The velocity, $V$, of a spherical particle falling slowly in a viscous liquid can be expressed as

$$ V = f(d, \mu, \rho, \eta) $$

where $d$ is the particle diameter, $\mu$ the liquid viscosity, and $\rho$ and $\eta$ the specific weight of the liquid and particle, respectively. Develop a set of dimensionless parameters that can be used to investigate this problem.

From the pi theorem, $5 - 3 = 2$ pi terms required. Use $d, \mu, \rho, \eta$ as repeating variables. Thus,

$$ \Pi_1 = \sqrt{\frac{d^a \mu^b \rho^c}{d^2 \rho}} $$

and

$$ (LT^{-1})(L)(FL^{-2}T)^b (FL^{-3})^c \equiv F^0L^0T^0 $$

so that

$$ b + c = 0 \quad \text{(for F)} $$
$$ 1 + a - 2b - 3c = 0 \quad \text{(for L)} $$
$$ -1 + b = 0 \quad \text{(for T)} $$

It follows that $a = -2, b = 1, c = -1$, and therefore

$$ \Pi_1 = \frac{\sqrt{\mu}}{d^2 \rho} $$

Check dimensions using MLT system:

$$ \frac{\sqrt{\mu}}{d^2 \rho} = \frac{(LT^{-1})(ML^{-1}T^{-1})}{(L)^2(ML^{-2}T^{-2})} \equiv M^0L^0T^0 \quad \therefore \text{OK} $$

For $\Pi_2$:

$$ \Pi_2 = \sqrt{\frac{\rho^a \mu^b \eta^c}{(FL^{-3})(FL^{-3})^b (FL^{-3})^c}} $$

$$ (FL^{-3})^a (FL^{-3})^b (FL^{-3})^c \equiv F^0L^0T^0 $$

so that

$$ 1 + b + c = 0 \quad \text{(for F)} $$
$$ -3 + a - 2b - 3c = 0 \quad \text{(for L)} $$
$$ b = 0 \quad \text{(for T)} $$

It follows that $a = 0, b = 0, c = -1$, and therefore

$$ \Pi_2 = \frac{\eta}{\rho} $$

which is obviously dimensionless.

Thus,

$$ \frac{\sqrt{\mu}}{d^2 \rho} = \phi \left( \frac{\eta}{\rho} \right) $$