7.9 The pressure rise, $\Delta p$, across a pump can be expressed as

$$\Delta p = f(D, \rho, \omega, Q)$$

where $D$ is the impeller diameter, $\rho$ the fluid density, $\omega$ the rotational speed, and $Q$ the flowrate.

Determine a suitable set of dimensionless parameters.

$$\Delta p = FL^{-2} \quad D = L \quad \rho = FL^{-4}T^{-2} \quad \omega = T^{-1} \quad Q = L^3T^{-1}$$

From the pi theorem, $5 - 3 = 2$ pi terms required. Use $D, \rho,$ and $\omega$ as repeating variables. Thus,

$$\pi_1 = \Delta p D^a \rho^b \omega^c$$

and

$$(FL^{-2})(L)^a (FL^{-4}T^{-2})^b (T^{-1})^c = F^0L^0T^0$$

$$1 + b = 0$$
$$-2 + a - 4b = 0$$
$$2b - c = 0$$

It follows that $a = -2, \ b = -1, \ c = -2$, and therefore

$$\pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

Check dimensions using MLT system:

$$\frac{\Delta p}{D^2 \rho \omega^2} = \frac{ML^{-3}T^{-2}}{(L)^a(ML^3)(T^{-1})^c} = M^0L^0T^0 \therefore \text{OK}$$

For $\pi_2$:

$$\pi_2 = \phi D^a \rho^b \omega^c$$

$$(L^3T^{-1})(L)^a (FL^{-4}T^{-2})^b (T^{-1})^c = F^0L^0T^0$$

$$b = 0$$
$$3 + a - 4b = 0$$
$$-1 + 2b - c = 0$$

It follows that $a = -3, \ b = 0, \ c = -1$, and therefore

$$\pi_2 = \frac{\phi}{D^3 \omega}$$

Check dimensions using MLT system:

$$\frac{\phi}{D^3 \omega} = \frac{F^3T^{-3}L}{(L)^3(T^{-1})} = M^0L^0T^0 \therefore \text{OK}$$

Thus,

$$\frac{\Delta p}{D^2 \rho \omega^2} = \phi \left( \frac{\phi}{D^3 \omega} \right)$$