2.84 When the Tucurui dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.

\[
\sum F_{\text{vertical}} = 0
\]

so that

\[ T = F_B - gW \]

(1)

For a truncated cone,

\[
\text{Volume} = \frac{\pi h}{3} (r_1^2 + r_1 r_2 + r_2^2)
\]

where:
- \( r_1 = \) base radius
- \( r_2 = \) top radius
- \( h = \) height

Thus,

\[
V_{\text{tree}} = \left( \frac{\pi}{3} \right) \left( \frac{(4 \text{ ft})^2}{3} + (4 \text{ ft} \times 1 \text{ ft}) + (1 \text{ ft})^2 \right)
\]

\[ = 2200 \text{ ft}^3 \]

For buoyant force,

\[
F_B = \gamma_{H_2O} \times V_{\text{tree}} = (62.4 \text{ lb/ft}^3)(2200 \text{ ft}^3) = 137,000 \text{ lb}
\]

For weight,

\[
W = \gamma_{\text{tree}} \times V_{\text{tree}} = (0.6)(62.4 \text{ lb/ft}^3)(2200 \text{ ft}^3) = 82,400 \text{ lb}
\]

From Eq. (1)

\[
T = 137,000 \text{ lb} - 82,400 \text{ lb} = 54,600 \text{ lb}
\]