The pressure drop, \( \Delta p \), along a straight pipe of diameter \( D \) has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, \( \ell \), between pressure taps. Assume that \( \Delta p \) is a function of \( D \) and \( \ell \), the velocity, \( V \), and the fluid viscosity, \( \mu \). Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

\[
\Delta p = f(D, \ell, V, \mu)
\]

\( \Delta p = \frac{FL^2}{D} \quad D = L \quad \ell = L \quad V = LT^{-1} \quad \mu = FL^{-2}T \)

From the pi theorem, \( 5 - 3 = 2 \) pi terms required.

By inspection, for \( \Pi_1 \) (containing \( \Delta p \)):

\[
\Pi_1 = \frac{\Delta p D}{\mu V} = \frac{(FL^{-3})(L)}{(FL^{-2}T)(LT^{-1})} = F^0L^0T^0
\]

Check using MLT:

\[
\frac{\Delta p D}{\mu V} = \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} = M^0L^0T^0 : \text{ok}
\]

For \( \Pi_2 \) (containing \( \ell \)):

\[
\Pi_2 = \frac{\ell}{D}
\]

which is obviously dimensionless. Thus,

\[
\frac{\Delta p D}{\mu V} = f\left(\frac{\ell}{D}\right)
\] (1)

From the statement of the problem, \( \Delta p \propto \ell \) so that Eq. (1) must be of the form

\[
\frac{\Delta p D}{\mu V} = K \frac{\ell}{D}
\]

where \( K \) is some constant. It thus follows that

\[
\frac{\Delta p}{D^2} \propto \frac{1}{D^2}
\]

for a given velocity.