7.6 Water sloshes back and forth in a tank as shown in Fig. P7.6. The frequency of sloshing, \( \omega \), is assumed to be a function of the acceleration of gravity, \( g \), the average depth of the water, \( h \), and the length of the tank, \( L \). Develop a suitable set of dimensionless parameters for this problem using \( g \) and \( L \) as repeating variables.

\[
\omega = f \left( g, h, L \right)
\]

\[
\omega \equiv T^{-1} \quad g \equiv LT^{-2} \quad h \equiv L \quad L \equiv L
\]

From the pi theorem, \( 4 - 2 = 2 \) dimensionless parameters required. Use \( g \) and \( L \) as repeating variables. Thus,

\[
\Pi_1 = \omega g^a L^b
\]

and

\[
(T^{-1})(LT^{-2})^a(L)^b = L^0 T^0
\]

so that

\[
a + b = 0 \quad \text{(for } L)\]
\[1 - 2a = 0 \quad \text{(for } T)\]

It follows that \( a = -\frac{1}{2}, \ b = \frac{1}{2} \), and therefore

\[
\Pi_1 = \omega \frac{g^{\frac{1}{2}}}{L}\]

Check dimensions:

\[
\omega \frac{g^{\frac{1}{2}}}{L} = \frac{1}{T} \sqrt{\frac{L}{LT^{-2}}} = L^0 T^0 \quad \text{OK}
\]

For \( \Pi_2 \):

\[
\Pi_2 = h \frac{g^{a}}{L^{b}}
\]

\[
L (LT^{-2})^a(L)^b = L^0 T^0
\]

\[
1 + a + b = 0 \quad \text{(for } L)\]
\[-2a = 0 \quad \text{(for } T)\]

It follows that \( a = 0, \ b = -1 \), and therefore

\[
\Pi_2 = \frac{h}{L}
\]

and \( \Pi_2 \) is obviously dimensionless. Thus,

\[
\omega \frac{g^{\frac{1}{2}}}{L} = \phi \left( \frac{h}{L} \right)
\]