Water flows downward through a vertical smooth pipe. When the flowrate is 0.5 ft³/s there is no change in pressure along the pipe. Determine the diameter of the pipe.

\[ \frac{\rho g}{2g} + g z_1 + \frac{V^2}{2g} = \frac{\rho g}{2g} + g z_2 + \frac{V^2}{2g} + f \frac{V^2}{D} \]

where \( \rho_1 = \rho_2 \), \( V_1 = V_2 = V \), and \( z_1 - z_2 = l \)

Thus,

\[ l = f \frac{V^2}{D} \]

or

\[ l = \frac{V^2}{D} \]

Also,

\[ V = \frac{Q}{A} = \frac{Q}{\pi D^2} \] so that Eq. (1) becomes

\[ l = f \left( \frac{\rho g}{2g} \right) \frac{Q^2}{D^5} \]

or

\[ D^5 = \frac{8\pi f Q^2}{g} = \frac{8\pi f (0.5)^2}{32} \]

or

\[ D = 0.363 f V S \]

Also,

\[ Re = \frac{Q V D}{\mu} = 1.94 \left( \frac{\rho g}{2g} \right) \frac{Q^2}{D^5} \]

or

\[ Re = \frac{5.28 \times 10^4}{D} \]

From Fig. 8.20 with \( \frac{V}{D} = 0 \) we have \( f = f(Re, \frac{V}{D} = 0) \)

Trial and error solution: 3 unknowns \( (D, Re, f) \) and 3 equations

Assume \( f = 0.02 \) so from Eq. (2), \( D = 0.166 \) ft and from Eq. (3), \( Re = 3.18 \times 10^5 \). Thus, from Fig. 8.20, \( f = 0.014 \pm 0.02 \)

Assume \( f = 0.014 \) so that \( D = 0.155 \) ft and \( Re = 3.42 \times 10^5 \)

Thus, from Fig. 8.20, \( f = 0.14 \) which checks with the assumed value.

Thus, \( D = 0.155 \) ft

An alternative method is to use the Colebrooke equation, Eq. 8.35, with \( e/D = 0 \), rather than the Moody chart, Fig. 8.20. Thus, \( \frac{1}{D} = -2.0 \log (2.51/Re V f) \)

which combined with Eqs. (2) and (3) gives

\[ \frac{1}{(D/0.363)^{5/2}} = -2.0 \log \left[ 2.51 / (5.28 \times 10^4) \sqrt{D/0.363} \right] \]

Using a computer root-finding program gives the solution of Eq. (4) as \( D = 0.155 \) ft as obtaining by the above trial and error method.