7.16 Assume that the drag, \( D \), on an aircraft flying at supersonic speeds is a function of its velocity, \( V \), fluid density, \( \rho \), speed of sound, \( c \), and a series of lengths, \( l_1, \ldots, l_i \), which describe the geometry of the aircraft. Develop a set of pi terms that could be used to investigate experimentally how the drag is affected by the various factors listed. Form the pi terms by inspection.

\[
D = f (V, \rho, c, l_1, \ldots, l_i)
\]

\[
D = F \quad V = LT^{-1} \quad \rho = ML^{-3} \quad c = LT^{-1} \quad \text{all lengths, } l_i = L
\]

From the pi theorem, \((4+i)-3 = 1+i\) pi terms required, where \(i\) is the number of length terms \((i=1, 2, 3, \text{etc.})\).

By inspection, for \( P_1 \) (containing \( D \)):

\[
P_1 = \frac{\rho V^2 l_i^2}{(FL^{-4} T^2)(LT^{-1})^2(L)^2} = F_0 L_0 T_0
\]

Check using \( MLT \):

\[
\frac{D}{\rho V^2 l_i^2} = \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} = M^0 L^0 T^0 \quad \text{OK}
\]

For \( P_2 \) (containing \( c \)):

\[
P_2 = \frac{c}{V} \quad \text{or} \quad \frac{V}{c}
\]

and both are obviously dimensionless.

For all other pi terms containing \( l_i \):

\[
P_i = \frac{l_i}{L}
\]

and these terms involving the \( l_i \) are obviously dimensionless.

Thus,

\[
\frac{D}{\rho V^2 l_i^2} = \phi \left( \frac{V}{c}, \frac{l_i}{L} \right)
\]

where \( \frac{l_i}{L} \) is a series of pi terms, \( \frac{l_1}{L}, \frac{l_2}{L}, \text{etc.} \).