} = -197 \text{ kPa}$$
A 40-cm pipe abruptly expands to a 60-cm size. These pipes are horizontal, and the discharge of water from the smaller size to the larger is $1.0 \text{ m}^3/\text{s}$. What horizontal force is required to hold the transition in place if the pressure in the 40-cm pipe is 70 kPa gage? Also, what is the head loss?
7.48 Information and assumptions

provided in problem statement

Find
horizontal force required to hold transition in place and head loss.

Solution

\[
\begin{align*}
V_{10} &= Q/A_{10} = 1.0/((\pi/4) \times 0.40^2) = 7.962 \text{ m/s} \\
V_{30}^2/2g &= 3.231 \text{ m} \\
V_{60} &= V_{30} \times (4/6)^2 = 3.539 \text{ m/s} \\
V_{60}^2/2g &= 0.638 \text{ m} \\
h_L &= (V_{30} - V_{60})^2/2g = 0.997 \text{ m} \\
p_{10}/\gamma + V_{30}^2/2g &= p_{60}/\gamma + V_{60}^2/2g + h_L \\
p_{60} &= 70,000 + 9,810(3.231 - 0.638 - 0.997) = 85,657 \text{ Pa}
\end{align*}
\]

\[
\begin{align*}
\text{Momentum equation:} \\
\sum F_x &= \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i} \\
70,000 \times \pi/4 \times 0.4^2 - 85,657 \times \pi/4 \times (0.6^2) + F_x &= 1,000 \times 1.0 \times (3.539 - 7.962) \\
F_x &= -8,796 + 24,219 - 4,423 \\
&= 10,993 \text{ N} = 11.0 \text{ kN}
\end{align*}
\]
P3.176 In the spillway flow of Fig. P3.176, the flow is assumed uniform and hydrostatic at sections 1 and 2. If losses are neglected, compute (a) $V_2$ and (b) the force per unit width of the water on the spillway.
In the spillway flow of Fig. P3.176, the flow is assumed uniform and hydrostatic at sections 1 and 2. If losses are neglected, compute (a) $V_2$ and (b) the force per unit width of the water on the spillway.

**Solution:** For mass conservation,

$$V_2 = V_1 h_1 / h_2 = \frac{5.0}{0.7} V_1 = 7.14 V_1$$

(a) Now apply Bernoulli from 1 to 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_1 \approx \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_2; \quad \text{or:} \quad 0 + \frac{V_1^2}{2g} + 5.0 \approx 0 + \frac{(7.14 V_1)^2}{2g} + 0.7$$

Solve for $V_1^2 = \frac{2(9.81)(5.0 - 0.7)}{[(7.14)^2 - 1]}$, or $V_1 = 1.30 \frac{m}{s}$, $V_2 = 7.14 V_1 = 9.28 \frac{m}{s}$ \textit{Ans. (a)}

(b) To find the force on the spillway ($F \leftarrow$), put a CV around sections 1 and 2 to obtain

$$\Sigma F_x = -F + \frac{\gamma h_1^2}{2} - \frac{\gamma h_2^2}{2} = m(V_2 - V_1), \quad \text{or, using the given data,}$$

$$F = \frac{1}{2}(9790)[(5.0)^2 - (0.7)^2] - 998[(1.30)(5.0)](9.28 - 1.30) \approx 68300 \frac{N}{m} \text{ \textit{Ans. (b)}}$$
6.4 The three components of velocity in a flow field are given by
\[ u = x^2 + y^2 + z^2 \]
\[ v = xy + yz + z^2 \]
\[ w = -3xz - z^2/2 + 4 \]

(a) Determine the volumetric dilatation rate, and interpret the results. (b) Determine an expression for the rotation vector. Is this an irrotational flow field?

(a) Volumetric dilatation rate \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \) (Eq. 6.9)

Thus, for velocity components given\[ \text{volumetric dilatation rate} = 2x + (x+z) + (-3x-z) = 0 \]

This result indicates that there is no change in the volume of a fluid element as it moves from one location to another.

(b) From Eqs. 6.12, 6.13, and 6.14 with the velocity components given:
\[ \omega_x = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} \left( y - 2y \right) = -\frac{y}{2} \]
\[ \omega_y = \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) = \frac{1}{2} \left[ 0 - (y+2z) \right] = -\frac{(y+z)}{2} \]
\[ \omega_z = \frac{1}{2} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = \frac{1}{2} \left[ 2z - (-3z) \right] = \frac{5z}{2} \]

Thus, \[ \vec{\omega} = -\left( \frac{y}{2} + z \right) \hat{\imath} + \frac{5z}{2} \hat{j} - \frac{y}{2} \hat{k} \]

Since \( \vec{\omega} \) is not zero everywhere the flow field is not irrotational. No.
6.11 The velocity components of an incompressible, two-dimensional velocity field are given by the equations
\[ u = y^2 - x(1 + x) \]
\[ v = y(2x + 1) \]

Show that the flow is irrotational and satisfies conservation of mass.

If the two-dimensional flow is irrotational,
\[ \omega_z = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0 \]

For the velocity distribution given,
\[ \frac{\partial u}{\partial x} = 2y \quad \frac{\partial u}{\partial y} = 2y \]

Thus,
\[ \omega_z = \frac{1}{2} (2y - 2y) = 0 \]
and the flow is irrotational.

To satisfy conservation of mass,
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

Since,
\[ \frac{\partial u}{\partial x} = -1 - 2x \quad \frac{\partial v}{\partial y} = 2x + 1 \]
then
\[ -1 - 2x + 2x + 1 = 0 \]
and conservation of mass is satisfied.
6.77 A viscous, incompressible fluid flows between the two infinite, vertical, parallel plates of Fig. P6.77. Determine, by use of the Navier-Stokes equations, an expression for the pressure gradient in the direction of flow. Express your answer in terms of the mean velocity. Assume that the flow is laminar, steady, and uniform.

With the coordinate system shown \( u=0, w=0 \) and from the continuity equation \( \frac{\partial u}{\partial y} = 0 \). Thus, from the \( y \)-component of the Navier-Stokes equations (Eq. 6.1276), with \( g_y = -g \),

\[
0 = -\frac{\partial P}{\partial y} - \rho g + \mu \frac{\partial^2 u}{\partial x^2}
\]

Since the pressure is not a function of \( x \), Eq. (1) can be written as

\[
\frac{\partial^2 u}{\partial x^2} = \frac{P}{\mu x} = \frac{P}{\mu} x + C_1
\]

(Where \( \frac{\partial P}{\partial y} + \rho g \)) and integrated to obtain

\[
\frac{\partial u}{\partial x} = \frac{P}{\mu} x + C_1
\]

From symmetry \( \frac{\partial u}{\partial x} = 0 \) at \( x = 0 \) so that \( C_1 = 0 \). Integration of Eq. (2) yields

\[
u = \frac{P}{\mu} \frac{x^2}{2} + C_2
\]

Since at \( x = \pm h \), \( \nu = 0 \) it follows that \( C_2 = -\frac{P}{2\mu} \frac{h^2}{2} \)

and therefore

\[
u = \frac{P}{2\mu} \left( x^2 - \frac{h^2}{2} \right)
\]

The flowrate per unit width in the \( z \)-direction can be expressed as

\[
q = \int_{-h}^{h} v \, dx = \int_{-h}^{h} \frac{P}{2\mu} \left( x^2 - \frac{h^2}{2} \right) \, dx = -\frac{2}{3} \frac{P h^3}{\mu}
\]

Thus, with \( V \) (mean velocity) given by the equation

\[
V = \frac{q}{2h} = -\frac{1}{3} \frac{P h^2}{\mu}
\]

it follows that

\[
\frac{\partial P}{\partial y} = -\frac{3\mu V}{h^2} - \rho g
\]
6.80 An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in Fig. P6.80. The two plates move in opposite directions with constant velocities, \( U_1 \) and \( U_2 \), as shown. The pressure gradient in the \( x \) direction is zero and the only body force is due to the fluid weight. Use the Navier–Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar flow.

For the specified conditions, \( \nu = 0 \), \( \omega = 0 \), \( \frac{\partial P}{\partial x} = 0 \), and \( \frac{\partial P}{\partial y} = 0 \), so that the \( x \)-component of the Navier–Stokes equations (Eq. 6.127a) reduces to

\[
\frac{d^2 u}{dy^2} = 0
\]  

(1)

Integration of Eq. (1) yields

\[
u = C_1 y + C_2
\]  

(2)

For \( y = 0 \), \( u = -U_2 \) and therefore from Eq. (2)

\[
C_2 = -U_2
\]

For \( y = b \), \( u = U_1 \) so that

\[
U_1 = C_1 b - U_2
\]

or

\[
C_1 = \frac{U_1 + U_2}{b}
\]

Thus,

\[
n = \left( \frac{U_1 + U_2}{b} \right) y - U_2
\]
6.83 A viscous fluid (specific weight = 80 lb/ft³, viscosity = 0.03 lb·s/ft²) is contained between two infinite, horizontal parallel plates as shown in Fig. P6.83. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity $U$ while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.1 in. If the upper plate moves with a velocity of 0.02 ft/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.

\[ u = U \frac{y}{b} + \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) (y^2 - b^2) \]  

(Eq. 6.140)

**Maximum velocity will occur at distance $y_m$ where $\frac{du}{dy} = 0$.**

Thus,

\[ \frac{du}{dy} = \frac{U}{b} + \frac{1}{2\mu} \left( \frac{\partial P}{\partial x} \right) (2y - b) \]

and for $\frac{du}{dy} = 0$,

\[ y_m = -\frac{\mu U}{b \left( \frac{\partial P}{\partial x} \right)} + \frac{b}{2} \]  

(1)

For manometer (see figure to right),

\[ \frac{P_1 - P_2}{\gamma F} \Delta h - \gamma_{oil} \Delta h = P_1 \]

or

\[ \frac{P_1 - P_2}{\gamma F} = (\gamma_{oil} - \gamma_F) \Delta h \]

\[ = \left( 100 \frac{lb}{ft^3} - 80 \frac{lb}{ft^3} \right) \left( \frac{0.11 lb}{12 in.} \right) = 0.167 \frac{lb}{ft^2} \]

Also,

\[ -\frac{\partial P}{\partial x} = \frac{P_1 - P_2}{\gamma F} = 0.167 \frac{lb}{ft^2} = 0.334 \frac{lb}{ft^3} \]

Thus, from Eq. (1),

\[ u_{max} = -\left( 0.03 \frac{lb/s}{ft^2} \right) (0.02 \frac{ft}{s}) \left( \frac{1.0 \text{ in.}}{12 \text{ in.}} \right) - 0.334 \frac{lb}{ft^3} \]

\[ = 0.0632 \frac{ft}{s} \left( \frac{12 \text{ in.}}{ft} \right) = 0.759 \text{ in.} \]  

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6. 90

A simple flow system to be used for steady flow tests consists of a constant head tank connected to a length of 4-mm-diameter tubing as shown in Fig. 6.90. The liquid has a viscosity of 0.015 N s/m², a density of 1200 kg/m³, and discharges into the atmosphere with a mean velocity of 2 m/s. (a) Verify that the flow is laminar. (b) The flow is fully developed in the last 3 m of the tube. What is the pressure at the pressure gage? (c) What is the magnitude of the wall shearing stress, \( \tau_w \), in the fully developed region?

**FIGURE P6.90**

(a) Check Reynolds number to determine if flow is laminar:

\[
Re = \frac{\rho V (2R)}{\mu} = \frac{(1200 \text{ kg/m}^3)(2 \text{ m/s})(0.004 \text{ m})}{0.015 \text{ N s/m}^2} = 640
\]

Since the Reynolds number is well below 2100, the flow is laminar.

(b) For laminar flow,

\[
V = \frac{R^2}{8\mu} \frac{\Delta p}{\ell}
\]

Since \( \Delta p = P_1 - P_2 = P_1 - 0 \) (see figure)

\[
P_1 = \frac{8\mu V^2}{R^2} = \frac{8}{(0.015 \text{ N s/m}^2)(2 \text{ m/s})(3 \text{ m})} = 180 \text{ N/m}^2
\]

(c) \( \tau_w = \mu \frac{\partial V}{\partial r} \)

For fully developed pipe flow, \( V = 0 \), so that

\[
\tau_w = \mu \frac{\partial V}{\partial r}
\]

Also,

\[
V_z = V_{max} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]
\]

and with \( V_{max} = 2V \), where \( V \) is the mean velocity

\[
\tau_w = 2V\mu \left( - \frac{2r}{R^2} \right)
\]

Thus, at the wall, \( r = R \),

\[
(\tau_w)_{wall} = \left| \frac{4V^2}{R} \right| = \left| \frac{4 (2 \text{ m/s})(0.015 \text{ N s/m}^2)}{(0.004 \text{ m})^2} \right| = 60.0 \text{ N/m}^2
\]
7.6 Water sloshes back and forth in a tank as shown in Fig. P7.6. The frequency of sloshing, \( \omega \), is assumed to be a function of the acceleration of gravity, \( g \), the average depth of the water, \( h \), and the length of the tank, \( L \). Develop a suitable set of dimensionless parameters for this problem using \( g \) and \( L \) as repeating variables.

\[
\omega = f(g, h, L)
\]

\[
\omega = T^{-1} \quad g = LT^{-2} \quad h = L \quad L = L
\]

From the pi theorem, \( 4 - 2 = 2 \) dimensionless parameters required. Use \( g \) and \( L \) as repeating variables, Thus,

\[
\Pi_1 = \omega g^a L^b
\]

and

\[
(T^{-1})(LT^{-2})^a(L)^b = L^0 T^0
\]

so that

\[
a + b = 0 \quad \text{(for } L) \\
-1 - 2a = 0 \quad \text{(for } T)
\]

It follows that \( a = -\frac{1}{2}, b = \frac{1}{2} \), and therefore

\[
\Pi_1 = \omega \sqrt{\frac{g}{L}}
\]

Check dimensions:

\[
\omega \sqrt{\frac{g}{L}} = \frac{1}{T} \sqrt{\frac{L}{LT^{-2}}} = L^0 T^{-1} \quad \text{OK}
\]

For \( \Pi_2 \):

\[
\Pi_2 = h \quad g^a L^b
\]

\[
(LT^{-2})^a(L)^b = L^0 T^0
\]

\[
1 + a + b = 0 \quad \text{(for } L) \\
-2a = 0 \quad \text{(for } T)
\]

It follows that \( a = 0, b = -1, \) and therefore

\[
\Pi_2 = \frac{h}{L}
\]

and \( \Pi_2 \) is obviously dimensionless. Thus,

\[
\omega \sqrt{\frac{g}{L}} = f \left( \frac{h}{L} \right)
\]
7.9 The pressure rise, \( \Delta p \), across a pump can be expressed as
\[
\Delta p = f(D, \rho, \omega, Q)
\]
where \( D \) is the impeller diameter, \( \rho \) the fluid density, \( \omega \) the rotational speed, and \( Q \) the flowrate. Determine a suitable set of dimensionless parameters.

\[
\Delta p \approx FL^{-2} \quad D \approx L \quad \rho \approx FL^{-4}T^{-2} \quad \omega \approx T^{-1} \quad Q \approx L^3T^{-1}
\]

From the pi theorem, \( 5 - 3 = 2 \) pi terms required. Use \( D, \rho, \) and \( \omega \) as repeating variables. Thus,
\[
\Pi_1 = \Delta p D^a \rho^b \omega^c
\]
and
\[
(FL^{-2})(L)^a(FL^{-4}T^{-2})^b(T^{-1})^c : F^0L^0T^0
\]
so that
\[
1 + b = 0 \quad \text{(for F)}
\]
\[-2 + a - 4b = 0 \quad \text{(for L)}
\]
\[2b - c = 0 \quad \text{(for T)}
\]
It follows that \( a = -2, b = -1, c = -2 \), and therefore
\[
\Pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}
\]

Check dimensions using MLT system:
\[
\frac{\Delta p}{D^2 \rho \omega^2} = \frac{ML^{-1}T^{-2}}{(L)^2(ML^{-3})(T^{-1})^2} = M^0L^0T^0 \quad \therefore \text{OK}
\]

For \( \Pi_2 \):
\[
\Pi_2 = \frac{\Delta p}{D^3 \omega}
\]
\[
(FL^{-1})(L)^a(FL^{-4}T^{-2})^b(T^{-1})^c : F^0L^0T^0
\]
\[b = 0 \quad \text{(for F)}
\]
\[3a - 4b = 0 \quad \text{(for L)}
\]
\[-1 + 2b - c = 0 \quad \text{(for T)}
\]
It follows that \( a = -3, b = 0, c = -1 \), and therefore
\[
\Pi_2 = \frac{\Delta p}{D^3 \omega}
\]

Check dimensions using MLT system:
\[
\frac{\Pi_2}{D^3 \omega} = \frac{L^{-3}T^{-1}}{(L)^3(T^{-1})} = M^0L^0T^0 \quad \therefore \text{OK}
\]

Thus,
\[
\frac{\Delta p}{D^2 \rho \omega^2} = \phi \left( \frac{\Pi_2}{D^3 \omega} \right)
\]
The pressure drop, \( \Delta p \), along a straight pipe of diameter \( D \) has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, \( \ell \), between pressure taps. Assume that \( \Delta p \) is a function of \( D \) and \( \ell \), the velocity, \( V \), and the fluid viscosity, \( \mu \). Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

\[
\Delta p = f(D, \ell, V, \mu)
\]

\( \Delta p \equiv FL^{-2} \quad D \equiv L \quad \ell \equiv L \quad V \equiv LT^{-1} \quad \mu \equiv FL^{-2}T \)

From the pi theorem, \( 5 - 3 = 2 \) pi terms required.

By inspection, for \( \Pi_1 \) (containing \( \Delta p \)):

\[
\Pi_1 = \frac{\Delta p D}{\mu V} = \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} = F L^0 T^0
\]

Check using MLT:

\[
\frac{\Delta p D}{\mu V} = \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} = M^0 L^0 T^0 \quad \text{OK}
\]

For \( \Pi_2 \) (containing \( \ell \)):

\[
\Pi_2 = \frac{\ell}{D}
\]

which is obviously dimensionless. Thus,

\[
\frac{\Delta p D}{\mu V} = f \left( \frac{\ell}{D} \right)
\]

(1)

From the statement of the problem, \( \Delta p \propto \ell \) so that Eq. (1) must be of the form

\[
\frac{\Delta p D}{\mu V} = k \frac{\ell}{D}
\]

where \( k \) is some constant. It thus follows that

\[
\Delta p \propto \frac{1}{D^2}
\]

for a given velocity.
7.22 The height, $h$, that a liquid will rise in a capillary tube is a function of the tube diameter, $D$, the specific weight of the liquid, $\gamma$, and the surface tension, $\sigma$. Perform a dimensional analysis using both the FLT and MLT systems for basic dimensions. Note: The results should obviously be the same regardless of the system of dimensions used. If your analysis indicates otherwise, go back and check your work giving particular attention to the required number of reference dimensions.

\[ h = f (D, \gamma, \sigma) \]

Using FLT system:

\[ \frac{h}{D} = L \quad D = L \quad \gamma = FL^3 \quad \sigma = FL^{-1} \]

From the \( \pi \) theorem, \( 3-2 = 1 \) \( \pi \) terms required.

By inspection, for \( \Pi_1 \) (containing $h$):

\[ \Pi_1 = \frac{h}{D} \]

Which is obviously dimensionless.

For \( \Pi_2 \) (containing $\gamma$ and $\sigma$):

\[ \Pi_2 = \frac{\sigma}{\gamma D^2} = \frac{FL^{-1}}{(FL^3)(L)^2} = F^0L^0 \]

Thus,

\[ \frac{h}{D} = f \left( \frac{\sigma}{\gamma D^2} \right) \]

Using MLT system:

\[ \frac{h}{D} = L \quad D = L \quad \gamma = ML^{-2}T^{-2} \quad \sigma = MT^{-2} \]

Although there appears to be 3 reference dimensions, only 2 reference dimensions are actually required (L and MT$^{-2}$) to describe the variables. By inspection, for \( \Pi_1 \) (see above)

\[ \Pi_1 = \frac{h}{D} \]

and for \( \Pi_2 \) (containing $\gamma$ and $\sigma$):

\[ \Pi_2 = \frac{\sigma}{\gamma D^2} = \frac{MT^{-2}}{(ML^{-2}T^{-2})(L)^2} = M^0L^0T^0 \]

Thus, (as above)

\[ \frac{h}{D} = f \left( \frac{\sigma}{\gamma D^2} \right) \]
The drag on a small, completely submerged solid body having a characteristic length of 2.5 mm and moving with a velocity of 10 m/s through water is to be determined with the aid of a model. The length scale is to be 50, which indicates that the model is to be larger than the prototype. Investigate the possibility of using either an unpressurized wind tunnel or a water tunnel for this study. Determine the required velocity in both the wind and water tunnels, and the relationship between the model drag and the prototype drag for both systems. Would either type of test facility be suitable for this study?

As demonstrated in Eq. 7.19, for flow around immersed bodies, Reynolds number similarity is required so that

\[ \frac{V_m l_m}{V_m} = \frac{V}{V} \]

or

\[ V_m = \frac{V}{V} \times \frac{v}{l_m} \]

If model tests are run in unpressurized wind tunnel, then

\[ V_m (\text{standard air}) = 1.46 \times 10^{-5} \text{ m}^2/\text{s} \]

and \[ V (\text{water}) = 1.12 \times 10^{-6} \text{ m}^2/\text{s} \]

so that

\[ V_m = \left( 1.46 \times 10^{-5} \text{ m}^2/\text{s} \right) \left( \frac{1}{50} \right) \left( 10 \text{ m/s} \right) = \frac{2.61 \text{ m/s}}{50} \quad \text{(for wind tunnel)} \]

If model tests are run in water tunnel with \[ V_m = V \], then

\[ V_m = \left( 1 \right) \left( \frac{1}{50} \right) \left( 10 \text{ m/s} \right) = \frac{0.200 \text{ m/s}}{50} \quad \text{(for water tunnel)} \]

Since \[ V_m \] is reasonable in both cases, either the wind tunnel or the water tunnel could be used. With geometric and dynamic similarity, it follows that

\[ \frac{D}{\rho V^2 l^2} = \frac{D_m}{\rho_m V_m^2 l_m^2} \]

or

\[ \frac{D}{D_m} = \frac{\rho}{\rho_m} \left( \frac{V}{V_m} \right)^2 \left( \frac{l}{l_m} \right)^2 \]

Thus, for wind tunnel tests

\[ \frac{D}{D_m} = \left( \frac{999 \text{ kg/m}^3}{1.23 \text{ kg/m}^3} \right) \left( \frac{10 \text{ m/s}}{3.91 \text{ m/s}} \right)^2 \left( \frac{1}{50} \right)^2 = 2.13 \quad \text{(for wind tunnel)} \]

and for water tunnel tests

\[ \frac{D}{D_m} = \left( 1.0 \right) \left( \frac{10 \text{ m/s}}{0.300 \text{ m/s}} \right)^2 \left( \frac{1}{50} \right)^2 = 0.444 \quad \text{(for water tunnel)} \]
7.60 River models are used to study many different types of flow situations. (See, for example, Video V7.6.) A certain small river has an average width and depth of 60 ft and 4 ft, respectively, and carries water at a flow rate of 700 ft³/s. A model is to be designed based on Froude number similarity so that the discharge scale is 1/250. At what depth and flow rate would the model operate?

For Froude number similarity

\[
\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}
\]

where \( \ell \) is some characteristic length, and with \( g_m = g \)

\[
\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}
\]

Since the flow rate is \( Q = VA \), where \( A \) is the appropriate cross-sectional area,

\[
\frac{Q_m}{Q} = \frac{V_m A_m}{V A} = \sqrt{\frac{\ell_m}{\ell}} \frac{A_m}{A}
\]

Also,

\[
\frac{A_m}{A} = \left(\frac{\ell_m}{\ell}\right)^2
\]

so that

\[
\frac{Q_m}{Q} = \left(\frac{\ell_m}{\ell}\right)^{5/2} = \frac{1}{250} \tag{1}
\]

Thus,

\[
\frac{\ell_m}{\ell} = 0.110
\]

and for a prototype depth of 4 ft the corresponding model depth is

\[
\ell_m = (0.110)(4 \text{ ft}) = 0.440 \text{ ft}
\]

The model flow rate is obtained from Eq. (1):

\[
Q_m = \left(\frac{1}{250}\right) \sqrt[5]{700 \frac{\text{ft}^3}{\text{s}}} = 2.80 \frac{\text{ft}^3}{\text{s}}
\]
8.16 Water at 20 °C flows through a horizontal 1-mm-diameter tube to which are attached two pressure taps a distance 1 m apart. (a) What is the maximum pressure drop allowed if the flow is to be laminar? (b) Assume the manufacturing tolerance on the tube diameter is \( D = 1.0 \pm 0.1 \) mm. Given this uncertainty in the tube diameter, what is the maximum pressure drop allowed if it must be assured that the flow is laminar?

\[ t \quad D = 10^{-3} \text{ m} \hspace{1cm} l = 1 \text{ m} \]

From Table 8.2, \( \nu = 1.0 \times 10^{-6} \text{ m}^2/\text{s} \), \( \mu = 1.0 \times 10^{-3} \text{ Ns/m}^2 \)

\( a) \text{ Maximum } \Delta p \text{ corresponds to maximum } V, \text{ or} \]
\[ Re = \frac{V D}{\nu} = 2100 \]
Thus, \( V = \frac{2100 \nu}{D} = \frac{2100(1 \times 10^{-6} \text{ m}^2/\text{s})}{10^{-3} \text{ m}} = 2.10 \frac{\text{m}}{\text{s}} \)

For laminar flow
\[ V = \frac{\Delta p D^2}{32 \mu L} \hspace{1cm} \text{or} \hspace{1cm} \Delta p = \frac{32 \mu L V}{D^2} = \frac{32(1 \times 10^{-3} \text{ Ns/m}^2)(1 \text{ m})(2.10 \frac{\text{m}}{\text{s}})}{(10^{-3} \text{ m})^2} \]
Thus,
\[ \Delta p = 6.72 \times 10^6 \frac{\text{N}}{\text{m}^2} \]

\( b) \text{ Since } V = \frac{2100 \nu}{D} \text{ and } \Delta p = \frac{32 \mu L V}{D^2} \text{ it follows that} \]
\[ \Delta p = \frac{32 \mu L (2100 \nu)}{D^2} \hspace{1cm} \text{Thus, the larger the diameter, the smaller the } \Delta p \text{ allowed to maintain laminar flow} \]

Thus, consider \( D = 1.1 \text{ mm} = 1.1 \times 10^{-3} \text{ m} \), or
\[ \Delta p = \frac{32(1 \times 10^{-3} \text{ Ns/m}^2)(1 \text{ m})(2100)(1 \times 10^{-6} \text{ m}^3)}{(1.1 \times 10^{-3} \text{ m})^3} = 5.05 \times 10^6 \frac{\text{N}}{\text{m}^2} \]
8.18 A fluid flows through a horizontal 0.1-in.-diameter pipe. When the Reynolds number is 1500, the head loss over a 20-ft length of the pipe is 6.4 ft. Determine the fluid velocity.

\[ h_L = \frac{fL}{D} \frac{V^2}{2g}, \text{ where since } Re = 1500 < 2100 \text{ the flow is laminar.} \]

\[ \text{Thus, } f = \frac{64}{Re} = \frac{64}{1500} = 0.0427 \text{ so that} \]

\[ 6.4 \text{ ft} = 0.0427 \frac{20 \text{ ft}}{(0.1/2 \text{ ft})} \frac{V^2}{2(32.2 \text{ ft/s}^2)} \]

\[ \text{or} \]

\[ V = \frac{2.01 \text{ ft}}{3} \]

8.19 A viscous fluid flows in a 0.10-m-diameter pipe such that its velocity measured 0.012 m away from the pipe wall is 0.8 m/s. If the flow is laminar, determine the centerline velocity and the flowrate.

For laminar flow in a pipe

\[ U(r) = V_c \left[ 1 - \left( \frac{r}{D/2} \right)^2 \right], \text{ where } D = 0.1 \text{ m and } U = 0.8 \frac{m}{s} \text{ at} \]

\[ r = \frac{0.1 \text{ m}}{2} = 0.05 \text{ m} = 0.03 \text{ m} \]

\[ 0.8 \frac{m}{s} = V_c \left[ 1 - \left( \frac{2(0.03 \text{ m})}{0.1 \text{ m}} \right)^2 \right] \text{ or } V_c = \frac{1.89 \frac{m}{s}}{2} \]

so that

\[ Q = \frac{\pi}{4} D^2 V = \frac{\pi}{4} D^2 (0.5 V_c) = \frac{\pi}{4} (0.1 \text{ m})^2 (0.5) (1.89 \frac{m}{s}) = 7.42 \times 10^{-3} \frac{m^3}{s} \]
As shown in Video V8.3 and Fig. P8.26, the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at \( Re = 10,000 \) the velocity profile can be approximated by the power-law profile shown in the figure. (a) For laminar flow, determine at what radial location you would place a Pitot tube if it is to measure the average velocity in the pipe. (b) Repeat part (a) for turbulent flow with \( Re = 10,000 \).

**Figure P8.26**

![Figure P8.26](image)

For laminar or turbulent flow,

\[
Q = AV = \pi R^2 V = \int u \, dA = \int u (2\pi r \, dr) = 2\pi \int_0^R r u \, dr
\]

**a) Laminar flow:**

\[
\pi R^2 V = 2\pi V_c \int_0^R \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \, dr = 2\pi V_c \left[ \frac{R^2}{2} - \frac{R^2}{4} \right] = \frac{\pi R^2}{2} V_c
\]

Thus, \( V = \frac{1}{2} V_c \). For \( u = V = \frac{V_c}{2} \), the equation for \( \frac{u}{V_c} \) gives

\[
\frac{u}{V_c} = \frac{1}{2} = 1 - \left( \frac{r}{R} \right)^2,
\]

or \( \left( \frac{r}{R} \right)^2 = \frac{1}{2} \). Thus, \( r = \frac{\sqrt{2}}{2} R = 0.707R \)

**b) Turbulent flow**

\[
\pi R^2 V = 2\pi V_c \int_0^R \left[ 1 - \left( \frac{r}{R} \right) \right] \, \frac{V_S}{V_c} \int_0^1 d\left( \frac{r}{R} \right)
\]

Let \( y = 1 - \left( \frac{r}{R} \right) \) so that \( \left( \frac{r}{R} \right) = 1 - y \) and \( d\left( \frac{r}{R} \right) = -dy \)

Thus,

\[
\pi R^2 V = 2\pi R^2 V_c \int_0^1 (1-y) \, \frac{V_S}{V_c} \int_0^1 (1-y) \, \frac{V_S}{V_c} \, dy = 2\pi R^2 V_c \left[ \frac{V_S}{V_c} \right] \int_0^1 (1-y)^2 \, dy
\]

\[
= 2\pi R^2 V_c \left[ \frac{V_S}{V_c} \right] \int_0^1 (1-y)^2 \, dy
\]

\[
= \frac{2\pi R^2 V_c \cdot V_S}{V_c} \left( \frac{2}{3} \right)
\]

or \( V = \frac{2\pi R^2 V_c}{3} \) For \( u = V = \frac{50}{60} \), the equation for \( \frac{u}{V_c} \) gives

\[
\frac{u}{V_c} = \frac{50}{60} = \left[ 1 - \left( \frac{r}{R} \right) \right] \frac{V_S}{V_c}
\]

or \( \frac{r}{R} = 0.750 \) so that \( r = 0.750R \).
8.27 Water at 80 °F flows in a 6-in.-diameter pipe with a flowrate of 2.0 cfs. What is the approximate velocity at a distance 2.0 in. away from the wall? Determine the centerline velocity.

\[
V = \frac{Q}{A} = \frac{2.0 \, \frac{ft^3}{s}}{\pi \left( \frac{6}{2} \right)^2} = 10.2 \, \frac{ft}{s}
\]

so that \( Re = \frac{VD}{V} = \frac{(10.2)^2 \left( \frac{ft}{s} \right)}{9.26 \times 10^{-3} \frac{ft}{s}} = 5.51 \times 10^5 \)

The flow is turbulent with \( \frac{\bar{u}}{V_c} = (1 - \frac{r}{R})^n \), where \( n \approx 8.3 \) (see Fig. 8.17)

Thus, (see Example 8.4)

\[
\frac{V}{V_c} = \frac{2n^2}{(n+1)(2n+1)} = \frac{2(8.3)^2}{(8.3+1)(2\times8.3+1)} = 0.842
\]

or

\[
V_c = \frac{10.2}{0.842} = 12.1 \, \frac{ft}{s}
\]

Also, at \( r = 3 \, in. - 2.0 \, in. = 1.0 \, in. \), \( \bar{u} = V_c (1 - \frac{r}{R})^n = 12.1 \frac{ft}{s} (1 - \frac{1.0 \, in.}{3 \, in.})^{8.3} = 11.5 \frac{ft}{s} \)
8.30 Water flows through a 6-in.-diameter horizontal pipe at a rate of 2.0 cfs and a pressure drop of 4.2 psi per 100 ft of pipe. Determine the friction factor.

For a horizontal pipe, \( \Delta \rho = f \cdot \frac{D}{2} \cdot \frac{1}{2} \rho V^2 \),
where \( V = \frac{Q}{A} = \frac{2.0 \frac{cfs}{ft^2}}{\frac{\pi (6 \frac{ft}{2})^2}{4}} = 10.2 \frac{ft}{s} \)

Thus,
\[
f = \frac{2 \Delta \rho}{\rho \cdot V^2} = \frac{2(\frac{5}{2} \frac{ft}{s})(4.2 \times 144 \frac{lb}{ft^2})}{(1.94 \times 105 \frac{lb}{ft^3})(100 \frac{ft}{10})(10.2 \frac{ft}{s})^2} = 0.0300
\]

8.31 Water flows in a cast-iron pipe of 200-mm diameter at a rate of 0.10 m³/s. Determine the friction factor for this flow.

For a \( D = 0.200 \text{ m} \) cast iron pipe, \( \frac{c}{D} = \frac{0.26 \text{ mm}}{200 \text{ mm}} = 1.3 \times 10^{-3} \) (Table 8.1)

Also,
\( \text{Re} = \frac{VD}{\nu} \), where \( V = \frac{Q}{A} = \frac{0.10 \frac{m^3}{s}}{\frac{\pi (0.2 \text{ m})^2}{4}} = 3.18 \frac{m}{s} \)

Hence,
\( \text{Re} = \frac{(3.18 \frac{m}{s})(0.2 \text{ m})}{1.12 \times 10^{-4} \frac{m^2}{s}} = 5.68 \times 10^5 \), so from Fig. 8.20 we obtain
\[f = 0.0215\]
8.58 Air flows through a rectangular galvanized iron duct of size 0.30 m by 0.15 m at a rate of 0.068 m³/s. Determine the head loss in 12 m of this duct.

\[ h_l = f \frac{L}{D_h} \frac{V^2}{2g} \]
where
\[ D_h = \frac{4A}{P} = \frac{4 \times 0.3 \times 0.15}{2 \times (0.3 + 0.15)} = 0.2 \text{ m} \]
and
\[ V = \frac{Q}{A} = \frac{0.068 \times 0.3^2}{0.3 \times 0.15} = 1.51 \text{ m/s} \]
Also, \( R_e_h = \frac{V D_h}{\nu} = \frac{(1.51)0.2}{1.46 \times 10^{-5}} = 2.07 \times 10^5 \)
and from Table 8.1,
\[ \frac{e_{D_h}}{D_h} = 0.15 \times 10^{-5} \text{ m} = 7.5 \times 10^{-6} \]
so that
\[ h_l = (0.027) \frac{12}{0.2} \frac{(1.51)^2}{2(9.81 \times 10^{-6})} = 0.188 \text{ m} \]

8.59 Air at standard conditions flows through a horizontal 1 ft by 1.5 ft rectangular wooden duct at a rate of 5000 ft³/min. Determine the head loss, pressure drop, and power supplied by the fan to overcome the flow resistance in 500 ft of the duct.

\[ h_l = f \frac{L}{D_h} \frac{V^2}{2g} \]
where
\[ V = \frac{Q}{A} = \frac{(5000 \text{ ft}^3/\text{min}) (1 \text{ min})}{(1 \text{ ft})(1.5 \text{ ft})} = 55.6 \text{ ft/s} \]
and
\[ D_h = \frac{4A}{P} = \frac{4 \times 1 \times (1.5)}{2 \times (1 \times 1.5)} = 1.2 \text{ ft} \]
Also, \( R_e_h = \frac{V D_h}{\nu} = \frac{(55.6)1.2}{1.57 \times 10^{-7}} = 4.25 \times 10^5 \) and from Table 8.1,
\[ e_{D_h \text{ ft}} = 0.0006 \text{ ft to 0.003 ft. Use an "average" } e = 0.0018 \text{ ft so that } e_{D_h} = \frac{0.0018 \text{ ft}}{1.2 \text{ ft}} = 0.0015 \text{ ft} \]
Thus, from Fig. 8.20 \( f = 0.022 \), or
\[ h_l = (0.022) \frac{500 \text{ ft}^3/\text{min}}{1.2 \text{ ft}} \frac{(55.6)^3}{(32.2)^2} = 440 \text{ ft} \]
For this horizontal pipe \( f_l + \frac{V^2}{2g} + z_1 = f_h + \frac{V^2}{2g} + z_2 + h_l \), where \( z_1 = z_2 \)
and \( V_f = V_h \).
Thus, \( f_l - f_h = \frac{\gamma h_l = (7.65 \times 10^{-2} \text{ lb/ft}) (440 \text{ ft})}{(33.7 \text{ lb/ft}^2)} = 33.7 \text{ ft}^2/\text{s} \)
\[ P = \gamma Q h_l = Q (f_l - f_h) = (5000 \text{ ft}^3/\text{min}) (1 \text{ min}) (33.7 \text{ lb/ft}^2) = 2810 \left( \frac{\text{ft} \cdot \text{lb}}{\text{s}^2} \right) \left( \frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb} / \text{s}} \right) \]
or
\[ P = 5.11 \text{ hp} \]
8.61 What horsepower is added to water to pump it vertically through a 200-ft-long, 1.0-in.-diameter drawn tubing at a rate of 0.060 ft³/s if the pressures at the inlet and outlet are the same?

\[ \frac{Q^2}{2g} + \frac{V_1^2}{2g} + Z_1 + h_p = \frac{Q^2}{2g} + \frac{V_2^2}{2g} + Z_2 + f \frac{L}{D} \frac{V_2^2}{2g} \quad \text{where} \quad \rho_1 = \rho_2 \]

and \( V_1 = V_2 \). Thus,

\[ h_p = Z_2 - Z_1 + f \frac{L}{D} \frac{V_2^2}{2g} \quad \text{where} \quad Z_2 - Z_1 = L = 200 \text{ ft and} \]

\[ V = \frac{Q}{A} = \frac{0.06 \text{ ft}^3}{\frac{1}{4} \pi (\frac{1}{4})^2} = 11.0 \text{ ft} \]

Also, \( \frac{d}{D} = \frac{5 \times 10^{-6} \text{ ft}}{(1/12 \text{ ft})} = 6 \times 10^{-5} \) (Table 8.1)

and \( \text{Re} = \frac{VD}{V} = \frac{(11.0 \text{ ft})(1/4 \text{ ft})}{1.2 \times 10^{-5} \text{ ft}} = 7.58 \times 10^4 \) we obtain from Fig. 8.20

\[ f = 0.019 \]

From Eq. (1)

\[ h_p = 200 \text{ ft} + (0.019) \left( \frac{200 \text{ ft}}{1/4 \text{ ft}} \right) \left( \frac{11.0 \text{ ft}}{2 \text{ ft}} \right)^2 = 286 \text{ ft} \]

Thus,

\[ \rho = \gamma Q h_p = (62.4 \text{ ft/lb}) (0.06 \text{ ft/s}) (286 \text{ ft}) = 1071 \frac{\text{ft}^3}{\text{s}} \left( \frac{1 \text{ hp}}{550 \text{ ft}^3/\text{s}} \right) = 1.95 \text{ hp} \]

8.62 Water flows from a lake as shown in Fig. P8.62 at a rate of 4.0 cfs. Is the device inside the building a pump or a turbine? Explain and determine the horsepower of the device. Neglect all minor losses and assume the friction factor is 0.025.

\[ \frac{Q^2}{2g} + \frac{V_1^2}{2g} + Z_1 + h_p = \frac{Q^2}{2g} + \frac{V_2^2}{2g} + Z_2 + h_4 + f \frac{L}{D} \frac{V_2^2}{2g} \quad \text{where} \quad \rho_1 = \rho_2 = 0, \quad V_1 = 0 \]

Assume the device is a pump (\( h_4 = 0 \)).

Thus, \( Z_1 + h_p = \frac{V_2^2}{2g} (1 + f \frac{L}{D}) + Z_2 \), or

\[ h_p = 495 \text{ ft} - 525 \text{ ft} + \frac{(31.8 \text{ ft})^2}{2(32.2 \text{ ft})} (1 + 0.025 \left( \frac{300 \text{ ft}}{0.4 \text{ ft}} \right)) = 280 \text{ ft} \]

Note: Since \( h_p > 0 \) the device is a pump.

Also, \( \rho = g \text{ in } h_p = \gamma Q h_p = (62.4 \text{ lb/ft}^3) (4 \text{ ft}^3/\text{s}) (280 \text{ ft}) = 69,900 \frac{\text{ft}^4}{\text{s}^2} \left( \frac{1 \text{ hp}}{550 \text{ ft}^3/\text{s}} \right) = 12.7 \text{ hp} \)

or

\[ \rho = 12.7 \text{ hp} \]

8.54
Water flows downward through a vertical smooth pipe. When the flowrate is 0.5 ft³/s there is no change in pressure along the pipe. Determine the diameter of the pipe.

\[
\rho_1 \frac{V_1^2}{2g} + z_1 + \frac{V_2^2}{2g} = \rho_2 \frac{V_2^2}{2g} + z_2 + \frac{V_1^2}{2g} + \frac{f}{D} \frac{V_1^2}{2g}
\]

where \( \rho_1 = \rho_2 \), \( V_1 = V_2 = V \), and \( z_1 - z_2 = l \)

Thus,

\[
l = \frac{f}{D} \frac{V^2}{2g}, \quad \text{or} \quad l = \frac{f}{D} \frac{V^2}{2g} \tag{1}
\]

Also,

\[
V = \frac{Q}{A} = \frac{Q}{\pi D^2} \quad \text{so that Eq. (1) becomes} \quad l = \frac{f}{D} \left( \frac{4Q}{\pi D^2} \right)^2
\]

or

\[
D^5 = \frac{8f Q^2}{g} = \frac{8}{\pi^2} f \left( \frac{0.5Q}{2g} \right)^2 \quad \text{or} \quad D = 0.363 f V_5 \tag{2}
\]

Also,

\[
Re = \frac{QVD}{\mu} = 1.94 \left( \frac{4Q}{\pi D^2} \right) \frac{D}{\mu} = 1.94 \left( \frac{4(0.5Q)}{2.34 \times 10^{-5} D} \right) \quad \text{or} \quad Re = \frac{5.28 \times 10^4}{D} \tag{3}
\]

From Fig. 8.20 with \( \frac{D}{D} = 0 \) we have \( f = f(Re, \frac{D}{D} = 0) \)

Trial and error solution: 3 unknowns (D, Re, f) and 3 equations (2, 3, and Fig. 8.20).

Assume \( f = 0.02 \) so from Eq. (2), \( D = 0.166 \) ft and from Eq. (3), \( Re = 3.18 \times 10^5 \). Thus, from Fig. 8.20, \( f = 0.014 \) ± 0.02

Assume \( f = 0.014 \) so that \( D = 0.155 \) ft and \( Re = 3.42 \times 10^5 \)

Thus, from Fig. 8.20, \( f = 0.14 \) which checks with the assumed value.

Thus, \( D = 0.155 \) ft

An alternative method is to use the Colebrook equation, Eq. 8.35, with \( e/D = 0 \), rather than the Moody chart, Fig. 8.20. Thus \( \frac{1}{f} = -2.0 \log \left( 2.51/Re^{1/4} \right) \)

which combined with Eqs. (2) and (3) gives

\[
\frac{1}{(D/0.363)^{5/4}} = -2.0 \log \left[ 2.51D/(5.28 \times 10^4 (D/0.363)^{5/4}) \right] \tag{4}
\]

Using a computer root-finding program gives the solution of Eq. (4) as \( D = 0.155 \) ft as obtaining by the above trial and error method.
8.77 The pressure at section (2) shown in Fig. P8.77 is not to fall below 60 psi when the flow rate from the tank varies from 0 to 1.0 cfs and the branch line is shut off. Determine the minimum height, \( h \), of the water tank under the assumption that (a) minor losses are negligible, (b) minor losses are not negligible.

\[
\frac{\Delta H}{g} + \frac{V^2}{2g} + z_i = \frac{\Delta H}{g} + \frac{V^2}{2g} + z_2 + (f \frac{l}{D} + \Sigma K_L) \frac{V^2}{2g}, \quad \text{where } \rho = 0, V_i = 0, z_1 = 16 \text{ ft } h,
\]

and \( z_2 = 0 \). Thus, with \( V = V_2 \)

\[
16 + h = \frac{\Delta H}{g} + \frac{V^2}{2g} + (f \frac{l}{D} + \Sigma K_L) \frac{V^2}{2g}. \quad \text{Note: } h \text{ must be no less than that with } \rho = \text{60 psi and } Q_{\text{max}} = 1 \text{ cfs, or } V_2 = V = \frac{Q}{A} = \frac{1.0}{A} (\text{ft}^3 \text{ s}^{-1}) = 5.09 \text{ ft s}^{-1}
\]

Hence,

\[
h = -16 + \frac{(60 \text{ psi})(144 \text{ in}^2)}{62,400 \text{ in}^2} + (1 + f \left( \frac{h + 6 + 600 + 900}{\frac{1}{2}} \right) + \Sigma K_L \frac{(5.09 \text{ ft s}^{-1})^2}{2(32.2 \text{ ft s}^{-1})^2})
\]

or

\[
h = 122.5 + (1 + f \left( \frac{150.6 + h}{0.5} \right) + \Sigma K_L \frac{(0.402)}{\text{ft}}, \quad \text{where } h \text{ in ft}
\]

With \( \frac{Q}{A} = 0 \) and \( \text{Re} = \frac{VD}{\nu} = \frac{(5.09 \text{ ft s}^{-1}) \left( \frac{1}{2} \text{ ft} \right)}{1.21 \times 10^{-5} \text{ ft s}^{-1}} = 2.10 \times 10^5 \) we obtain

\[
f = 0.0155 \text{ (see Fig. 8.20)}
\]

\( a) \) Neglect minor losses (\( \Sigma K_L = 0 \)):

From Eq. (1)

\[
h = 122.5 + (1 + (0.0155) (\frac{150.6 + h}{0.5})) (0.402)
\]

or

\[
h = 143 \text{ ft}
\]

\( b) \) Include minor losses:

\[
\Sigma K_L = K_L \text{ entrance } + 15 K_L \text{ elbow } + K_L \text{ tee } = 0.5 + 15 (0.3) + 0.2 = 5.2
\]

(see Table 8.2, assume flanged fittings)

Thus, from Eq. (1)

\[
h = 122.5 + (1 + (0.0155) (\frac{150.6 + h}{0.5}) + 5.2) (0.402)
\]

or

\[
h = 145 \text{ ft}
\]

Note: For this case minor losses are not very important.
When water flows from the tank shown in Fig. P8.85, the water depth in the tank as a function of time is as indicated. Determine the cross-sectional area of the tank. The total length of the 0.60-in.-diameter pipe is 20 ft, and the friction factor is 0.03. The loss coefficients are: 0.50 for the entrance, 1.5 for each elbow, and 10 for the valve.

\[
\frac{\rho}{\rho_i} + Z + \frac{V_i^2}{2g} - h_L = \frac{\rho}{\rho_i} + Z + \frac{V_2^2}{2g},
\]

where
\[
\rho = \rho_2 = 0, \ Z = 0, \ Z_1 = 3ft + h, \ V_1 = 0, \ V_2 = V \text{ and } h_L = (f \frac{L}{D} + \sum K_i) \frac{V^2}{2g} \text{ with } \sum K_i = 0.5 + 5(1.5) + 10 = 18
\]

Thus,
\[
Z_i = h_L + \frac{V_2^2}{2g} = \left(f \frac{L}{D} + \sum K_i + f_i\right) \frac{V^2}{2g}
\]

Consider the flow when \( h = 1.5 \text{ ft} \) so that \( Z_1 \leq 4.5 \text{ ft} \).

Hence,
\[
4.5 \text{ ft} = \left(0.03 \frac{20 \text{ ft}}{\left(\frac{0.6 \text{ ft}}{\text{ft}^2}\right) + 18 + 1}\right) \frac{V^2}{(32.2 \text{ ft/s})}
\]

or
\[
V = 3.06 \text{ ft/s} \text{ so that } Q = \frac{\pi}{4} \left(\frac{0.6 \text{ ft}}{2}\right)^2 (3.06 \text{ ft/s}) = 0.0060 \text{ ft}^3/\text{s}
\]

But \( Q = A_{\text{tank}} \left(-\frac{dh}{dt}\right) \)

where from the graph
\[
\frac{dh}{dt} \approx \frac{-1 \text{ ft}}{250 \text{ s}} = -0.004 \text{ ft/s}
\]

Hence,
\[
0.0060 \text{ ft}^3/\text{s} = A_{\text{tank}} \left(0.004 \text{ ft/s}\right)
\]

or
\[
A_{\text{tank}} = 1.50 \text{ ft}^2
\]
Air enters a square duct through a 1-ft opening as is shown in Fig. P9.15. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant $U = 2$ ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, $d$, as a function of $x$ for $0 \leq x \leq 10$ ft if $U$ is to remain constant. Assume laminar flow.

For incompressible flow $Q_0 = Q(x)$ where $Q_0 = \text{flowrate into the duct}$ and $Q(x) = UA$, where $A = (d - 2\delta^+)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_0 = U(d - 2\delta^+)^2 \quad \text{or} \quad d = 1\text{ ft} + 2\delta^+,$$

where

$$\delta^+ = 1.721 \sqrt{\frac{V_x}{U}} = 1.721 \left[ \frac{(1.57 \times 10^{-10}) x}{2\text{ ft}} \right]^{\frac{1}{2}} = 0.0152 \sqrt{x} \text{ ft}, \text{ where } x \text{-ft}$$

Hence, from Eq. (1)

$$d = 1 + 0.0304 \sqrt{x} \text{ ft}$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.
9.16 A smooth, flat plate of length $\ell = 6$ m and width $b = 4$ m is placed in water with an upstream velocity of $U = 0.5$ m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

$$\delta = 5 \sqrt{\frac{v_e}{U}} = 5 \sqrt{\frac{(1.12 \times 10^6 \text{ m}^2)}{0.5 \text{ m}^2}} = 7.48 \times 10^{-3} \sqrt{x} \text{ m, where } x \sim m$$

and

$$\tau_w = 0.332 U^{\frac{3}{2}} \frac{\rho U}{x} = 0.332 (0.5 \text{ m})^{\frac{3}{2}} \sqrt{\frac{(999 \text{ kg/m}^3)(1.12 \times 10^{-3} \text{ m/s})}{x}}$$

$$= \frac{0.124}{\sqrt{x}} \frac{N}{\text{ m}^2}, \text{ where } x \sim m$$

Thus, at $x = 3$ m

$$\delta = 7.48 \times 10^{-3} \sqrt{3} = 0.0130 \text{ m}$$

$$\tau_w = \frac{0.124}{\sqrt{3}} = 0.0716 \frac{N}{\text{ m}^2}$$

while at $x = 6$ m

$$\delta = 7.48 \times 10^{-3} \sqrt{6} = 0.0183 \text{ m}$$

$$\tau_w = \frac{0.124}{\sqrt{6}} = 0.0506 \frac{N}{\text{ m}^2}$$
If the drag on one side of a flat plate parallel to the upstream flow is $\mathcal{D}$ when the upstream velocity is $U$, what will the drag be when the upstream velocity is $2U$; or $U/2$? Assume laminar flow.

For laminar flow, $\mathcal{D} = \frac{1}{2} \rho U^2 C_D A$, where $C_D = \frac{1.328}{\sqrt{U/V}}$.

Thus,

$\mathcal{D} = \frac{1}{2} \rho U^2 \frac{1.328 \sqrt{V}}{V} A = 0.664 \rho A \frac{V}{V} U^{3/2} \sim U^{3/2}$

Hence,

$\frac{\mathcal{D}_U}{\mathcal{D}_{2U}} = \frac{U^{3/2}}{(2U)^{3/2}}$ or $\mathcal{D}_{2U} = 2.63 \mathcal{D}_U$

and

$\frac{\mathcal{D}_U}{\mathcal{D}_{U/2}} = \frac{U^{3/2}}{(U/2)^{3/2}}$ or $\mathcal{D}_{U/2} = 0.354 \mathcal{D}_U$
9.34 Water flows past a triangular flat plate oriented parallel to the free stream as shown in Fig. P9.34. Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow.

\[
\mathcal{D} = \int \tau_w \, dA \quad \text{where} \quad \tau_w = 0.332 \, \frac{U^{3/2} \sqrt{\rho \mu}}{x}
\]

Thus,

\[
\mathcal{D} = 0.332 \, U^{3/2} \sqrt{\rho \mu} \int_{x=0}^{x=0.5} \int_{y=0}^{y=0.5-x} \frac{dy}{\sqrt{x}} \, dx
\]

\[
= 0.332 \, U^{3/2} \sqrt{\rho \mu} \left( \frac{2}{3} \right) \int_{x=0}^{0.5} \frac{0.5-x}{\sqrt{x}} \, dx
\]

\[
= 0.332 \, U^{3/2} \sqrt{\rho \mu} \left( \frac{2}{3} \right) \left[ 0.5 \left( \frac{2}{3} x^{1/2} \right) - \frac{2}{3} x^{3/2} \right]_{0}^{0.5}
\]

\[
= 0.664 (0.2 \, \frac{m}{s})^{3/2} \sqrt{999 \, \frac{kg}{m^3} (1.12 \times 10^{-3} \, N \cdot s/m^2)} \left[ \sqrt{0.5} - \frac{2}{3} (0.5)^{3/2} \right]
\]

or

\[
\mathcal{D} = 0.0296 \, N
\]
9.64 A turbulent boundary layer develops from the leading edge of a flat plate with water at 20°C flowing tangentially past the plate with a free-stream velocity of 5 m/s. Determine the thickness of the viscous sublayer, $\delta'$, at a distance 1 m downstream from the leading edge. Would a roughness element 100 μm high affect the local skin friction coefficient? Why?

9.64 Information and assumptions

- From Table $\mu = 10^{-6}$ m²/s provided in problem statement

Find

thickness of viscous sublayer 1 m downstream from leading edge.

Solution

$$\delta' = \frac{5\nu}{u_*}$$

where $u_* = (\tau_0/\rho)^{0.5}$ and $\tau_0 = c_f \rho U^2/2$

$$\tau_0/\rho = (0.058/\text{Re}^{0.2})U_0^2/2$$
$$\text{Re}_x = U_0x/\nu$$
$$= (5)(1)/10^{-6} = 5 \times 10^6$$
$$\text{Re}_x^{0.2} = 21.87$$

Then

$$\tau_0/\rho = (0.058/21.87)(25/2)$$
$$\tau_0/\rho = 0.0332 \text{ m}^2/\text{s}^2$$
$$u_* = (\tau_0/\rho)^{0.5} = 0.1822 \text{ m/s}$$

Finally

$$\delta' = \frac{5\nu}{u_*} = (5)(10^{-6})/(0.1822)$$

$$\delta' = 27.4 \times 10^{-6} \text{ m}$$

Roughness element size of 100 microns is about 4 times greater than the thickness of the viscous sublayer; therefore, it would definitely affect the skin friction coefficient.
9.73 Consider the boundary layer next to the smooth hull of a ship. The ship is cruising at a speed of 30 ft/s in 60°F fresh water. Assuming that the boundary layer on the ship hull develops the same as on a flat plate, determine

a. The thickness of the boundary layer at a distance of 100 ft downstream from the bow.

b. The velocity of the water at a point in the boundary layer at \( y/\delta = 0.50 \).

c. The shear stress, \( \tau_0 \), adjacent to the hull at this position.

**9.73 Information and assumptions**

From Table H, \( \nu = 1.22 \times 10^{-5} \text{ ft}^2/\text{s} \) and \( \rho = 1.94 \text{ slug/ft}^3 \), provided in problem statement.

**Find**

a) thickness of boundary layer at 100 ft downstream, b) velocity of water at \( y/\delta = 0.5 \) and c) shear stress on hull.

**Solution**

\[
\text{Re}_x = \frac{Ux}{\nu} = \frac{(30)(100)}{(1.22 \times 10^{-5})} = 2.46 \times 10^8
\]

\[
c_f = \frac{0.455}{\ln^2(0.06 \text{ Re}_x)} = 0.00167
\]

\[
\tau_0 = c_f \rho \frac{U_0^2}{2} = (0.00167)(1.94)(30^2)/2 = 1.456 \text{ lbf/ft}^2 \quad (c)
\]

\[
u_* = \left( \frac{\tau_0}{\rho} \right)^{0.5} = (1.456/1.94)^{0.5} = 0.866 \text{ ft/s}
\]

\[
\frac{\delta}{x} = 0.16 \text{ Re}_x^{-1/7} = 0.010
\]

\[
\delta = (0.010)(100) = 1.0 \text{ ft} \quad (a)
\]

\[
\frac{\delta}{2} = 0.5 \text{ ft}
\]

From Fig. 9-12 at \( y/\delta = 0.50 \), \( (U_0 - u)u_* \approx 3.0 \). Then

\[
(30 - u)/0.866 = 3.0
\]

\[
u_{\delta/2} = 27.4 \text{ ft/s} \quad (b)
\]
A flat plate is oriented parallel to a 15 m/s air flow at 20°C and atmospheric pressure. The plate is 1 m long in the flow direction and 0.5 m wide. On one side of the plate, the boundary layer is tripped at the leading edge and on the other side there is no tripping device. Find the total drag force on the plate.

**PROBLEM 9.66**

**9.66 Information and assumptions**

provided in problem statement

**Find**

total drag force on plate.

**Solution**

The force due to shear stress is

\[ F_s = C_f \frac{1}{2} \rho U_c^2 BL \]

The density and kinematic viscosity of air at 20°C and atmospheric pressure is 1.2 kg/m³ and 1.5×10⁻⁵ N·s/m², respectively. The Reynolds number based on the plate length is

\[ Re_L = \frac{15 \times 1}{1.5 \times 10^{-5}} = 10^6 \]

The average shear stress coefficient on the “tripped” side of the plate is

\[ C_f = \frac{0.074}{(10^6)^{1/5}} = 0.0047 \]

The average shear stress on the “untripped” side is

\[ C_f = \frac{0.523}{\ln^2(0.06 \times 10^6) - \frac{1520}{10^6}} = 0.0028 \]

The total force is

\[ F_s = \frac{1}{2} \times 1.2 \times 15^2 \times 1 \times 0.5 \times (0.0047 + 0.0028) = 0.506 \text{ N} \]
A supertanker has length, breadth, and draught (fully loaded) dimensions of 325 m, 48 m, and 19 m, respectively. In open seas the tanker normally operates at a speed of 15 kt (1 kt = 0.515 m/s). For these conditions, and assuming that flat-plate boundary-layer conditions are approximated, estimate the skin-friction drag of such a ship steaming in 10°C water. What power is required to overcome the skin-friction drag? What is the boundary-layer thickness at 300 m from the bow?

Information and assumptions

From Table 9.14 \( \frac{B}{2} \nu = 1.4 \times 10^{-6} \text{ m}^2/\text{s} \) and \( \rho = 1026 \text{ kg/m}^3 \).

Find

- skin friction drag,
- power required,
- and boundary layer thickness at 300 m from bow.

Solution

\[
\text{Re}_L = \frac{U_0 L}{\nu} = \left(15 \times 0.515\right) \times 325/(1.4 \times 10^{-6}) = 1.79 \times 10^9
\]

From Fig. 9.15 \( C_f = 0.00153 \). Then

\[
F_s = C_f \rho U_0^2/2
\]

\[
F_s = 0.00153 \times 325(48 + 38) \times 10.26 \times (15 \times 0.515)^2/2 = 1.309 \text{ MN}
\]

\[
P = 1.309 \times 10^6 \times 15 \times 0.515 = 10.1 \text{ MW}
\]

To find \( \delta \) at \( x = 300 \text{ m} \)

\[
\text{Re}_{300} = \frac{U_0 x}{\nu} = 15 \times 0.515 \times 0.515 \times 300/(1.4 \times 10^{-6})
\]

\[
= 1.66 \times 10^9
\]

\[
\delta/x = 0.16/\text{Re}_x^{1/7} = 0.0077
\]

\[
\delta = 300 \times 0.0077 = 2.31 \text{ m}
\]
9.41 A small spherical water drop of diameter 0.002 in. exists in the atmosphere at 5000-ft altitude. Will the drop rise or fall if it is in a thermal (an upward flowing column of air) having a speed of 4 ft/s? Repeat for speeds of 1 ft/s and 0.1 ft/s.

In stationary air the particle falls with speed $U$ such that $D + F_B = W$, where

If $Re = \frac{UD}{\mu} < 1$ then

$$D = \text{drag} = 3\pi DU \mu.$$ Also, $W = \gamma_{H_2O} V = \gamma_{H_2O} \frac{4\pi}{3} \left(\frac{D}{2}\right)^3 = \text{weight}$

and $F_B = \gamma_{air} V = \gamma_{air} \frac{4\pi}{3} \left(\frac{D}{2}\right)^3 = \text{buoyant force}$

Since $\gamma_{air} \ll \gamma_{H_2O}$ we can neglect the buoyant force.

That is, $D = W$, or

$$3\pi DU = \gamma_{H_2O} \frac{4\pi}{3} \left(\frac{D}{2}\right)^3$$

so that

$$U = \frac{\left(62.4 \frac{\mu}{\gamma_{H_2O}}\right) \left(0.002 \text{ ft}^3\right)}{18 \times 3.637 \times 10^{-7} \frac{\text{lb} \cdot s}{\text{ft}^2}} = 0.265 \frac{\text{ft}}{s}$$

Thus, the drop will rise if the upward velocity is $4 \frac{\text{ft}}{s}$ or $1 \frac{\text{ft}}{s}$, but it will fall if it is $0.1 \frac{\text{ft}}{s}$.

Note: The above is correct if $Re < 1$. Since $Re = \frac{UD}{\mu}$

or $Re = \frac{(2.048 \times 10^{-3} \frac{\text{lb}}{\text{ft}^2})(0.265 \frac{\text{ft}}{s})(0.002 \text{ ft}^3)}{3.637 \times 10^{-7} \frac{\text{lb} \cdot s}{\text{ft}^2}} = 0.249$ the low $Re$ drag equation, Eq. (1), is valid.
9.53 Estimate the wind velocity necessary to knock over a 10-lb garbage can that is 3 ft tall and 2 ft in diameter. List your assumptions.

If the can is about to tip around corner O, then \( \sum M_O = 0 \), or \( 1.5 \theta = 1 \ W \)

or \( 1.5 C_D \frac{1}{2} \rho U^2 A = W \)  

A typical value of \( C_D \) for a cylinder is \( C_D = 1 \) (see Fig. 9.21)

Thus,

\[
(1.5 \text{ ft})(1)(\frac{1}{2})(0.0023 \text{ lbs ft/s}^2 \text{ ft}) U^2 (2 \text{ ft})(3 \text{ ft}) = 10 \text{ ft-lb}, \text{ where } U = \frac{ft}{s}
\]

or \( U = 30.6 \frac{ft}{s} \)
As shown in Video V9.8 and Fig. P9.56, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from \( C_D = 0.96 \) to \( C_D = 0.70 \) corresponds to a reduction of how many horsepower needed at a highway speed of 65 mph?

\[ P = \text{power} = \delta U \]

\[ \delta = \frac{1}{2} \rho U^2 C_D A \]

Thus, \( \Delta P = \text{reduction in power} \]

\[ = P_b - P_a \]

\[ = \frac{1}{2} \rho U^2 A \left[ C_{D_b} - C_{D_a} \right] \]

With \( U = 65 \text{ mph} = 95.3 \text{ fps} \),

\[ \Delta P = \frac{1}{2} \left( 0.00238 \frac{\text{slugs}}{\text{ft}^2} \right) \left( 95.3 \frac{\text{ft}}{\text{sec}} \right)^2 (10 \text{ ft})(12 \text{ ft}) \left[ 0.96 - 0.70 \right] \]

\[ = 32,100 \frac{\text{ft}-\text{lb}}{\text{sec}} \left( \frac{1 \text{ hp}}{550 \frac{\text{ft}-\text{lb}}{\text{sec}}} \right) = 58.4 \text{ hp} \]
A Boeing 747 aircraft weighing 580,000 lb when loaded with fuel and 100 passengers takes off with an airspeed of 140 mph. With the same configuration (i.e., angle of attack, flap settings, etc.) what is its takeoff speed if it is loaded with 372 passengers. Assume each passenger with luggage weighs 200 lb.

For steady flight \( L = C_L \frac{1}{2} \rho U^2 A = W \)

Let \( (.)_{100} \) denote conditions with 100 passengers and \( (.)_{372} \) with 372 passengers. Thus, with \( C_{L_{100}} = C_{L_{372}} \), \( A_{100} = A_{372} \), and \( \rho_{100} = \rho_{372} \), Eq. (1) gives

\[
\frac{L_{100}}{L_{372}} = \frac{U_{100}^2}{U_{372}^2} \quad \text{or} \quad U_{372} = U_{100} \left( \frac{580,000 \text{ lb}}{580,000 \text{ lb}} \right)^{1/2}, \quad \text{with} \ U_{100} = 140 \text{ mph}
\]

Thus, \( U_{372} = 146 \text{ mph} \)

Show that for unpowered flight (for which the lift, drag, and weight forces are in equilibrium) the glide slope angle, \( \theta \), is given by \( \tan \theta = C_D / C_L \).

For steady unpowered flight
\[ \sum F_x = 0 \text{ gives } -D = W \sin \theta \]
\[ \sum F_y = 0 \text{ gives } L = W \cos \theta \]

Thus,
\[
\frac{d\theta}{dL} = \frac{W \sin \theta}{W \cos \theta} = \tan \theta, \quad \text{where} \quad \frac{d\theta}{dL} = \frac{1}{2} \frac{\rho U^2 A_D}{C_D} = \frac{C_D}{C_L}
\]

Hence, \( \tan \theta = \frac{C_D}{C_L} \)